BINOMIAL DISTRIBUTION

$$P(X=r) = {}^nC_r\, p^r q^{n-r}$$

where p = probability of event happening once.

q = probability of event not happening
$= 1-p$

r = No. of times the event should happen

n = Total attempts.

Q.1) In a hurdle race, a player has to cross 10 hurdles. The probability that he will clear each hurdle is $5/6$. What is the probability that he will knock down less than 2 hurdles.

(2) A die is thrown 8 times & it is required to find the probability that 3 will show (i) exactly 2 times (ii) at least seven times (iii) at least once.

(3) If 10% bolts produced by a machine are defective, calculate the probability that out of a sample selected at random, of 7 bolts, not more than 1 bolt will be defective.

(4) In 100 sets of 10 tosses of a coin, in how many cases do you expect
(i) 7 heads & 3 tails (ii) at least 7 heads.

(5) Out of 2000 families with 4 children each, how many would you expect to have
(i) at least a boy (ii) 1 or 2 girls
(iii) no girls (iv) having 2 boys.

(6) Assume that on average one telephone number out of 15 called between 12 p.m. to 3 p.m. on week days is busy. What is the probability that if 6 randomly selected telephone numbers are called:
(i) at least 3 of them are busy.
(ii) not more than 3 are busy.

(7) Ten coins are thrown simultaneously. Find the probability of getting at least seven heads.
[Ans: 176/1024]

(8) A & B play a game in which their chances of winning are in the ratio 3:2. Find A's chance of winning at least three games out of five games played.
[Ans: 0·68]

win A B $\frac{3}{5}$ $\frac{2}{5}$

(9) A coffee connoisseur claims that he can distinguish between a cup of instant coffee & a cup of percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chances of having the claim (i) accepted, (ii) rejected, when he does have the ability he claims. [Ans: 0.534; 0.466]

(10) A multiple-choice test consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced die & checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4 & the third answer if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction? [Ans: 0.0197]

(11) An irregular six-faced die is thrown & the expectation that in 10 throws it will give five even numbers is twice the expectation that it will give four even numbers. How many items in 10,000 sets of 10 throws each, would you expect it to give no even number.

[Ans : ≅ 1]

(12) A department in a works has 10 machines which may need adjustment from time to time during the day. Three of these machines are old, each having a probability of 1/11 of needing adjustment during the day, & 7 are new, having corresponding probabilities of 1/21.

Assuming that no machine needs adjustment twice on the same day, determine the probabilities that on a particular day

(i) just 2 old & no new machines need adjustment.

(ii) If just 2 machines need adjustment, they are of the same type.

[Ans: 0.016; 0.044]

(13) The probability of a man hitting a target is $1/4$.
(i) If he fires 7 times what is the probability of his hitting the target at least twice?
(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $2/3$?
[Ans: $\frac{4547}{8192}$; 4]

(14) In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? [Ans: 11]

(15) In a binomial distribution consisting 5 independent trials, probabilities of 1 & 2 successes are 0.4096 & 0.2048 respectively. Find the parameter 'p' of the distribution
[Ans: 0.2]

(16) Find p if $n = 6$ & $9P(X=4) = P(X=2)$ [Ans: $p = 1/4$]

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TYPE II:

- Moments about the origin of binomial distribution:

$$\mu_1' = \text{mean} = np$$

$$\mu_2' = n^2p^2 + npq$$

$$\mu_3' = n(n-1)(n-2)p^3 + 3n^2p^2 + 3npq + np$$

$$\mu_4' = n(n-1)(n-2)(n-3)p^4 + 6[n(n-1)(n-2)p^3 + 3np(np+q) + np] + 7np(np+q) + np$$

- Moments about the mean of Binomial Distribution:

$$\mu_1 = 0$$

$$\mu_2 = \text{variance} = npq$$

NOTE: Standard deviation $\sigma = \sqrt{\text{variance}} = \sqrt{\mu_2} = \sqrt{npq}$

$$\mu_3 = npq(q-p) = npq^2 - np^2q$$

$$\mu_4 = npq[1+3(n-2)pq]$$

- Karl Pearson's co-efficient of Kurtosis

$\beta_1 = \text{measure of skewness} = \dfrac{\mu_3^2}{\mu_2^3}$

β_2 = measure of flatness, peakedness of single humped distribution

$$= \frac{\mu_4}{\mu_2^2}$$

$$\gamma_1 = \sqrt{\beta_1} \qquad \gamma_2 = \beta_2 - 3$$

Q.1) A random variable X has a binomial probability distribution with parameters n & p. The mean value is 3 & the variance is 1.2. Find the values of n & p & calculate $P(X<4)$. [Ans: n = 5; p = 0.6; 0.663]

(2) Is the following statement true or false
The mean of binomial distribution is 6 & its standard deviation 4. [Ans: False]

(3) Find standard deviation of binomial deviation whose mean is 5, the second moment about zero is 27. [Ans: 1.414]

(4) Fit a Binomial distribution to the following:

x	0	1	2	3	4	5	6
f	6	20	28	12	8	6	0

(5) Five dice are thrown 96 times. The number of times 4, 5, 6 was actually thrown in the experiment is given below:

No. of dice showing 4, 5, 6	0	1	2	3	4	5
observed frequency	1	10	24	35	18	8

Find the expected (theoretical) frequencies, if distribution is assumed to be binomial. What is the theoretical mean & standard deviation? Also calculate the mean & s.d. from observed data. Does the observed data represent binomial distribution?

(6) The mean & variance of binomial distribution are 4 & 4/3 respectively. Find $P(X \geq 1)$. [Ans: 0.99863]

(7) Seven coins are tossed & the number of heads noted. The experiment is repeated 128 times & the following distribution is obtained:

No. of heads	0	1	2	3	4	5	6	7
Frequencies	7	6	19	35	30	23	7	1

Fit a binomial distribution assuming, (i) The coin is unbiased (ii) The nature of the coin is not known.

(8) Show that for $p = 0.5$, the binomial distribution has a maximum probability at $X = \frac{1}{2}n$, if n is even, & at $X = \frac{1}{2}(n-1)$ as well as $X = \frac{1}{2}(n+1)$, if n is odd.

[NOTE: MODE at $(n+1)p$
If not an integer, then unique mode is the lower integer.
If an integer then its bimodal with mode = that integer & next lower integer.]