LAPLACE TRANSFORM

Q1) $f(t) = t^2 - e^{-2t} + e^t$

$L[f(t)] = L[t^2] - L[e^{-2t}] + L[e^t]$

$= \frac{2}{s^3} - \frac{1}{s+2} + \frac{1}{s-1}$

Q2) $f(t) = \frac{t}{4} - \frac{\sin t}{3} + \frac{\sin 2t}{24}$

$L[f(t)] = \frac{1}{4} L[(t)] - \frac{1}{3} L[\sin t]$

$+ \frac{1}{24} L[\sin 2t]$

$= \frac{1}{4}\left(\frac{1}{s^2}\right) - \frac{1}{3}\left(\frac{1}{s^2+1}\right) + \frac{1}{24}\left(\frac{2}{s^2+4}\right)$

$= \frac{1}{4s^2} - \frac{1}{3(s^2+1)} + \frac{1}{12(s^2+4)}$

Q3) $f(t) = (t^2+1)^2$

$f(t) = t^4 + 2t^2 + 1$

$L[f(t)] = L[t^4] + 2L[t^2] + L[1]$

$= \frac{4!}{s^5} + 2 \times \frac{2}{s^3} + \frac{1}{s}$

$= \frac{24}{s^5} + \frac{4}{s^3} + \frac{1}{s}$

Q4) $f(t) = (\sin 2t - \cos 2t)^2$

$f(t) = \sin^2 2t + \cos^2 2t - 2\sin 2t \cos 2t$

$f(t) = 1 - \sin 4t$ $\{ \because \sin 2\theta = 2\sin\theta\cos$

$L[f(t)] = L[1] - L[\sin 4t]$

$= \frac{1}{s} - \frac{4}{s^2+16}$

Q5) $f(t) = a\cos^2 2bt$

$f(t) = a\left(\frac{1+\cos 4bt}{2}\right)$ $\left\{ \because \cos^2\theta = \frac{1+\cos 2\theta}{2} \right\}$

$$L[f(t)] = \frac{9}{2}\left\{L[1] + L[\cos 4bt]\right\}$$

$$= \frac{9}{2}\left\{\frac{1}{s} + \frac{s}{s^2+16b^2}\right\}$$

Q6) $f(t) = \cosh^2 4t$

$f(t) = \dfrac{1+\cosh 8t}{2}$ $\left\{\because \cosh^2 t = \dfrac{1+\cosh 2t}{2}\right\}$

$$L[f(t)] = \frac{1}{2}\left\{L[1] + L[\cosh 8t]\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} + \frac{s}{s^2-64}\right\}$$

Q7) $f(t) = \sin^3 t$

$f(t) = \dfrac{3\sin t - \sin 3t}{4}$ $\left\{\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta\right\}$

$$L[f(t)] = \frac{3}{4}L[\sin t] - \frac{1}{4}L[\sin 3t]$$

$$L[f(t)] = \frac{3}{4}\left(\frac{1}{s^2+1}\right) - \frac{1}{4}\cdot\left(\frac{3}{s^2+9}\right)$$

$$= \frac{3}{4(s^2+1)} - \frac{3}{4(s^2+9)}$$

Q8) $f(t) = \sin(\omega t + \alpha)$

$f(t) = \sin\omega t\cos\alpha + \cos\omega t\sin\alpha$

$L[f(t)] = \cos\alpha\, L[\sin\omega t] + \sin\alpha\, L[\cos\omega t]$

$$= \cos\alpha \cdot \frac{\omega}{s^2+\omega^2} + \sin\alpha \cdot \frac{s}{s^2+\omega^2}$$

$$= \frac{\omega\cos\alpha}{s^2+\omega^2} + \frac{s\sin\alpha}{s^2+\omega^2}$$

Q9) $f(t) = \cos t\cos 2t$

$f(t) = \frac{1}{2}(2\cos t\cos 2t)$

$$= \frac{1}{2}(\cos 3t + \cos t)$$

$$L[f(t)] = \frac{1}{2}\left\{L[\cos 3t] + L[\cos t]\right\}$$

$$= \frac{1}{2}\left\{\frac{s}{s^2+9} + \frac{s}{s^2+1}\right\}$$

$$= \frac{s}{2}\left(\frac{2s^2+10}{(s^2+9)(s^2+1)}\right)$$

$$= \frac{s(s^2+5)}{(s^2+9)(s^2+1)}$$

Q10) $f(t) = t^2 - 3t + 5$

$$L[f(t)] = L[t^2] - 3L[t] + L[5]$$

$$= \frac{2}{s^3} - \frac{3}{s^2} + \frac{5}{s}$$

Q11) $f(t) = t^{5/2}$

$$L[f(t)] = L[t^{5/2}]$$

$$= \frac{\overline{|\tfrac{5}{2}+1}}{s^{\frac{5}{2}+1}} = \frac{\overline{|7/2}}{s^{7/2}} = \frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\overline{|\tfrac{1}{2}}}{s^{7/2}}$$

$$L[f(t)] = \frac{15\sqrt{\pi}}{8.s^{7/2}}$$

Q12) $f(t) = t + t^2 + t^3$

$$L[f(t)] = L[t] + L[t^2] + L[t^3]$$

$$= \frac{1}{s^2} + \frac{2}{s^3} + \frac{3!}{s^4}$$

$$= \frac{1}{s^2} + \frac{2}{s^3} + \frac{6}{s^4}$$

Q13) $f(t) = \sin^3 2t$

$$f(t) = \frac{3\sin 2t - \sin 6t}{4}$$

$$L[f(t)] = \frac{3}{4} L[\sin 2t] - \frac{1}{4} L[\sin 6t]$$

$$= \frac{3}{4}\left(\frac{2}{s^2+4}\right) - \frac{1}{4}\left(\frac{6}{s^2+36}\right)$$

$$L[f(t)] = \frac{3}{2(s^2+4)} - \frac{3}{2(s^2+36)}$$

Q14) $f(t) = \sin 2t \cos 3t$

$$f(t) = \frac{1}{2}(2\sin 2t \cos 3t)$$

$$= \frac{1}{2}[\sin 5t + \sin(-t)]$$

$$= \frac{1}{2}(\sin 5t - \sin t)$$

$$L[f(t)] = \frac{1}{2}\{L[\sin 5t] - L[\sin t]\}$$

$$= \frac{1}{2}\left\{\frac{5}{s^2+25} - \frac{1}{s^2+1}\right\}$$

Q15) $f(t) = \sin t \cos t$

$$f(t) = \frac{1}{2}(2\sin t \cos t)$$

$$f(t) = \frac{1}{2}(\sin 2t)$$

$$L[f(t)] = \frac{1}{2} L[\sin 2t]$$

$$= \frac{1}{2}\left(\frac{2}{s^2+4}\right) = \frac{1}{s^2+4}$$

Q16) $f(t) = \cos^2 t$

$$= \frac{1+\cos 2t}{2}$$

$$L[f(t)] = \frac{1}{2}\left\{L[1] + L[\cos 2t]\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} + \frac{s}{s^2+4}\right\}$$

Q17) $f(t) = \sin 2t \sin 3t$

$$f(t) = \frac{1}{2}\left\{2 \sin 2t \sin 3t\right\}$$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$f(t) = \frac{1}{2}(\cos t - \cos 5t)$

$L[f(t)] = \frac{1}{2}\{L[\cos t] - L[\cos 5t]\}$

$= \frac{1}{2}\left\{\frac{s}{s^2+1} - \frac{s}{s^2+25}\right\}$

Q18) $f(t) = t^{-1/2}$

$L[f(t)] = L[t^{-1/2}]$

$= \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}} = \frac{\Gamma(1/2)}{s^{1/2}} = \sqrt{\frac{\pi}{s}}$

Q19) $f(t) = \cos(\omega t + \alpha)$

$= \cos\omega t \cos\alpha - \sin\omega t \sin\alpha$

$L[f(t)] = \cos\alpha\, L[\cos\omega t] - \sin\alpha\, L[\sin\omega t]$

$= \cos\alpha\left(\frac{s}{s^2+\omega^2}\right) - \sin\alpha\left(\frac{\omega}{s^2+\omega^2}\right)$

$= \frac{s\cos\alpha - \omega\sin\alpha}{s^2+\omega^2}$

Q20) $f(t) = \sin\sqrt{t}$

$$\left\{ \because \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - - - - \right\}$$

$$\therefore f(t) = \sqrt{t} - \frac{t^{3/2}}{3!} + \frac{t^{5/2}}{5!} - - - -$$

$$L[f(t)] = L[t^{1/2}] - \frac{1}{3!} L[t^{3/2}] + \frac{1}{5!} L[t^{5/2}] - - - -$$

$$= \frac{\Gamma(3/2)}{s^{3/2}} - \frac{1}{3!}\frac{\Gamma(5/2)}{s^{5/2}} + \frac{1}{5!}\frac{\Gamma(7/2)}{s^{7/2}} - - - -$$

$$= \frac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{s^{3/2}} - \frac{1}{6}\frac{\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{s^{5/2}} + \frac{1}{120}\frac{\frac{5}{2}\cdot\frac{3}{2}\cdot\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{s^{7/2}} \cdots$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} - \frac{\sqrt{\pi}}{8s^{5/2}} + \frac{\sqrt{\pi}}{64s^{7/2}} - - - - -$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\left[1 - \frac{1}{4s} + \frac{1}{32s^2} - - - \right]$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\left[1 - \frac{1}{2^2 \cdot s} + \frac{1}{2!}\left(\frac{1}{2^2 \cdot s}\right)^2 - - \right]$$

$$L[f(t)] = \frac{\sqrt{\pi}}{2s^{3/2}}\left[e^{-\frac{1}{2^2 s}}\right]$$

$$\left\{\because e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right\}$$

$$L[f(t)] = \frac{\sqrt{\pi}}{2s^{3/2}}\left(e^{-\frac{1}{4s}}\right)$$

Q21) $f(t) = \dfrac{\cos\sqrt{t}}{\sqrt{t}}$

$$\left\{\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right\}$$

$$f(t) = \frac{1}{\sqrt{t}}\left(1 - \frac{t}{2!} + \frac{t^2}{4!} - \frac{t^3}{6!} + \cdots\right)$$

$$= t^{-1/2} - \frac{t^{1/2}}{2!} + \frac{t^{3/2}}{4!} - \cdots$$

$$L[f(t)] = L[t^{-1/2}] - \frac{1}{2}L[t^{1/2}] + \frac{1}{24}L[t^{3/2}] \ldots$$

$$L[f(t)] = \frac{\Gamma\frac{1}{2}}{s^{1/2}} - \frac{1}{2}\cdot\frac{\Gamma 3/2}{s^{3/2}} + \frac{1}{24}\frac{\Gamma 5/2}{s^{5/2}} \cdots$$

$$= \frac{\sqrt{\pi}}{s^{1/2}} - \frac{1}{2}\frac{\frac{1}{2}\Gamma\frac{1}{2}}{s^{3/2}} + \frac{1}{24}\frac{\frac{3}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2}}{s^{5/2}} \cdots$$

$$= \frac{\sqrt{\pi}}{s^{1/2}} - \frac{\sqrt{\pi}}{4s^{3/2}} + \frac{\sqrt{\pi}}{32s^{5/2}} \cdots$$

$$= \frac{\sqrt{\pi}}{s^{1/2}}\left(1 - \frac{1}{4s} + \frac{1}{32s^2} - \cdots\right)$$

$$= \frac{\sqrt{\pi}}{s^{1/2}}\cdot e^{-\frac{1}{4s}} = \sqrt{\frac{\pi}{s}}\cdot e^{\frac{-1}{4s}}.$$

TYPE II: Using First Shifting Theorem:-

Q22)

Let $f(t) = \cos at$

$L[f(t)] = L[\cos at] = \dfrac{s}{s^2+a^2}$

By First Shifting theorem.

$L[e^{-bt}\cos at] = \dfrac{s+b}{(s+b)^2+a^2}$

Q23) Let $f(t) = t^2$

$L[f(t)] = L[t^2] = \dfrac{2!}{s^3} = \dfrac{2}{s^3}$

$$L[e^{3t}t^2] = \frac{2}{(s-3)^3}$$

Let $f(t) = 2\cos bt - 3\sin bt$ $\therefore L[f(t)] =$

Q24) $L[2\cos bt - 3\sin bt]$

$$= 2L[\cos bt] - 3L[\sin bt]$$

$$= \frac{2s}{s^2+b^2} - \frac{3b}{s^2+b^2}$$

$$= \frac{2s-3b}{s^2+b^2}$$

$$L[e^{at}(2\cos bt - 3\sin bt)]$$

$$= \frac{2(s-a)-3b}{(s-a)^2+b^2}$$

$$= \frac{2s-2a-3b}{(s-a)^2+b^2}$$

Q25) Let $f(t) = t^n$

$$L[f(t)] = L[t^n]$$

$$= \frac{\lceil n+1}{s^{n+1}}$$

$$L[e^{-at}t^n] = \frac{\overline{|n+1}}{(s+a)^{n+1}}$$

Q26) Let $f(t) = (t+1)^2$

$$= t^2+2t+1$$

$$L[f(t)] = L[t^2] + 2L[t] + L[1]$$

$$= \frac{2!}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

$$= \frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

$$L[e^t(t+1)^2] = \frac{2}{(s-1)^3} + \frac{2}{(s-1)^2} + \frac{1}{s-1}$$

Q27) $\sinh at \cos at = \left(\frac{e^{at}-e^{-at}}{2}\right)\cos at$

$$= \frac{\cancel{1}}{\cancel{2}} \cdot e^{at} \cdot \frac{\cos at}{2} - e^{-at}\frac{\cos at}{2}$$

Let $f(t) = \frac{\cos at}{2}$

$$L[f(t)] = \frac{1}{2} L[\cos at]$$

$$= \frac{1}{2}\left(\frac{s}{s^2+a^2}\right)$$

$$L[\sinh at \cos at] = L\left[e^{at}\frac{\cos at}{2}\right] - L\left[e^{-at}\frac{\cos at}{2}\right]$$

$$= \frac{1}{2}\left[\frac{s-a}{(s-a)^2+a^2}\right] - \frac{1}{2}\left[\frac{s+a}{(s+a)^2+a^2}\right]$$

Q28) Let $f(t) = 2\sin 4t \cos 2t$

$$= \sin 6t + \sin 2t$$

$$L[f(t)] = L[\sin 6t] + L[\sin 2t]$$

$$= \frac{6}{s^2+36} + \frac{2}{s^2+4}$$

$$L\left[e^{t}\, 2\sin 4t\cos 2t\right] = \frac{6}{(s-1)^2+36} + \frac{2}{(s-1)^2+4}$$

$$= \frac{6}{s^2-2s+37} + \frac{2}{s^2-2s+5}$$

Q29) Let $f(t) = t^{3/2}$

$$L[f(t)] = L\left[t^{3/2}\right] = \frac{\Gamma\left(\frac{3}{2}+1\right)}{s^{\frac{3}{2}+1}} = \frac{\Gamma\left(\frac{5}{2}\right)}{s^{5/2}}$$

$$= \frac{\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma\left(\frac{1}{2}\right)}{s^{5/2}} = \frac{3\sqrt{\pi}}{4s^{5/2}}$$

$$L\left[e^{4t}t^{3/2}\right] = \frac{3\sqrt{\pi}}{4(s-4)^{5/2}}$$

Q30) Let $f(t) = \sin^2 t$

$$= \frac{1-\cos 2t}{2}$$

$$L[f(t)] = \frac{1}{2}\left\{L[1] - L[\cos 2t]\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\}$$

$$L[e^{-t}\sin^2 t] = \frac{1}{2}\left[\frac{1}{s+1} - \frac{s+1}{(s+1)^2+4}\right]$$

$$= \frac{1}{2}\left\{\frac{\cancel{(s+1)^2} + 4 - \cancel{(s+1)^2}}{(s+1)(s^2+2s+5)}\right\}$$

$$= \frac{2}{(s+1)(s^2+2s+5)}$$

Q31) Let $f(t) = \cosh 5t$

$$L[f(t)] = \frac{s}{s^2-25}$$

$$L[e^{4t}\cdot\cosh 5t] = \frac{s-4}{(s-4)^2-25}$$

$$= \frac{s-4}{s^2-8s-9}$$

Q32) Let $f(t) = t^{-1/2}$

$$L[f(t)] = L[t^{-1/2}]$$

$$= \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}} = \frac{\Gamma\left(\frac{1}{2}\right)}{s^{1/2}}$$

$$= \sqrt{\frac{\pi}{s}}$$

$$L[e^{-3t}t^{-1/2}] = \sqrt{\frac{\pi}{s+3}}$$

Q33) Let $f(t) = \cosh t$

$$L[f(t)] = L[\cosh t]$$

$$= \frac{s}{s^2-1}$$

$$L[e^{-t}\cosh t] = \frac{s+1}{(s+1)^2-1}$$

$$= \frac{s+1}{s^2+2s}$$

Q34) $\cosh at \sin at = \left(\frac{e^{at}+e^{-at}}{2}\right)\sin at$

$$= e^{at}\left(\frac{\sin at}{2}\right) + e^{-at}\left(\frac{\sin at}{2}\right)$$

Let $f(t) = \frac{\sin at}{2}$

$$L[f(t)] = \frac{1}{2} L[\sin at]$$

$$= \frac{1}{2}\left(\frac{a}{s^2+a^2}\right)$$

$$L[\cosh at \sin at] = L\left[e^{at}\left(\frac{\sin at}{2}\right)\right] + L\left[e^{-at}\left(\frac{\sin at}{2}\right)\right]$$

$$= \frac{a}{2(s-a)^2+a^2} + \frac{a}{2(s+a)^2+a^2}$$

Q35) Let $f(t) = \sin t \cos t = \frac{\sin 2t}{2}$

$L[f(t)] = \frac{1}{2} L[\sin 2t] = \frac{1}{\cancel{2}}\left(\frac{1\cancel{2}}{s^2+4}\right)$

$L[e^t \sin t \cos t] = \frac{1}{(s-1)^2+4}$

$= \frac{1}{s^2-2s+5}$

Q36) Let $f(t) = t^n$

Same as (Q25)

$L[f(t)] = \frac{\overline{|n+1}}{s^{n+1}}$

$L[e^{-at} t^n] = \frac{\overline{|n+1}}{(s+a)^{n+1}}$

Q37) $\sinh at \sin at = \left(\frac{e^{at} - e^{-at}}{2}\right) \sin at$

$= e^{at} \frac{\sin at}{2} - e^{-at} \frac{\sin at}{2}$

Let $f(t) = \frac{\sin at}{2}$

$$L[f(t)] = \frac{1}{2} L[\sin at]$$

$$= \frac{1}{2}\left(\frac{a}{s^2+a^2}\right)$$

$$L[\sinh at \sin at] = L\left[e^{at}\left(\frac{\sin at}{2}\right)\right] - L\left[e^{-at}\left(\frac{\sin at}{2}\right)\right]$$

$$= \frac{a}{2[(s-a)^2+a^2]} - \frac{a}{2[(s+a)^2+a^2]}$$

$$= \frac{a}{2}\left\{\frac{1}{s^2-2as+2a^2} - \frac{1}{s^2+2as+2a^2}\right\}$$

$$= \frac{a}{2}\left\{\frac{4as}{(s^2+2a^2)^2-4a^2s^2}\right\}$$

$$= \frac{2a^2 s}{s^4+4a^4}$$

Q38) Let $f(t) = \cos 2t + \frac{1}{2}\sin 2t$

$L[f(t)] = L[\cos 2t] + \frac{1}{2}L[\sin 2t]$

$= \frac{s}{s^2+4} + \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{s^2+4}\right)$

$= \frac{s+1}{s^2+4}$

$L\left[e^{t}\left(\cos 2t + \frac{1}{2}\sin 2t\right)\right] = \frac{(s-1)+1}{(s-1)^2+4}$

$= \frac{s}{s^2-2s+5}$

Q39) Let $f(t) = t^3$

$L[f(t)] = \frac{3!}{s^4} = \frac{6}{s^4}$

$L[e^{-2t}t^3] = \frac{6}{(s+2)^4}$

Q40) Let $f(t) = 2t$

$$L[f(t)] = 2L[t] = \frac{2}{s^2}$$

$$L[e^{2t} \cdot 2t] = \frac{2}{(s-2)^2}$$

Q41) Let $f(t) = t^2$

$$L[f(t)] = \frac{2}{s^3}$$

$$L[e^{t} t^2] = \frac{2}{(s-1)^3}$$

Q42) Let $f(t) = 4t^3$

$$L[f(t)] = \frac{4 \times 3!}{s^4} = \frac{24}{s^4}$$

$$L[e^{-2t} \cdot 4t^3] = \frac{24}{(s+2)^4}$$

Q43) Let $f(t) = \frac{t^4}{2}$.

$L[f(t)] = \frac{4!}{2s^5} = \frac{12}{s^5}$

$L[e^{-3t}\frac{t^4}{2}] = \frac{12}{(s+3)^5}$

Q44) Let $f(t) = \cos t$

$L[f(t)] = \frac{s}{s^2+1}$

$L[e^t \cos t] = \frac{(s-1)}{(s-1)^2+1} = \frac{s-1}{s^2-2s+2}$

Q45) Let $f(t) = 3\sin 2t$

$L[f(t)] = 3\left(\frac{2}{s^2+4}\right) = \frac{6}{s^2+4}$

$L[e^{2t} 3\sin 2t] = \frac{6}{(s-2)^2+4} = \frac{6}{s^2-4s+8}$

Q46) Let $f(t) = 5\cos 3t$

$$L[f(t)] = 5\left(\frac{s}{s^2+9}\right)$$

$$L[e^{-2t}\, 5\cos 3t] = \frac{5(s+2)}{(s+2)^2+9}$$

$$= \frac{5(s+2)}{s^2+\cancel{2}4s+13}$$

Q47) Let $f(t) = 4\sin t$

$$L[f(t)] = \frac{4}{s^2+1}$$

$$L[e^{-5t}\, 4\sin t] = \frac{4}{(s+5)^2+1} = \frac{4}{s^2+10s+26}$$

Q48) Let $f(t) = 2\sin^2 t = \cancel{2}\left(\frac{1-\cos 2t}{\cancel{2}}\right)$

$$L[f(t)] = L[1] - L[\cos 2t]$$

$$L[f(t)] = \frac{1}{s} - \frac{s}{s^2+4} = \frac{4}{s(s^2+4)}$$

$$L[e^t 2\sin^2 t] = \frac{4}{(s-1)[(s-1)^2+4]}$$

$$= \frac{4}{(s-1)(s^2-2s+5)}$$

Q49) Let $f(t) = \frac{1}{2}\cos^2 t = \frac{1+\cos 2t}{4}$

$$L[f(t)] = \frac{1}{4}\left\{L[1] + L[\cos 2t]\right\}$$

$$= \frac{1}{4}\left\{\frac{1}{s} + \frac{s}{s^2+4}\right\}$$

$$L\left[e^{3t}\frac{\cos^2 t}{2}\right] = \frac{1}{4}\left\{\frac{1}{s-3} + \frac{s-3}{(s-3)^2+4}\right\}$$

$$= \frac{1}{4}\left\{\frac{1}{s-3} + \frac{s-3}{s^2-6s+13}\right\}$$

Q50) Let $f(t) = \sinh t$

$$L[f(t)] = L[\sinh t] = \frac{1}{s^2-1}$$

$$L[e^t \sinh t] = \frac{1}{(s-1)^2-1} = \frac{1}{s^2-2s\cancel{+2}}$$

Q51) Let $f(t) = 3\cosh 4t$

$$L[f(t)] = 3L[\cosh 4t]$$

$$= \frac{3s}{s^2-16}$$

$$L[e^{2t} 3\cosh 4t] = \frac{3(s-2)}{(s-2)^2-16}$$

$$= \frac{3(s-2)}{s^2-4s-12}$$

Q52) Let $f(t) = 2\sinh 3t$

$$L[f(t)] = 2L[\sinh 3t]$$

$$= 2\left(\frac{3}{s^2-9}\right) = \frac{6}{s^2-9}$$

$$L\left[e^{-t}2\sinh 3t\right] = \frac{6}{(s+1)^2-9}$$
$$= \frac{6}{s^2+2s-8}$$

Q53) Let $f(t) = \frac{\cosh 2t}{4}$

$$L[f(t)] = \frac{1}{4} L[\cosh 2t]$$
$$= \frac{1}{4}\left(\frac{s}{s^2-4}\right)$$
$$L\left[e^{-3t}\frac{\cosh 2t}{4}\right] = \frac{1}{4}\left[\frac{s+3}{(s+3)^2-4}\right]$$
$$= \frac{s+3}{4(s^2+6s+5)}$$

Q54) Let $f(t) = 2(\cos 3t - 3\sin 3t)$
$$= 2\cos 3t - 6\sin 3t$$

$$L[f(t)] = 2L[\cos 3t] - 6L[\sin 3t]$$
$$= \frac{2s}{s^2+9} - \frac{6\times 3}{s^2+9}$$
$$= \frac{2s}{s^2+9} - \frac{18}{s^2+9}$$
$$= \frac{2(s-9)}{s^2+9}$$
$$L[2e^t(\cos 3t - 3\sin 3t)] = \frac{2[(s-1)-9]}{(s-1)^2+9}$$
$$= \frac{2(s-10)}{s^2-2s+10}$$

Q55) Let $f(t) = 3(\sinh 2t - 2\cosh 2t)$
$$= 3\sinh 2t - 6\cosh 2t$$
$$L[f(t)] = 3L[\sinh 2t] - 6L[\cosh 2t]$$
$$= \frac{3\times 2}{s^2-4} - \frac{6s}{s^2-4}$$

$$L[f(t)] = \frac{6(1-s)}{s^2-4}$$

$$L[e^{-2t}3(\sinh 2t - 2\cosh 2t)]$$
$$= \frac{6(1-s-2)}{(s+2)^2-4}$$
$$= \frac{-6(s+1)}{s^2+4s} = \frac{-6(s+1)}{s(s+4)}$$

Type III: Using Second Shifting Theorem.

Q86) $F(t) = (t-1)^3$

$f(t) = t^3$ {Eliminate a(1) from F(t) to get f(t)}

$$L[f(t)] = \frac{3!}{s^4} = \frac{6}{s^4}$$

By SST $L[F(t)] = e^{-s}\cdot\left(\frac{6}{s^4}\right)$

857) $f(t) = \cos(t-\alpha)$

$g(t) = \cos t$

$L[g(t)] = L[\cos t] = \dfrac{s}{s^2+1}$

$L[f(t)] = e^{-\alpha s}\left(\dfrac{s}{s^2+1}\right)$

858) i) $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$= \int_0^b e^{-st} a\,dt + \int_b^\infty e^{-st} \cdot 0\,dt$

$= a\int_0^b e^{-st}\,dt = a\left[\dfrac{e^{-st}}{-s}\right]_0^b$

$= a\left(\dfrac{e^{-sb}-1}{-s}\right) = \dfrac{a}{s}\left(1-e^{-bs}\right)$

ii) $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$$L[f(t)] = \int_0^4 e^{-st}\cdot t\,dt + \int_4^\infty e^{-st}\cdot 5\,dt$$

$$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_0^4 + 5\left[\frac{e^{-st}}{-s}\right]_4^\infty$$

$$= \left(-\frac{4e^{-4s}}{s} - \frac{e^{-4s}}{s^2} + \frac{1}{s^2}\right) + 5\left(0 + \frac{e^{-4s}}{s}\right)$$

$$= \frac{e^{-4s}}{s} - \frac{e^{-4s}}{s^2} + \frac{1}{s^2}$$

(iii) $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$$= \int_0^1 e^{-st}\,0\,dt + \int_1^2 e^{-st}\,t\,dt + \int_2^\infty e^{-st}\,0\,dt$$

$$= \left[\frac{te^{-st}}{-s} - \frac{e^{-st}}{s^2}\right]_1^2$$

$$= \frac{2e^{-2s}}{-s} + \frac{e^{-s}}{s} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s^2}$$

$$\int_{x_1}^{x_2} e^{at}\sin bt\,dt = \left[\frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt)\right]_{x_1}^{x_2}$$

$$L[f(t)] = e^{-s}\left(\frac{1}{s}+\frac{1}{s^2}\right) - e^{-2s}\left(\frac{2}{s}+\frac{1}{s^2}\right)$$

(iv) $$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^\pi e^{-st}\sin 2t\,dt + \int_\pi^\infty e^{-st}\,0\,dt$$

$$= \left[\frac{e^{-st}}{s^2+4}(-s\sin 2t - 2\cos 2t)\right]_0^\pi$$

$$= \left[\frac{e^{-\pi s}}{s^2+4}(-2) - \frac{1}{s^2+4}(-2)\right]$$

$$= \frac{2(1-e^{-\pi s})}{s^2+4}$$

(v) $$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^1 e^{-st}\,0\,dt + \int_1^\infty e^{-st}(t-1)^2\,dt$$

$$\int_{x_1}^{x_2} e^{at}\cos bt\,dt = \left[\frac{e^{at}}{a^2+b^2}(a\cos bt + b\sin bt)\right]_{x_1}^{x_2}$$

$$= \left[(t-1)^2\frac{e^{-st}}{-s} - 2(t-1)\frac{e^{-st}}{s^2} + 2\frac{e^{-st}}{-s^3}\right]_1^{\infty}$$

$$= \frac{2e^{-s}}{s^3}$$

Type IV

Q59) Let $f(t) = \frac{\sinh at}{2a}$

$$L[f(t)] = \frac{1}{2a} L[\sinh at]$$

$$= \frac{1}{2\cancel{a}}\left(\frac{\cancel{a}}{s^2-a^2}\right) = \frac{1}{2(s^2-a^2)}$$

$$L\left[\frac{t}{2a}\sinh at\right] = (-1)^1 \frac{d}{ds}\left[\frac{1}{2(s^2-a^2)}\right]$$

$$= \frac{(-1)}{\cancel{2}}\frac{(-1)}{(s^2-a^2)^2}(\cancel{2}s) = \frac{s}{(s^2-a^2)^2}$$

Q60) $f(t) = \cos at$

$$L[f(t)] = \frac{s}{s^2+a^2}$$

$$L[t\cos at] = (-1)^1 \frac{d}{ds}\left(\frac{s}{s^2+a^2}\right)$$

$$= (-1)\left[\frac{(s^2+a^2) - s(2s)}{(s^2+a^2)^2}\right]$$

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$

$$L[t^2\cos at] = (-1)^1 \frac{d}{ds}\left[\frac{(s^2-a^2)}{(s^2+a^2)^2}\right]$$

$$= (-1)\left[\frac{(s^2+a^2)^{\cancel{2}}(2s) - 2(s^2-a^2)\cancel{(s^2+a^2)}(2s)}{(s^2+a^2)^{\cancel{4}3}}\right]$$

$$= (-1)\left[\frac{2s^3+2a^2s-4s^3+4a^2s}{(s^2+a^2)^3}\right]$$

$$= \frac{2s^3-6a^2s}{(s^2+a^2)^3} = \frac{2s(s^2-3a^2)}{(s^2+a^2)^3}$$

Q61) Let $f(t) = 2\sin 3t - 3\cos 3t$

$$L[f(t)] = 2L[\sin 3t] - 3L[\cos 3t]$$

$$= 2\left(\frac{3}{s^2+9}\right) - 3\left(\frac{s}{s^2+9}\right)$$

$$= \frac{6-3s}{s^2+9} = 3\left(\frac{2-s}{s^2+9}\right)$$

$$L[t(2\sin 3t - 3\cos 3t)]$$

$$= (-1)^1 \frac{d}{ds}\left[3\left(\frac{2-s}{s^2+9}\right)\right]$$

$$= -3\left[\frac{(s^2+9)(-1)-(2-s)(2s)}{(s^2+9)^2}\right]$$

$$= -3\left[\frac{-s^2-9-4s+2s^2}{(s^2+9)^2}\right]$$

$$= 3\left[\frac{-s^2+4s+9}{(s^2+9)^2}\right]$$

Q62) Same as (Q60)

Q63) $f(t) = \sin 2t$

$L[f(t)] = \dfrac{2}{s^2+4}$

$L[t \sin 2t] = (-1)^1 \dfrac{d}{ds}\left(\dfrac{2}{s^2+4}\right)$

$= -2\left[-\dfrac{1}{(s^2+4)^2} \cdot 2s\right]$

$= \dfrac{4s}{(s^2+4)^2}$

$L[e^{3t} t \sin 2t] = \dfrac{4(s-3)}{[(s-3)^2+4]^2}$

$= \dfrac{4(s-3)}{(s^2-6s+13)^2}$

Q64) Let $f(t) = \sin at$

$L[f(t)] = \dfrac{a}{s^2+a^2}$

$$L[t\sin at] = (-1)^1 \frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$$

$$= -a\left[\frac{-1}{(s^2+a^2)^2}(2s)\right]$$

$$= \frac{2as}{(s^2+a^2)^2}$$

$$L[t^2\sin at] = (-1)^1 \frac{d}{ds}\left[\frac{2as}{(s^2+a^2)^2}\right]$$

$$= -2a\frac{d}{ds}\left[\frac{s}{(s^2+a^2)^2}\right]$$

$$= -2a\left[\frac{(s^2+a^2)^{\cancel{2}}(1) - 2s\cancel{(s^2+a^2)}2s}{(s^2+a^2)^{\cancel{4}\,3}}\right]$$

$$= -2a\left[\frac{s^2+a^2-4s^2}{(s^2+a^2)^3}\right]$$

$$= 2a\left[\frac{3s^2-a^2}{(s^2+a^2)^3}\right]$$

Q65) $L[f(t)] = \bar{f}(s)$

$L[tf(t)] = (-1)^1 \frac{d}{ds}\bar{f}(s)$

$= -\bar{f}'(s)$

$L[e^t t f(t)] = -\bar{f}'(s-1)$

TYPE V :-

Q66) $f(t) = \sin at$

$L[f(t)] = L[\sin at]$

$= \frac{a}{s^2+a^2}$

$L[\sin t] = \frac{1}{s^2+1}$

$L\left[\frac{\sin t}{t}\right] = \int_s^\infty \frac{ds}{s^2+1}$

$= [\tan^{-1}s]_s^\infty$

$= \tan^{-1}\infty - \tan^{-1}s = \frac{\pi}{2} - \tan$

By fundamental def$\underline{n}$

$$\int_0^\infty e^{-st} f(t)\,dt = L[f(t)]$$

$$\int_0^\infty e^{-st} \frac{\sin t}{t}\,dt = \frac{\pi}{2} - \tan^{-1} s$$

Put $s=0$ on both side

$$\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}.$$

57) $f(t) = 1-\cos at$

$$L[f(t)] = L[1] - L[\cos at]$$

$$= \frac{1}{s} - \frac{s}{s^2+a^2}$$

$$L\left[\frac{1}{t}(1-\cos at)\right] = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+a^2}\right) ds$$

$$= \int_s^\infty \left(\frac{ds}{s} - \frac{1}{2}\int_s \frac{2s}{s^2+a^2}\right) ds$$

$$= \left[\log s - \frac{1}{2}\log(s^2+a^2) \right]_s^\infty$$

$$= \left[\frac{1}{2}\left\{ 2\log s - \log(s^2+a^2) \right\} \right]_s^\infty$$

$$= \left[\frac{1}{2}\left\{ \log s^2 - \log(s^2+a^2) \right\} \right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2}{s^2+a^2} \right) \right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1}{1+\frac{a^2}{s^2}} \right) \right]_s^\infty$$

$$= \frac{1}{2}\left[\log 1 - \log\left(\frac{1}{1+\frac{a^2}{s^2}} \right) \right]$$

$$= \frac{1}{2}\left[-\log\left(\frac{s^2}{s^2+a^2} \right) \right]$$

$$= \frac{1}{2}\log\left(\frac{s^2+a^2}{s^2} \right)$$

68) $f(t) = e^{at} - e^{bt}$

$L[f(t)] = \frac{1}{s-a} - \frac{1}{s-b}$

$L\left[\frac{1}{t}(e^{ab} - e^{bt})\right] = \int_s^\infty \left(\frac{1}{s-a} - \frac{1}{s-b}\right) ds$

$= \left[\log(s-a) - \log(s-b)\right]_s^\infty$

$= \left[\log\left(\frac{s-a}{s-b}\right)\right]_s^\infty$

$= \left[\log\left(\frac{1-a/s}{1-b/s}\right)\right]_s^\infty$

$= \log 1 - \log\left(\frac{1-a/s}{1-b/s}\right)$

$= -\log\left(\frac{s-a}{s-b}\right) = \log\left(\frac{s-b}{s-a}\right)$

869) $f(t) = \cos at - \cos bt$

$L[f(t)] = L[\cos at] - L[\cos bt]$

$= \dfrac{s}{s^2+a^2} - \dfrac{s}{s^2+b^2}$

$L\left[\dfrac{1}{t}(\cos at - \cos bt)\right]$

$= \dfrac{1}{2}\int_s^\infty \left(\dfrac{2s}{s^2+a^2} - \dfrac{2s}{s^2+b^2}\right) d$

$= \dfrac{1}{2}\left[\log(s^2+a^2) - \log(s^2+b^2)\right.$

$= \dfrac{1}{2}\left[\log\left(\dfrac{s^2+a^2}{s^2+b^2}\right)\right]_s^\infty$

$= \dfrac{1}{2}\left[\log\left(\dfrac{1+a^2/s^2}{1+b^2/s^2}\right)\right]_s^\infty$

$= \dfrac{1}{2}\left[\log(1) - \log\left(\dfrac{1+a^2/s^2}{1+b^2/s^2}\right)\right]$

$= \dfrac{1}{2}\left[-\log\left(\dfrac{s^2+a^2}{s^2+b^2}\right)\right] = \dfrac{1}{2}\log\left(\dfrac{s}{s}\right.$

70) $f(t) = \sinh t$

$L[f(t)] = L[\sinh t] = \dfrac{1}{s^2-1}$

$L\left[\frac{1}{t}(\sinh t)\right] = \int_s^\infty \frac{ds}{s^2-1} = \frac{1}{2}\left[\log\left(\frac{s-1}{s+1}\right)\right]_s^\infty$

$= \frac{1}{2}\left[\log\left(\frac{1-1/s}{1+1/s}\right)\right]_s^\infty$

$= \frac{1}{2}\left[\log(1) - \log\left(\frac{1-1/s}{1+1/s}\right)\right]$

$= \frac{1}{2}\left[-\log\left(\frac{s-1}{s+1}\right)\right]$

$= \frac{1}{2}\left[\log\left(\frac{s+1}{s-1}\right)\right]$

71) $f(t) = \sin t$

$L[f(t)] = \dfrac{1}{s^2+1}$

$$L\left[\frac{1}{t}\sin t\right] = \int_s^\infty \frac{ds}{s^2+1}$$

$$= \left[\tan^{-1}(s)\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}s$$

$$L\left[e^{-t}\frac{\sin t}{t}\right] = \frac{\pi}{2} - \tan^{-1}(s+1)$$

$$= \cot^{-1}(s+1)$$

Q72) $f(t) = \sin^2 t = \dfrac{1-\cos 2t}{2}$

$$L[f(t)] = \frac{1}{2}\left\{L[1] - L[\cos 2t]\right\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} - \frac{s}{s^2+4}\right\}$$

$$L\left[\frac{\sin^2 t}{t}\right] = \frac{1}{2}\int_s^\infty \left[\frac{1}{s} - \frac{1}{2}\cdot\frac{2s}{s^2+4}\right] ds$$

$$= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty$$

$$= \frac{1}{4}\left[2\log s - \log(s^2+4)\right]$$

$$= \frac{1}{4}\left[\log s^2 - \log(s^2+4)\right]$$

$$= \frac{1}{4}\left[\log\left(\frac{s^2}{s^2+4}\right)\right]_s^\infty$$

$$= \frac{1}{4}\left[\log\left(\frac{1}{1+\frac{4}{s^2}}\right)\right]_s^\infty$$

$$= \frac{1}{4}\left[\log(1) - \log\left(\frac{1}{1+4/s^2}\right)\right]$$

$$= \frac{1}{4}\left[-\log\left(\frac{s^2}{s^2+4}\right)\right]$$

$$= \frac{1}{4}\left[\log\left(\frac{s^2+4}{s^2}\right)\right]$$

Q73) $f(t) = 1 - \cos t$

$$L[f(t)] = L[1] - L[\cos t]$$

$$= \frac{1}{s} - \frac{s}{s^2+1}$$

$$L\left[\frac{1-\cos t}{t}\right] = \int_s^\infty \left(\frac{1}{s} - \frac{1}{2}\cdot\frac{2s}{s^2+1}\right) ds$$

$$= \left[\log s - \frac{1}{2}\log(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[2\log s - \log(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log s^2 - \log(s^2+1)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2}{s^2+1}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1}{1+1/s^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{1}{1+1/s^2}\right)\right]$$

$$= \frac{1}{2}\left[-\log\left(\frac{s^2}{s^2+1}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+1}{s^2}\right)\right]$$

$$L\left[\frac{1-\cos t}{t^2}\right] = \frac{1}{2}\int_s^\infty \log\left(\frac{s^2+1}{s^2}\right)\cdot 1\,ds$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+1}{s^2}\right)\cdot s\right]_s^\infty - \frac{1}{2}\int_s^\infty \frac{s^2}{s^2+1}\frac{d}{ds}\left(1+\frac{1}{s^2}\right)s$$

$$= \frac{1}{2}\left[s\log\left(\frac{s^2+1}{s^2}\right)\right]_s^\infty - \frac{1}{2}\int_s^\infty \frac{\cancel{s^3}}{s^2+1}\left(-\frac{2}{\cancel{s^3}}\right)$$

$$= \frac{1}{2}\left[s\log\left(1+\frac{1}{s^2}\right)\right]_s^\infty + \frac{1}{2}\int_s^\infty \frac{2}{s^2+1}$$

$$\frac{1}{2}\left[\infty\log 1 - s\log\left(\frac{s^2+1}{s^2}\right)\right]$$

$$= + \frac{2}{2}\left[\tan^{-1}(s)\right]_s^\infty = -\frac{s}{2}\log\left(\frac{s^2+1}{s^2}\right) + \frac{\pi}{2} - \tan^{-1}(s)$$

Q74) $f(t) = 1 - e^{t}$

$$L[f(t)] = L[1] - L[e^{t}]$$

$$= \frac{1}{s} - \frac{1}{s-1}$$

$$L\left[\frac{1-e^{t}}{t}\right] = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s-1}\right) ds$$

$$= \left[\log s - \log(s-1)\right]_s^\infty$$

$$= \left[\log\left(\frac{s}{s-1}\right)\right]_s^\infty = \left[\log\left(\frac{1}{1-\frac{1}{s}}\right)\right]_s^\infty$$

$$= \log 1 - \log\left(\frac{s}{s-1}\right) = \log\left(\frac{s-1}{s}\right)$$

Q75) $f(t) = e^{-at} - e^{-bt}$

$$L[f(t)] = L[e^{-at}] - L[e^{-bt}]$$

$$= \frac{1}{s+a} - \frac{1}{s+b}$$

$$L\left[\frac{e^{-at}-e^{-bt}}{t}\right] = \int_s^{\infty}\left(\frac{1}{s+a}-\frac{1}{s+b}\right)ds$$

$$= \left[\log(s+a)-\log(s+b)\right]_s^{\infty}$$

$$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^{\infty} = \left[\log\left(\frac{1+a/s}{1+b/s}\right)\right]_s^{\infty}$$

$$= \left[\log(1)-\log\left(\frac{s+a}{s+b}\right)\right]$$

$$= \log\left(\frac{s+b}{s+a}\right)$$

876) $f(t) = \sin 2t$

$$L[f(t)] = L[\sin 2t] = \frac{2}{s^2+4}$$

$$L\left[\frac{\sin 2t}{t}\right] = 2\int_s^{\infty}\frac{ds}{s^2+4} = \frac{2}{2}\left[\tan^{-1}\left(\frac{s}{2}\right)\right]_s^{\infty}$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{2}\right) = \cot^{-1}\left(\frac{s}{2}\right)$$

Q77) $f(t) = \sin^3 t = \dfrac{3\sin t - \sin 3t}{4}$

$$L[f(t)] = \frac{3}{4} L[\sin t] - \frac{1}{4} L[\sin 3t]$$

$$= \frac{3}{4(s^2+1)} - \frac{3}{4(s^2+9)}$$

$$L\left[\frac{\sin^3 t}{t}\right] = \frac{3}{4}\int_s^\infty \left(\frac{1}{s^2+1} - \frac{1}{s^2+9}\right) ds$$

$$= \frac{3}{4}\left[\tan^{-1}(s) - \frac{1}{3}\tan^{-1}\left(\frac{s}{3}\right)\right]_s^\infty$$

$$= \frac{3}{4}\left[\frac{\pi}{2} - \frac{\pi}{6} - \tan^{-1}s + \frac{1}{3}\tan^{-1}\left(\frac{s}{3}\right)\right]$$

$$= \frac{3}{4}\left[\frac{\pi}{3} - \tan^{-1}(s) + \frac{1}{3}\tan^{-1}\left(\frac{s}{3}\right)\right]$$

Q78) Same as (Q66)

Type VI :-

Q79) $L[f(t)] = \dfrac{8+12s-2s^2}{(s^2+4)^2}$

$L[f(2t)] = \dfrac{1}{2}\left[\dfrac{8+12\left(\frac{s}{2}\right)-2\left(\frac{s}{2}\right)^2}{\left[\left(\frac{s}{2}\right)^2+4\right]^2}\right]$

$= \dfrac{1}{\cancel{2}}\left[\dfrac{8+6s-s^2/2}{\dfrac{(s^2+16)^2}{4\times \cancel{4}_2}}\right]$

$= \dfrac{4(16+12s-s^2)}{(s^2+16)^2}$

Q80) $L[f(t)] = \dfrac{1}{s}e^{-4/s}$

$L[f(3t)] = \dfrac{1}{\cancel{3}}\cdot\dfrac{1}{s/\cancel{3}}\cdot e^{-3/s} = \dfrac{1}{s}e^{-3/s}$

$L[e^{-t}f(3t)] = \dfrac{1}{(s+1)}e^{-3/(s+1)}$

Q81) $L[\mathrm{erf}\sqrt{t}] = \dfrac{1}{s\sqrt{s+1}}$

$L[\mathrm{erf}\sqrt{4t}] = \dfrac{1}{\cancel{4}}\,\dfrac{1}{\frac{s}{\cancel{4}}\sqrt{\frac{s}{4}+1}} = \dfrac{2}{s\sqrt{4+s}}$

$L[t\,\mathrm{erf}(2\sqrt{t})] = (-1)^1 \dfrac{d}{ds}\left(\dfrac{2}{s\sqrt{s+4}}\right)$

$= \dfrac{+2}{s^2(s+4)}\left[\sqrt{s+4} + \dfrac{s}{2\sqrt{s+4}}\right]$

$= \dfrac{+\cancel{2}}{s^2(s+4)}\left[\dfrac{2(s+4)+s}{\cancel{2}\sqrt{s+4}}\right]$

$= +\left[\dfrac{3s+8}{s^2(s+4)^{3/2}}\right]$

Q82) $L[f(t)] = \dfrac{s^2-s+1}{(2s+1)^2(s-1)}$

$L[f(2t)] = \dfrac{1}{2}\left[\dfrac{s^2/4 - s/2 + 1}{\left(\cancel{2}\frac{s}{\cancel{2}}+1\right)^2\left(\frac{s}{2}-1\right)}\right]$

$$= \frac{1}{\cancel{2}}\left[\frac{s^2-2s+4}{4(s+1)^2\frac{(s-2)}{\cancel{2}}}\right]$$

TYPE VII

883) $f_1(u) = u$ & $f_2(t-u) = e^{a(t-u)}$

$$= e^{at} \cdot e^{-au}$$

$$f_1(t) = t$$

$$L[f_1(t)] = L[t] = \frac{1}{s^2}$$

$$f_2(t) = e^{at}$$

$$L[f_2(t)] = L[e^{at}] = \frac{1}{s-a}$$

$$L[f_1(t)] \cdot L[f_2(t)] = \frac{1}{s^2(s-a)} \quad \text{—— (I)}$$

$$I = \int_0^t f_1(u) f_2(t-u)\,du$$

$$= \int_0^t u \cdot e^{at} \cdot e^{-au}\,du$$

$$I = e^{at}\int_0^t u e^{-au}\,du = e^{at}\left[u\left(\frac{e^{-au}}{-a}\right) - 1\left(\frac{e^{-au}}{a^2}\right)\right]$$

$$= e^{at}\left[-\frac{te^{-at}}{a} - \frac{e^{-at}}{a^2} + \frac{1}{a^2}\right]$$

$$= -\frac{t}{a} - \frac{1}{a^2} + \frac{e^{at}}{a^2}$$

$$L[I] = L\left[-\frac{t}{a} - \frac{1}{a^2} + \frac{e^{at}}{a^2}\right]$$

$$= -\frac{1}{a}L[t] - \frac{1}{a^2}L[1] + \frac{1}{a^2}L[e^{at}]$$

$$= -\frac{1}{as^2} - \frac{1}{a^2 s} + \frac{1}{a^2(s-a)}$$

$$= \frac{-a(s-a) - s(s-a) + s^2}{a^2 s^2 (s-a)}$$

$$= \frac{-\cancel{as} + a^2 - \cancel{s^2} + \cancel{as} + \cancel{s^2}}{a^2 s^2 (s-a)}$$

$$= \frac{1}{s^2(s-a)} \quad \text{---(II)} \qquad \therefore (I) = (II)$$

... is verified

Q84) $f_1(u) = u^2$; $f_2(t-u) = e^{-a(t-u)}$
$= e^{-at} \cdot e^{au}$

$f_1(t) = t^2$
$L[f_1(t)] = \frac{2}{s^3}$

$f_2(t) = e^{-at}$
$L[f_2(t)] = \frac{1}{s+a}$

$L[f_1(t)] \cdot L[f_2(t)] = \frac{2}{s^3(s+a)}$ — (I)

$I = \int_0^t f_1(u) \cdot f_2(t-u)\,du$
$= \int_0^t u^2 \cdot e^{-at} \cdot e^{au}\,du$
$= e^{-at} \int_0^t u^2 e^{au}\,du$
$= e^{-at}\left[u^2\left(\frac{e^{au}}{a}\right) - 2u\left(\frac{e^{au}}{a^2}\right) + 2\left(\frac{e^{au}}{a^3}\right)\right]_0^t$

$$= e^{-at}\left[\frac{t^2 e^{at}}{a} - \frac{2te^{at}}{a^2} + \frac{2e^{at}}{a^3} - \frac{2}{a^3}\right]$$

$$= \frac{t^2}{a} - \frac{2t}{a^2} + \frac{2}{a^3} - \frac{2e^{-at}}{a^3}$$

$$L[I] = \frac{1}{a}L[t^2] - \frac{2}{a^2}L[t] + \frac{2}{a^3}L[1] - \frac{2}{a^3}L[e^{-at}]$$

$$= \frac{2}{as^3} - \frac{2}{a^2s^2} + \frac{2}{a^3s} - \frac{2}{a^3(s+a)}$$

$$= \frac{2a^2(s+a) - 2as(s+a) + 2s^2(s+a) - 2s^3}{a^3s^3(s+a)}$$

$$= \frac{\cancel{2a^2s} + 2a^3 - \cancel{2as^2} - \cancel{2a^2s} + \cancel{2s^3} + \cancel{2as^2} - \cancel{2s^3}}{a^3s^3(s+a)}$$

$$= \frac{2}{s^3(s+a)} \quad \text{— (II)}$$

$\therefore$ (I) = (II) $\therefore$ Convol. theo. is verified

Type VIII

385) $LHS = L\left[\int_0^t u^2 e^{-u} du\right]$

$= L\left\{\left[u^2\left(\frac{e^{-u}}{-1}\right) - 2u\left(\frac{e^{-u}}{1}\right) + 2\left(\frac{e^{-u}}{-1}\right)\right]_0^t\right\}$

$= L\left\{-t^2e^{-t} - 2te^{-t} - 2e^{-t} + 2\right\}$

$= -L\left[t^2e^{-t}\right] - 2L\left[te^{-t}\right] - 2L\left[e^{-t}\right] + 2L[1]$

$= -1(-1)^2\frac{d^2}{ds^2}\left(\frac{1}{s+1}\right) - 2(-1)^1\frac{d}{ds}\left(\frac{1}{s+1}\right) - \frac{2}{s+1} + \frac{2}{s}$

$= (-1)\left[\frac{2}{(s+1)^3}\right] + \frac{(-1)2}{(s+1)^2} - \frac{2}{s+1} + \frac{2}{s}$

$= -\frac{2}{(s+1)^3} - \frac{2}{(s+1)^2} - \frac{2}{s+1} + \frac{2}{s}$

$$2\left[\frac{-s^2-2s-s^3-2s^2-s+s^3+1+3s^2+3s}{s(s+1)^3}\right] = \frac{2}{s(s+1)^3}$$

$$= 2\left[-\frac{1}{(s+1)^3} - \frac{1}{(s+1)^2} - \frac{1}{s+1} + \frac{1}{s}\right]$$

$$= 2\left[\frac{-s - s(s+1) - s(s+1)^2 + (s+1)^3}{s(s+1)^3}\right]$$

$$= 2\left[\frac{-s - s^2 - s - s(s^2+2s+1) + (s^3+1+3s^2+3s)}{s(s+1)^3}\right]$$

$$= \cancel{2\left[\frac{s^2+s^3+2s^2+s-s^3-1-3s^2-3s}{s(s+1)^3}\right]}$$

$$= \cancel{2\left[\frac{-2s^3-4s^2-4s-1}{s(s+1)^3}\right]}$$

$$RHS = \frac{1}{s} L\left[t^2 e^{-t}\right]$$

$$= \frac{1}{s}\left\{(-1)^2 \frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)\right\}$$

$$= \frac{1}{s}\left\{\frac{d}{ds}\left[\frac{-1}{(s+1)^2}\right]\right\}$$

$$= \frac{1}{s}\left\{\frac{2}{(s+1)^3}\right\} = \frac{2}{s(s+1)^3}$$

886) $f(t) = \sin t$

$L[f(t)] = L[\sin t] = \frac{1}{s^2+1}$

$L\left[\cancel{e^t}\frac{\sin t}{t}\right] = \int_s^\infty \frac{ds}{s^2+1}$

$= \left[\tan^{-1}(s)\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}(s)$

$= \cot^{-1}(s)$

$L\left[e^t \frac{\sin t}{t}\right] = \cot^{-1}(s-1)$

$L\left[\int_0^t e^t \frac{\sin t}{t} dt\right] = \frac{1}{s}\cot^{-1}(s-1)$

887) $L[\cosh x] = \frac{s}{s^2-1}$

$L[x\cosh x] = (-1)^1 \frac{d}{ds}\left(\frac{s}{s^2-1}\right)$

$= (-1)\left[\cancel{-2s} \ldots\right] = \cancel{2s}$

$$L[x\cosh x] = (-1)^1 \left[\frac{(s^2-1) - s(2s)}{(s^2-1)^2}\right]$$

$$= \frac{s^2+1}{(s^2-1)^2}$$

$$L\left[\int_0^t x\cosh x\,dx\right] = \frac{1}{s}\left[\frac{s^2+1}{(s^2-1)^2}\right]$$

Q88) $L[\cosh x] = \dfrac{s}{s^2-1}$

$$L[e^x \cosh x] = \frac{(s-1)}{(s-1)^2-1}$$

$$= \frac{s-1}{s^2-2s}$$

$$L\left[\int_0^t e^x \cosh x \, dx\right] = \frac{s-1}{s(s^2-2s)}$$

$$= \frac{s-1}{s^2(s-2)}$$

$$L\left[\cosh t \int_0^t e^x \cosh x \, dx\right]$$

$$= L\left[\left(\frac{e^t+e^{-t}}{2}\right)\int_0^t e^x \cosh x \, dx\right]$$

$$= \frac{1}{2} L\left[e^t \int_0^t e^x \cosh x \, dx\right] + \frac{1}{2} L\left[e^{-t}\int_0^t e^x \cosh x \, dx\right]$$

$$= \frac{1}{2}\left[\frac{s-1-1}{(s-1)^2(s-1-2)}\right] + \frac{1}{2}\left[\frac{s+1-1}{(s+1)^2(s+1-2)}\right]$$

$$= \frac{1}{2}\left[\frac{s-2}{(s-1)^2(s-3)}\right] + \frac{1}{2}\left[\frac{s}{(s+1)^2(s-1)}\right]$$

$f(u) = u^3$

889) $L[f(u)] = L[u^3] = \frac{3!}{s^4}$

$= \frac{6}{s^4}$

$L[e^u u^3] = \frac{6}{(s-1)^4}$

$L\left[\int_0^t e^u u^3\, du\right] = \frac{6}{s(s-1)^4}$.

890) $f(u) = \cos u$

$L[f(u)] = L[\cos u] = \frac{s}{s^2+1}$

$L[e^u \cos u] = \frac{s-1}{(s-1)^2+1} = \frac{s-1}{s^2-2s+2}$

$L\left[\int_0^t e^u \cos u\, du\right] = \frac{s-1}{s(s^2-2s+2)}$

891) $f(x) = 1 - e^{-x}$

$L[f(x)] = L[1] - L[e^{-x}]$

$$= \frac{1}{s} - \frac{1}{s+1}$$

$$L\left[\frac{1-e^{-x}}{x}\right] = \int_s^{\infty}\left(\frac{1}{s} - \frac{1}{s+1}\right) ds$$

$$= \Big] \log s - \log(s+1) \Big]_s^{\infty}$$

$$= \left[\log\left(\frac{s}{s+1}\right)\right]_s^{\infty}$$

$$= \left[\log\left(\frac{1}{1+1/s}\right)\right]_s^{\infty} = \log 1 - \log\left(\frac{s}{s+1}\right)$$

$$= \log\left(\frac{s+1}{s}\right)$$

$$L\left[\int_0^t \frac{1-e^{-x}}{x} dx\right] = \frac{1}{s} \log\left(\frac{s+1}{s}\right)$$

Q92) $f(x) = \sin 3x$

$$L[f(x)] = L[\sin 3x] = \frac{3}{s^2+9}$$

$$L[e^{-4x}\sin 3x] = \frac{3}{(s+4)^2+9} = \frac{3}{s^2+8s+25}$$

$$L\left[\int_0^t e^{-4x}\sin 3x\,dx\right] = \frac{3}{s(s^2+8s+25)}$$

$$L\left[t\int_0^t e^{-4x}\sin 3x\,dx\right] = (-1)^1\frac{d}{ds}\left(\frac{3}{s(s^2+8s+25)}\right)$$

$$= -3\left\{\frac{-1}{s^2(s^2+8s+25)^2}\left(s^2+8s+25+s(2s+8)\right)\right\}$$

$$= \frac{3}{s^2(s^2+8s+25)^2}\left[3s^2+16s+25\right]$$

893) $f(t) = \sin 3t$

$$L[f(t)] = L[\sin 3t] = \frac{3}{s^2+9}$$

$$L[t\sin 3t] = (-1)^1\frac{d}{ds}\left(\frac{3}{s^2+9}\right)$$

$$= -3\left[\frac{-1}{(s^2+9)^2}\cdot 2s\right]$$

$$= \frac{6s}{(s^2+9)^2}$$

$$L\left[\int_0^t t\sin 3t\,dt\right] = \frac{1}{8}\,\frac{6s}{(s^2+9)^2} = \frac{6}{(s^2+9)^2}$$

$$L\left[e^{-3t}\int_0^t t\sin 3t\,dt\right] = \frac{6}{[(s+3)^2+9]^2}$$

$$= \frac{6}{(s^2+6s+18)^2}$$

Q94) $f(t) = \sin 2t$

$$L[f(t)] = L[\sin 2t] = \frac{2}{s^2+4}$$

$$L[t\sin 2t] = (-1)^1 \frac{d}{ds}\left(\frac{2}{s^2+4}\right)$$

$$= -2\left[-\frac{1}{(s^2+4)^2}\,2s\right] = \frac{4s}{(s^2+4)^2}$$

$$L\left[e^{-4t}\cdot t\sin 2t\right] = \frac{4(s+4)}{\left[(s+4)^2+4\right]^2}$$

$$= \frac{4(s+4)}{(s^2+8s+20)^2}$$

$$\cancel{L\left[e^{-4t}\, t\sin 2t\right] = \frac{4(s+4+4)}{\left[(s+4)^2+8(s+4)+20\right]^2}}$$

$$\cancel{= \frac{4(s+8)}{\left[s^2+8s+16+8s+32+20\right]^2}}$$

$$\cancel{= \frac{4(s+8)}{(s^2+16s+68)^2}}$$

$$\cancel{L\left[\int_0^t t\, e^{-4t}\sin 2t\, dt\right] = \frac{4(s+8)}{s(s^2+16s+68)^2}}$$

$$L\left[\int_0^t e^{-4t}\, t\sin 2t\right] = \frac{4(s+4)}{s(s^2+8s+20)^2}$$

Type IX

$L[y] = \bar{y}(s)$

$L\left[\frac{dy}{dt}\right] = s\bar{y}(s) - y(0)$

$L\left[\frac{d^2y}{dt^2}\right] = s^2\bar{y}(s) - sy(0) - y'(0)$

895) $L\left[\frac{d^2y}{dt^2}\right] - 3L\left[\frac{dy}{dt}\right] + 5L[y]$

$= s^2\bar{y}(s) - sy(0) - y'(0)$
$\quad - 3[s\bar{y}(s) - y(0)]$
$\quad + 5\bar{y}(s)$

$= s^2\bar{y}(s) - 2s + 4 - 3(s\bar{y}(s) - 2)$
$\quad + 5\bar{y}(s)$

$= \bar{y}(s)(s^2 - 3s + 5) - 2s + 4 + 6$

$= \bar{y}(s)(s^2 - 3s + 5) - 2s + 10$

896) Let $f(t) = 2\sqrt{\dfrac{t}{\pi}}$

$$L[f(t)] = \frac{1}{s^{3/2}} = \bar{f}(s)$$

$$f'(t) = \frac{2}{\sqrt{\pi}} \frac{d}{dt}(t^{1/2})$$

$$= \frac{2}{2\sqrt{\pi}} \cdot \frac{1}{\sqrt{t}} = \frac{1}{\sqrt{\pi t}}$$

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$f(0) = 2\sqrt{\frac{0}{\pi}} = 0$$

$$L[f'(t)] = \frac{s}{s^{3/2}} = \frac{1}{\sqrt{s}}$$

897) $f(t) = \sin t$

$$L[f(t)] = L[\sin t] = \frac{1}{s^2+1}$$

$$L\left[\frac{\sin t}{t}\right] = \int_s^{\infty} \frac{ds}{s^2+1} = [\tan^{-1} s]_s^{\infty}$$

$$= \frac{\pi}{2} - \tan^{-1}(s) = \cot^{-1}(s)$$

$$L\left[\frac{d}{dt}\left(\frac{\sin t}{t}\right)\right] = sL\left[\frac{\sin t}{t}\right] - \lim_{t \to 0} \frac{\sin t}{t}$$

$$= s\cot^{-1}s - 1$$

Q98) By fund. def.

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^2 e^{-st}(t+1)\,dt + \int_2^\infty 3e^{-st}\,dt$$

$$= \left[(t+1)\left(\frac{e^{-st}}{-s}\right) - 1\left(\frac{e^{-st}}{s^2}\right)\right]_0^2 + 3\left[\left(\frac{e^{-st}}{-s}\right)\right]_2^\infty$$

$$= \cancel{\frac{-3e^{-2s}}{s}} - \frac{e^{-2s}}{s^2} + \frac{1}{s} + \frac{1}{s^2} + \cancel{\frac{3e^{-2s}}{s}}$$

$$L[f'(t)] = sL[f(t)] - f(0)$$
$$= s\left[\frac{1}{s} + \frac{1}{s^2} - \frac{e^{-2s}}{s^2}\right] - 1.$$

Type X :-

899) $f(t) = 5 + 2\cos 3t$

$$L[f(t)] = 5L[1] + 2L[\cos 3t]$$
$$= \frac{5}{s} + \frac{2s}{s^2+9}$$

$$\lim_{t \to 0} f(t) = \lim_{t \to 0} (5 + 2\cos 3t)$$
$$= 5 + 2 = 7$$

$$\lim_{s \to \infty} s\bar{f}(s) = \lim_{s \to \infty} \left(5 + \frac{2s^2}{s^2+9}\right)$$
$$= 5 + \lim_{s \to \infty} \frac{2}{1 + 9/s^2} = 5 + 2 = 7$$

100 i) $f(t) = 3 - 4\sin t$

$L[f(t)] = 3L[1] - 4L[\sin t]$

$= \frac{3}{s} - \frac{4}{s^2+1}$

$\lim_{t \to 0} f(t) = 3$

$\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \left(3 - \frac{4s}{s^2+1}\right)$

$= 3 - \lim_{s \to \infty} \left(\frac{4/s}{1+1/s^2}\right) = 3 - 0$

$= 3$

ii) $(t-4)^2 = t^2 - 8t + 16$

$L[f(t)] = L[t^2] - 8L[t] + 16L[1]$

$= \frac{2!}{s^3} - \frac{8}{s^2} + \frac{16}{s}$

$\lim_{t \to 0} f(t) = 16$

$$\lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left(\frac{2}{s^2} - \frac{8}{s} + 16\right) = 16$$

TYPE XI

10| i) $f(t) = 2 + 3e^{-2t}\sin 4t$

$$L[f(t)] = \frac{2}{s} + \frac{12}{(s+2)^2+16}$$

$$\lim_{t\to\infty} f(t) = 2 + 0 = 2$$

$$\lim_{s\to 0} sF(s) = 2 + \lim_{s\to 0}\frac{12s}{(s+2)^2+16} = 2$$

ii) $f(t) = 4 + e^{-2t}(\sin t + \cos t)$

$$L[f(t)] = \frac{4}{s} + \frac{1}{(s+2)^2+1} + \frac{s+2}{(s+2)^2+1}$$

$$\lim_{t\to\infty} f(t) = 4 + 0 = 4$$

$$\lim_{s\to 0} s\bar{f}(s) = 4 + \lim_{s\to 0} \frac{s(s+3)}{(s+2)^2+1}$$
$$= 4 + 0 = 4$$

MISCELLANEOUS :-

102) $L[J_0(t)] = \dfrac{1}{\sqrt{1+s^2}}$

1) $L[J_0(at)] = \dfrac{1}{a} \dfrac{1}{\sqrt{1+\frac{s^2}{a^2}}} = \dfrac{1}{\sqrt{s^2+a^2}}$

$$L[t\,J_0(at)] = (-1)^1 \frac{d}{ds}\left(\frac{1}{\sqrt{s^2+a^2}}\right)$$
$$= \frac{1}{2}\cdot\left[\frac{1}{(s^2+a^2)^{3/2}}\cdot 2s\right] = \frac{s}{(s^2+a^2)^{3/2}}$$

(ii) $L[e^{-at} J_0(at)] = \dfrac{1}{\sqrt{(s+a)^2+a^2}}$

$$= \frac{1}{\sqrt{s^2+2as+2a^2}}$$

iii) $\int_0^\infty e^{-st} J_0(t)\,dt = \frac{1}{\sqrt{1+s^2}}$

Put $s=0$ in both sides

$$\int_0^\infty J_0(t)\,dt = 1$$

iv) $L[J_0(4t)] = \frac{1}{4}\frac{1}{\sqrt{1+\frac{s^2}{16}}} = \frac{1}{\sqrt{s^2+16}}$

$$L[tJ_0(4t)] = (-1)\frac{d}{ds}\left(\frac{1}{\sqrt{s^2+16}}\right)$$

$$= \frac{1}{2}\left[\frac{1}{(s^2+16)^{3/2}} \cdot 2s\right] = \frac{s}{(s^2+16)^{3/2}}$$

$$\int_0^\infty e^{-st}\, t\, J_0(4t)\,dt = \frac{s}{(s^2+16)^{3/2}}$$

Put $s=3$ on both sides

$$\int_0^\infty e^{-3t}\, t\, J_0(4t)\,dt = \frac{3}{25^{3/2}} = \frac{3}{125}$$

(103) i) $L[\sin t] = \frac{1}{s^2+1}$

$$L[t\sin t] = (-1)\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= (-1)\left[\frac{-1}{(s^2+1)^2}\right] \cdot 2s$$

$$= \frac{2s}{(s^2+1)^2}$$

$$L[t^2\sin t] = (-1)^1 \frac{d}{ds}\left[\frac{2s}{(s^2+1)^2}\right]$$

$$= -2\left[\frac{(s^2+1)^2 \cdot (1) - 2s(s^2+1)(2s)}{(s^2+1)^{4}}\right] = -2\left[\frac{(s^2+1) \cdot (1) - 2s(2s)}{(s^2+1)^{3}}\right]$$

$$\cancel{= -2\left[\frac{1-4s^2}{(s^2+1)^3}\right]}$$

$$\cancel{L\left[t^3 \sin t\right] = (-1)^1 \frac{d}{ds}\left[-2\left(\frac{1-4s^2}{(s^2+1)^3}\right)\right]}$$

$$\cancel{= 2\left[\frac{(s^2+1)^3(-8s) - 3(1-4s^2)(s^2+1)^2(2s)}{(s^2+1)^6}\right]}$$

$$= -2\left[\frac{1-3s^2}{(s^2+1)^3}\right]$$

$$L\left[t^3 \sin t\right] = (-1)^1 \frac{d}{ds}\left[-2\left(\frac{1-3s^2}{(s^2+1)^3}\right)\right]$$

$$= 2\left[\frac{(s^2+1)^3(-6s) - 3(1-3s^2)(s^2+1)^2(2s)}{(s^2+1)^6}\right]$$

$$= 2\left[\frac{-6s^3-6s-6s(1-3s^2)}{(s^2+1)^4}\right]$$

$$= 2\left[\frac{-6s^3-6s-6s+18s^3}{(s^2+1)^4}\right]$$

$$= 2\left[\frac{12s^3-12s}{(s^2+1)^4}\right] = \frac{24s(s^2-1)}{(s^2+1)^4}$$

$$\int_0^\infty e^{-st} t^3 \sin t\,dt = \frac{24s(s^2-1)}{(s^2+1)^4}$$

Put $s = 1$

$$\int_0^\infty e^{-t} t^3 \sin t\,dt = 0$$

(ii) $f(t) = \sin^3 t = \dfrac{3\sin t - \sin 3t}{4}$

$$L[\sin^3 t] = \frac{3}{4}\left(\frac{1}{s^2+1}\right) - \frac{1}{4}\left(\frac{3}{s^2+9}\right)$$

$$= \frac{3}{4(s^2+1)} - \frac{3}{4(s^2+9)}$$

$$\int_0^\infty e^{-st} \sin^3 t\,dt = \frac{3}{4(s^2+1)} - \frac{3}{4(s^2+9)}$$

Put $s = 2$

$$\int_0^\infty e^{-2t} \sin^3 t\,dt = \frac{3}{4(5)} - \frac{3}{4(13)}$$

$$= \frac{3}{4}\left[\frac{1}{5} - \frac{1}{13}\right] = \frac{3}{\cancel{4}} \times \frac{\cancel{8}^{2}}{5\times 13} = \frac{6}{65}$$

(iii) $f(t) = \sin t$

$$L[f(t)] = \frac{1}{s^2+1}$$

$$L[t \sin t] = (-1)^1 \frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= (-1)^1 \frac{(-1)}{(s^2+1)^2}(2s) = \frac{2s}{(s^2+1)^2}$$

$$\int_0^\infty e^{-st}\, t \sin t\, dt = \frac{2s}{(s^2+1)^2}$$

Put $s=3$

$$\int_0^\infty e^{-3t}\, t \sin t\, dt = \frac{6}{(10)^2} = \frac{6}{100} = \frac{3}{50}$$

(iv) Let $f(t) = \sinh t$

$$L[f(t)] = \frac{1}{s^2-1}$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \frac{ds}{s^2-1}$$

$$= \left[\frac{1}{2}\log\left(\frac{s-1}{s+1}\right)\right]_s^\infty$$

$$= \left[\frac{1}{2}\log\left(\frac{1-1/s}{1+1/s}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{s-1}{s+1}\right)\right]$$

$$= \frac{1}{2}\log\left(\frac{s+1}{s-1}\right)$$

$$\int_0^\infty e^{-st}\frac{\sinh t}{t}\,dt = \frac{1}{2}\log\left(\frac{s+1}{s-1}\right)$$

$$\int_0^\infty e^{-2t}\frac{\sinh t}{t}\,dt = \frac{1}{2}\log(3)$$

v) $f(t) = \cos 6t - \cos 4t$

$$L[f(t)] = \frac{s}{s^2+6} - \frac{s}{s^2+4}$$

$$= \frac{1}{2}\left[\frac{2s}{s^2+6} - \frac{2s}{s^2+4}\right]$$

$$L\left[\frac{f(t)}{t}\right] = \frac{1}{2}\int_s^\infty \left(\frac{2s}{s^2+6} - \frac{2s}{s^2+4}\right) ds$$

$$= \frac{1}{2}\left[\log(s^2+6) - \log(s^2+4)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+6}{s^2+4}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log\left(\frac{1+6/s^2}{1+4/s^2}\right)\right]_s^\infty$$

$$= \frac{1}{2}\left[\log(1) - \log\left(\frac{s^2+6}{s^2+4}\right)\right]$$

$$= \frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+6}\right)\right]$$

$$\int_0^\infty e^{-st}\left(\frac{\cos 6t - \cos 4t}{t}\right) dt = \frac{1}{2}\left[\log\left(\frac{s^2+4}{s^2+6}\right)\right]$$

Put $s = 0$

$$\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt = \frac{1}{2}\log$$

$$= \frac{1}{2}\log\left(\frac{2}{3}\right)$$

vi) $f(t) = \sin t$

$$L[f(t)] = \frac{1}{s^2+1}$$

$$L[t f(t)] = (-1)^1 \frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= \frac{2s}{(s^2+1)^2}$$

$$\int_0^\infty e^{-st}\, t \sin t\, dt = \frac{2s}{(s^2+1)^2}$$

Put $s = 3$

$$\int_0^\infty e^{-3t}\, t \sin t\, dt = \frac{6}{100}$$

(vii) $\because \cos 3t = 4\cos^3 t - 3\cos t$

$$\cos^3 t = \frac{3\cos t + \cos 3t}{4}$$

$$L[\cos^3 t] = \frac{3}{4} L[\cos t] + \frac{1}{4} L[\cos 3t]$$

$$= \frac{3s}{4(s^2+1)} + \frac{s}{4(s^2+9)}$$

$$\int_0^\infty e^{-st}\cos^3 t\,dt = \frac{3s}{4(s^2+1)} + \frac{s}{4(s^2+9)}$$

Put $s=3$

$$\int_0^\infty e^{-3t}\cos^3 t\,dt = \frac{9}{4\times 10} + \frac{3}{4\times 18}$$

$$= \frac{9}{40} + \frac{1}{24} = \frac{\cancel{256}^{\,32\,8\,4}}{{}_{5}\cancel{40}\times \cancel{24}_{\cancel{6}\,3}} = \frac{4}{15}$$

viii) $\displaystyle\int_0^\infty e^{-t}\left(\frac{1-e^{-2t}}{t}\right)dt$

$f(t) = 1 - e^{-2t}$; $L[f(t)] = \frac{1}{s} - \frac{1}{s+2}$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \left(\frac{1}{s} - \frac{1}{s+2}\right) ds$$

$$= \left[\log s - \log(s+2)\right]_s^\infty = \left[\log\left(\frac{s}{s+2}\right)\right]_s^\infty$$

$$= \left[\log\left(\frac{1}{1+2/s}\right)\right]_s^\infty = \log\left(\frac{s+2}{s}\right)$$

$$\int_0^\infty e^{-st}\left(\frac{1-e^{-2t}}{t}\right) dt = \log\left(\frac{s+2}{s}\right)$$

Put $s = 1$

$$\int_0^\infty e^{-t}\left(\frac{1-e^{-2t}}{t}\right) dt = \log 3$$

ix) $f(t) = \sin t$; $L[f(t)] = \frac{1}{s^2+1}$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \frac{ds}{s^2+1} = \left[\tan^{-1}(s)\right]_s^\infty = \cancel{\cot^{-1} s}$$

$$= \frac{\pi}{2} - \tan^{-1}(s)$$

$$\int_0^\infty e^{-st}\frac{\sin t}{t}\,dt = \cancel{\cot^{-1}(s)}\ \frac{\pi}{2} - \tan^{-1}(s)$$

Put $s=1$

$$\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt = \cancel{\cot^{-1}(1)} = \frac{\pi}{4}$$

x) $$\int_0^\infty \frac{e^{-2t}}{t}\left(\frac{e^{t}-e^{-t}}{2}\right)\sin t\,dt$$

$$= \frac{1}{2}\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt - \frac{1}{2}\int_s^\infty e^{-3t}\frac{\sin t}{t}\,dt$$

$$= I_1 - I_2$$

Let $f(t) = \sin t$

$$L[f(t)] = \frac{1}{s^2+1}$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \frac{ds}{s^2+1} = \left[\tan^{-1}(s)\right]_s^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}(s)$$

$$\int_0^\infty e^{-st}\frac{\sin t}{t}\,dt = \frac{\pi}{2} - \tan^{-1}(s)$$

Put $s=1$

$$\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$I_1 = \frac{1}{2}\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt = \frac{\pi}{8}$$

Put $s=3$

$$\int_0^\infty e^{-3t}\frac{\sin t}{t}\,dt = \frac{\pi}{2} - \tan^{-1}(3)$$

$$I_2 = \frac{1}{2}\int_0^\infty e^{-3t}\frac{\sin t}{t}\,dt = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}(3)$$

$$I_1 - I_2$$

$$= \frac{\pi}{8} - \frac{\pi}{4} + \frac{1}{2}\tan^{-1}(3)$$

$$= \frac{1}{2}\tan^{-1}(3) - \frac{\pi}{8}$$

INVERSE LAPLACE TRANSFORM

TYPE I:-

(1) Let $f(t) = L^{-1}\left[\frac{1}{s+4}\right] = e^{-4t}$

(2) $f(t) = L^{-1}\left[\frac{2s+6}{s^2+4}\right]$

$= 2L^{-1}\left[\frac{s}{s^2+4}\right] + \frac{6}{2}L^{-1}\left[\frac{2}{s^2+4}\right]$

$= 2\cos 2t + \frac{6}{2}\sin 2t = 2\cos 2t + 3\sin 2t$

(3) $f(t) = L^{-1}\left[\frac{1}{2s-3}\right] = \frac{1}{2}L^{-1}\left[\frac{1}{s-3/2}\right]$

$= \frac{1}{2}e^{\frac{3t}{2}}$

(5) $f(t) = L^{-1}\left[\frac{4s+15}{16s^2-25}\right]$

$= \frac{1}{16}L^{-1}\left[\frac{4s+15}{s^2-\frac{25}{16}}\right]$

$$= \frac{4}{16} L^{-1}\left[\frac{s}{s^2-\left(\frac{5}{4}\right)^2}\right] + \frac{15}{16}\times\frac{4}{5} L^{-1}\left[\frac{5/4}{s^2-(5/4)^2}\right]$$

$$= \frac{1}{4}\cosh\left(\frac{5t}{4}\right) + \frac{3}{4}\sinh\left(\frac{5t}{4}\right)$$

(4) $f(t) = L^{-1}\left[\frac{3s+5\sqrt{2}}{s^2+8}\right]$

$$= 3L^{-1}\left[\frac{s}{s^2+8}\right] + \frac{5\sqrt{2}}{\sqrt{8}} L^{-1}\left[\frac{\sqrt{8}}{s^2+8}\right]$$

$$= 3\cos 2\sqrt{2}t + \frac{5}{2}\sin 2\sqrt{2}t$$

(6) $f(t) = L^{-1}\left[\frac{3(s^4-2s^2+1)}{2s^5}\right] = L^{-1}\left[\frac{3}{2s} - \frac{3}{s^3} + \frac{3}{2s^5}\right]$

$$= \frac{3}{2}L^{-1}\left[\frac{1}{s}\right] - 3L^{-1}\left[\frac{1}{s^3}\right] + \frac{3}{2}L^{-1}\left[\frac{1}{s^5}\right]$$

$$= \frac{3}{2}(1) - \frac{3t^2}{2!} + \frac{3\cdot t^4}{2\times 4!}$$

$$= \frac{3}{2} - \frac{3t^2}{2} + \frac{\cancel{3}t^4}{2\times 4\times \cancel{3}\times 2}$$

$$= \frac{3}{2} - \frac{3t^2}{2} + \frac{t^4}{16}$$

(7) $f(t) = L^{-1}\left[\frac{1}{s^{3/2}}\right] = \frac{t^{3/2-1}}{\Gamma 3/2} = \frac{t^{1/2}}{\frac{1}{2}\Gamma 1/2} = 2\sqrt{\frac{t}{\pi}}$

(8) $f(t) = L^{-1}\left[\frac{s+1}{s^{4/3}}\right] = L^{-1}\left[\frac{1}{s^{1/3}}\right] + L^{-1}\left[\frac{1}{s^{4/3}}\right]$

$$= \frac{t^{1/3-1}}{\Gamma\frac{1}{3}} + \frac{t^{4/3-1}}{\Gamma 4/3} = \frac{t^{-2/3}}{\Gamma 1/3} + \frac{t^{1/3}}{\frac{1}{3}\Gamma\frac{1}{3}}$$

$$= \frac{1}{\Gamma 1/3}\left[t^{-2/3} + 3t^{1/3}\right]$$

(9) $f(t) = L^{-1}\left[\frac{1-2\sqrt{s}+s}{s^4}\right] = L^{-1}\left[\frac{1}{s^4} - \frac{2}{s^{7/2}} + \frac{1}{s^3}\right]$

$$= \frac{t^{4-1}}{\Gamma 4} - \frac{2t^{7/2-1}}{\Gamma 7/2} + \frac{t^{3-1}}{\Gamma 3}$$

$$= \frac{t^3}{6} - \frac{2t^{5/2}}{\frac{1}{2}\sqrt{\frac{1}{2}}\times\frac{5}{2}\times\frac{3}{2}} + \frac{t^2}{2} = \frac{t^3}{6} + \frac{t^2}{2} - \frac{16t^{5/2}}{15\sqrt{\pi}}$$

(10) $f(t) = \frac{1}{3}L^{-1}\left[\frac{3}{s^2+9}\right] = \frac{1}{3}\sin 3t$

(11) $f(t) = \frac{5}{3}L^{-1}\left[\frac{1}{s-1/3}\right] = \frac{5}{3}e^{+t/3}$

(12) $f(t) = 6L^{-1}\left[\frac{1}{s^3}\right] = 6\times\frac{t^{3-1}}{\lfloor 3} = \frac{6t^2}{2} = 3t^2$

(13) $f(t) = 3L^{-1}\left[\frac{1}{s^4}\right] = 3\frac{t^{4-1}}{\lfloor 4} = \frac{3t^3}{6} = \frac{t^3}{2}$

(14) $f(t) = 7L^{-1}\left[\frac{s}{s^2+4}\right] = 7\cos 2t$

(15) $f(t) = 4L^{-1}\left[\frac{s}{s^2-16}\right] = 4\cosh(4t)$

(16) $f(t) = \frac{3}{\sqrt{7}}L^{-1}\left[\frac{\sqrt{7}}{s^2-7}\right] = \frac{3}{\sqrt{7}}\sinh(\sqrt{7}t)$

(17) $f(t) = L^{-1}\left[\frac{1}{s+4}\right] = e^{-4t}$

(18) $f(t) = 2L^{-1}\left[\frac{s}{s^2-4}\right] + \frac{6}{2}L^{-1}\left[\frac{2}{s^2-4}\right]$

$= 2\cosh(2t) + 3\sinh(2t)$

TYPE II :-

(19) $f(t) = L^{-1}\left[\frac{(s+1)+6}{(s^2+2s+1)+4}\right]$

$= L^{-1}\left[\frac{(s+1)+6}{(s+1)^2+4}\right] = e^{-t}L^{-1}\left[\frac{s+6}{s^2+4}\right]$

$= e^{-t}L^{-1}\left[\frac{s}{s^2+4}\right] + \frac{6e^{-t}}{2}L^{-1}\left[\frac{2}{s^2+4}\right]$

$$= e^{-t}\cos 2t + 3e^{-t}\sin 2t$$

(20) $f(t) = L^{-1}\left[\dfrac{1}{(s-1)^5}\right] = e^{t} L^{-1}\left[\dfrac{1}{s^5}\right]$

$$= e^{t}\left[\frac{t^{5-1}}{\lceil 5}\right] = \frac{e^{t}t^4}{24}$$

(21) $f(t) = L^{-1}\left[\dfrac{1}{\sqrt{2s+3}}\right] = \dfrac{1}{\sqrt{2}} L^{-1}\left[\dfrac{1}{(s+\frac{3}{2})^{1/2}}\right]$

$$= \frac{1}{\sqrt{2}} e^{-\frac{3t}{2}} L^{-1}\left[\frac{1}{s^{1/2}}\right] = \frac{1}{\sqrt{2}} e^{-3t/2} \frac{t^{1/2-1}}{\lceil 1/2}$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{e^{-3t/2}}{\sqrt{t}} = \frac{e^{-3t/2}}{\sqrt{2\pi t}}$$

(22) $f(t) = L^{-1}\left[\dfrac{1}{(s+4)^{3/2}}\right] = e^{-4t} L^{-1}\left[\dfrac{1}{s^{3/2}}\right]$

$$= e^{-4t}\frac{t^{3/2-1}}{\lceil \frac{3}{2}} = 2e^{-4t}\sqrt{\frac{t}{\pi}}$$

(23) $f(t) = 4L^{-1}\left[\frac{s+3}{s^2+8s+16}\right]$

$= 4L^{-1}\left[\frac{(s+4)-1}{(s+4)^2}\right]$

$= 4e^{-4t} L^{-1}\left[\frac{s-1}{s^2}\right]$

$= 4e^{-4t}\left\{L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{1}{s^2}\right]\right\}$

$= 4e^{-4t}\left\{1 - \frac{t^{2-1}}{\lfloor 2}\right\}$

$= 4e^{-4t}[1-t]$

(24) $f(t) = L^{-1}\left[\frac{s+1}{\left(s^2+s+\frac{1}{4}\right)+\frac{3}{4}}\right]$

$= L^{-1}\left[\frac{(s+1/2)+1/2}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$

$$= e^{-t/2} L^{-1}\left[\frac{s+1/2}{s^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

$$= e^{-t/2}\left\{L^{-1}\left[\frac{s}{s^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right] + \frac{1}{2}\cdot\frac{2}{\sqrt{3}} L^{-1}\left[\frac{\sqrt{3}/2}{s^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]\right\}$$

$$= e^{-t/2}\left\{\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right\}$$

(125) $f(t) = 3L^{-1}\left[\frac{s+7/3}{(s^2-2s+1)-4}\right]$

$$= 3L^{-1}\left[\frac{(s-1)+10/3}{(s-1)^2-4}\right]$$

$$= 3e^{t} L^{-1}\left[\frac{s+10/3}{s^2-4}\right]$$

$$= 3e^{t}\left\{L^{-1}\left[\frac{s}{s^2-4}\right] + \frac{10}{3\times 2} L^{-1}\left[\frac{2}{s^2-4}\right]\right\}$$

$$= 3e^{t}\left\{\cosh 2t + \frac{5}{3}\sinh 2t\right\}$$

(126) $f(t) = L^{-1}\left[\dfrac{s-1}{(s^2+2s+1)-1}\right]$

$= L^{-1}\left[\dfrac{(s+1)-2}{(s+1)^2-1}\right]$

$= e^{-t} L^{-1}\left[\dfrac{s-2}{s^2-1}\right]$

$= e^{-t}\left\{L^{-1}\left[\dfrac{s}{s^2-1}\right] - 2L^{-1}\left[\dfrac{1}{s^2-1}\right]\right\}$

$= e^{-t}\left\{\cosh t - 2\sinh t\right\}$

$= e^{-t}\left\{\dfrac{e^{t}+e^{-t}}{2} - e^{t} + e^{-t}\right\}$

$= -\dfrac{1}{2} + \dfrac{3}{2}e^{-2t}.$

(127) $f(t) = 2L^{-1}\left[\dfrac{1}{(s-3)^5}\right] = 2e^{3t}L^{-1}\left[\dfrac{1}{s^5}\right]$

$= 2e^{3t}\dfrac{t^{5-1}}{\Gamma 5} = \cancel{2}e^{3t}\dfrac{t^4}{4\times 3\times \cancel{2}} = \dfrac{e^{3t}t^4}{12}$

(128) $f(t) = 3L^{-1}\left[\dfrac{1}{(s^2-4s+4)+9}\right]$

$= 3L^{-1}\left[\dfrac{1}{(s-2)^2+9}\right]$

$= \dfrac{3e^{2t}}{3} L^{-1}\left[\dfrac{3}{s^2+9}\right] = e^{2t}\sin 3t$

(129) $f(t) = 2L^{-1}\left[\dfrac{s+1}{(s^2+2s+1)+9}\right] = 2L^{-1}\left[\dfrac{s+1}{(s+1)^2+9}\right]$

$= 2e^{-t} L^{-1}\left[\dfrac{s}{s^2+9}\right] = 2e^{-t}\cos 3t$

(130) $f(t) = L^{-1}\left[\dfrac{(s-3)+3}{(s-3)^5}\right]$

$= e^{3t} L^{-1}\left[\dfrac{s+3}{s^5}\right] = e^{3t}\left\{L^{-1}\left[\dfrac{1}{s^4}\right] + 3L^{-1}\left(\dfrac{1}{s^5}\right)\right.$

$= e^{3t}\left\{\dfrac{t^{4-1}}{\lfloor 4} + \dfrac{3t^{5-1}}{\lfloor 5}\right\} = e^{3t}\left[\dfrac{t^3}{6} + \dfrac{\cancel{3}t^4}{\cancel{24}\,8}\right]$

$$= e^{3t} t^3 \left[\frac{1}{6} + \frac{t}{8}\right]$$

(31) $f(t) = L^{-1}\left[\frac{1}{(s+2)^2}\right] = e^{-2t} L^{-1}\left[\frac{1}{s^2}\right]$

$$= e^{-2t} \frac{t^{2-1}}{\Gamma 2} = t e^{-2t}$$

(32) $f(t) = L^{-1}\left[\frac{1}{(s^2+2s+1)+1}\right] = L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$

$$= e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right] = e^{-t} \sin t$$

(33) $f(t) = L^{-1}\left[\frac{(s+1)+6}{(s^2+2s+1)+1}\right]$

$$= L^{-1}\left[\frac{(s+1)+6}{(s+1)^2+1}\right] = e^{-t} L^{-1}\left[\frac{s+6}{s^2+1}\right]$$

$$= e^{-t}\left\{L^{-1}\left[\frac{s}{s^2+1}\right] + 6 L^{-1}\left[\frac{1}{s^2+1}\right]\right\}$$

$= e^{-t}(\cos t + 6\sin t)$

(34) $f(t) = 3L^{-1}\left[\frac{s+1/3}{(s+1)^4}\right]$

$= 3L^{-1}\left[\frac{(s+1)-2/3}{(s+1)^4}\right]$

$= 3e^{-t}\, L^{-1}\left[\frac{s-2/3}{s^4}\right]$

$= 3e^{-t}\left\{L^{-1}\left[\frac{1}{s^3}\right] - \frac{2}{3}L^{-1}\left[\frac{1}{s^4}\right]\right\}$

$= 3e^{-t}\left\{\frac{t^{3-1}}{\lceil 3} - \frac{2}{3}\frac{t^{4-1}}{\lceil 4}\right\}$

$= 3e^{-t}\left\{\frac{t^2}{2} - \frac{2}{3}\frac{t^3}{3\times 6}\right\}$

$= 3e^{-t}\left\{\frac{t^2}{2} - \frac{t^3}{9}\right\}$

(35) $f(t) = L^{-1}\left[\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}\right]$

Factor of $s^3-6s^2+11s-6$ is $(s-1)$

other factors by syn. $\div$

	1	−6	11	−6
	0	1	−5	6
s = 1	1	−5	6	0

$s^2-5s+6 = (s-3)(s-2)$

$f(t) = L^{-1}\left[\dfrac{2s^2-6s+5}{(s-1)(s-2)(s-3)}\right]$

$= L^{-1}\left[\dfrac{1}{2(s-1)} - \dfrac{1}{(s-2)} + \dfrac{5}{2(s-3)}\right]$

$= \dfrac{1}{2}e^{t} - e^{2t} + \dfrac{5}{2}e^{3t}$

(36) $f(t) = L^{-1}\left[\frac{3s+1}{(s-1)(s^2+1)}\right]$

$$\frac{3s+1}{(s-1)(s^2+1)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+1}$$

$3s+1 = A(s^2+1) + (Bs+C)(s-1)$

put $s=1$; $A = 2$

put $s=0$; $C = 1$

Eq. co-eff of s^2 ; $B = -2$

$$f(t) = 2L^{-1}\left[\frac{1}{s-1}\right] + L^{-1}\left[\frac{-2s}{s^2+1} + \frac{1}{s^2+1}\right]$$

$$= 2e^t - 2\cos t + \sin t$$

(37) $f(t) = L^{-1}\left[\frac{6s^3-21s^2+20s-7}{(s+1)(s-2)^3}\right]$

$$\frac{6s^3-21s^2+20s-7}{(s+1)(s-2)^3} = \frac{A}{s+1} + \frac{B}{(s-2)} + \frac{C}{(s-2)^2} + \frac{D}{(s-2)^3}$$

$$6s^3-21s^2+20s-7=A(s-2)^3+B(s+1)(s-2)^2$$
$$+C(s+1)(s-2)+D(s+1)$$

Put $s=-1$

$$-6-21-20-7=-27A$$
$$A=2$$

Put $s=2$

$$48-84+40-7=3D$$
$$88-91=3D \therefore D=-1$$

Put $s=0$

$$-7=-8A+4B-2C+D$$
$$-7=-16+4B-2C-1$$
$$4B-2C=10$$
$$2B-C=5 \quad \text{--- } (1)$$

Eq. s^3 co-eff

$$6=A+B \therefore B=4$$
$$C=2B-5=3$$

$$f(t)=L^{-1}\left[\frac{2}{s+1}+\frac{4}{s-2}+\frac{3}{(s-2)^2}-\frac{1}{(s-2)^3}\right]$$

(38) Let $s^2+2s = u$

$$\frac{u-4}{(u+5)(u+2)} = \frac{A}{u+5} + \frac{B}{u+2}$$

$$u-4 = A(u+2)+B(u+5)$$

Let $u=-2$, $B=-2$

Let $u=-5$, $A=3$

$$f(t) = L^{-1}\left[\frac{3}{s^2+2s+5} - \frac{2}{s^2+2s+2}\right]$$

$$= \frac{3}{2}L^{-1}\left[\frac{2}{(s+1)^2+4}\right] - 2L^{-1}\left[\frac{1}{(s+1)^2+1}\right]$$

$$= \frac{3}{2}e^{-t}\sin 2t - 2e^{-t}\,\cancel{\cos}\,t$$

sin

(39) $$\frac{1}{s(s+1)(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} + \frac{D}{s+3}$$

put $s=0$, $A=1/6$

$s=-1$, $B=-1/2$

$s = -2 \quad , \quad C = 1/2$

$s = -3 \quad , \quad D = -1/6$

$$f(t) = L^{-1}\left[\frac{1}{6s} - \frac{1}{2(s+1)} + \frac{1}{2(s+2)} - \frac{1}{6(s+3)}\right]$$

$$= \frac{1}{6} - \frac{e^{-t}}{2} + \frac{e^{-2t}}{2} - \frac{e^{-3t}}{6}$$

(40) $\frac{s^2+1}{s(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2}$

Put $s = 0 \quad , \quad A = 1/2$

$s = -1 \quad , \quad B = -2$

$s = -2 \quad , \quad C = 5/2$

$$f(t) = L^{-1}\left[\frac{1}{2s} - \frac{2}{s+1} + \frac{5}{2(s+2)}\right]$$

$$= \frac{1}{2} - 2e^{-t} + \frac{5}{2}e^{-2t}$$

41) $$\frac{s^2-3}{(s+2)(s-3)(s^2+2s+5)} = \frac{A}{s+2} + \frac{B}{s-3} + \frac{Cs+D}{s^2+2s+5}$$

$$s^2-3 = A(s-3)(s^2+2s+5) + B(s+2)(s^2+2s+5) + (Cs+D)(s+2)(s-3)$$

Put $s=-2$

$$1 = A(-5)(5) \quad \therefore A = -1/25$$

Put $s=+3$

$$6 = B(5)(20) \quad \therefore B = 3/50$$

Put $s=0$

$$-3 = -15A + 10B - 6D$$

$$-3 = \frac{\cancel{15}^{3}}{\cancel{25}_{5}} + \frac{3\cancel{0}}{5\cancel{0}} - 6D$$

$$-3 = \frac{6}{5} - 6D$$

$$\therefore 6D = \frac{6}{5} + 3 = \frac{21}{5}$$

$$\therefore D = \frac{\cancel{21}^{7}}{5 \times \cancel{6}_{2}} = \frac{7}{10}$$

Eq. co-eff of s^3

$$0 = A + B + C$$

$$0 = -\frac{1}{25} + \frac{3}{50} + C$$

$$0 = \frac{1}{50} + C \quad \therefore C = -\frac{1}{50}$$

$$f(t) = L^{-1}\left[-\frac{1}{25(s+2)} + \frac{3}{50(s-3)} - \frac{1}{50}\cdot\frac{s}{(s+1)^2+4} + \frac{7}{10[(s+1)^2+4]}\right]$$

$$= -\frac{e^{-2t}}{25} + \frac{3e^{3t}}{50} - \frac{1}{50}e^{-t}\cos 2t + \frac{7}{20}e^{-t}\sin 2t$$

TYPE IV :-

(62) Let $F_1(s) = \frac{1}{s^2}$ & $F_2(s) = \frac{1}{(s+1)^2}$

$f_1(t) = t$ & $f_2(t) = e^{-t}.t$

$$L^{-1}\left[\frac{1}{s^2(s+1)^2}\right] = \int_0^t u.e^{-(t-u)}(t-u)\,du$$

$$= e^{-t}\int_0^t (tu-u^2)e^u\,du$$

$$= e^{-t}\left[(tu-u^2)(e^u) - (t-2u)e^u + (-2)e^u\right]_0^t$$

$$= e^{-t}\left[-(-t)e^t - 2e^t + t + 2\right]$$

$$= t - 2 + te^{-t} + 2e^{-t}$$

(63) $F_1(s) = \frac{s}{s^2+a^2}$ & $F_2(s) = \frac{1}{s^2+a^2}$

$f_1(t) = \cos at$ & $f_2(t) = \frac{1}{a}\sin at$

$$L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{1}{a}\int_0^t \cos au \sin a(t-u)\,du$$

$$= \frac{1}{2a}\int_0^t 2\cos au \sin(at-au)\,du$$

$$= \frac{1}{2a}\int_0^t 2\sin(at-au)\cos au\,du$$

$$= \frac{1}{2a}\int_0^t \left[\sin at + \sin(at-2au)\right]du$$

$$= \frac{1}{2a}\left[\cancel{\frac{-\cos at}{a}}\; u\sin at - \frac{\cos(at-2au)}{-2a}\right]_0^t$$

$$= \frac{1}{2a}\left[t\sin at + 2a(\cancel{\cos at} - \cancel{\cos at})\right]$$

$$= \frac{t}{2a}\sin at.$$

(64) $\bar{f}_1(s) = \dfrac{1}{(s-2)^4}$ & $\bar{f}_2(s) = \dfrac{1}{s+3}$

$f_1(t) = e^{2t}\dfrac{t^3}{6}$ & $f_2(t) = e^{-3t}$

$$L^{-1}\left[\frac{1}{(s-2)^4(s+3)}\right] = \int_0^t e^{2u}\frac{u^3}{6}\cdot e^{-3(t-u)}\,du$$

$$= \frac{e^{-3t}}{6}\int_0^t e^{3u+2u}\cdot u^3\,du$$

$$= \frac{e^{-3t}}{6}\int_0^t u^3\cdot e^{5u}\,du$$

$$= \frac{e^{-3t}}{6}\left[u^3\left(\frac{e^{5u}}{5}\right) - 3u^2\left(\frac{e^{5u}}{25}\right) + 6u\left(\frac{e^{5u}}{125}\right) - 6\left(\frac{e^{5u}}{625}\right)\right]_0^t$$

$$= \frac{e^{-3t}}{6}\left[\frac{t^3e^{5t}}{5} - \frac{3t^2e^{5t}}{25} + \frac{6te^{5t}}{125} - \frac{6e^{5t}}{625} + \frac{6}{625}\right]$$

$$= \frac{t^3e^{2t}}{30} - \frac{3t^2e^{2t}}{150} + \frac{6te^{2t}}{\cancel{750}\,125}$$

$$- \frac{\cancel{6}e^{2t}}{} + \frac{e^{-3t}}{}$$

$$* \quad = \frac{e^{2t}}{6}\left[\frac{t^3}{5} - \frac{3t^2}{25} + \frac{6t}{125} - \frac{6}{625}\right] + \frac{e^{-3t}}{625}.$$

(65) $\bar{f}_1(s) = \frac{a}{s}$ & $\bar{f}_2(s) = \frac{1}{s-a}$

$f_1(t) = a$ & $f_2(u) = e^{at}$

$$L^{-1}\left[\frac{a}{s(s-a)}\right] = \int_0^t a\, e^{a(t-u)}\, du$$

$$= a e^{at} \int_0^t e^{-au}\, du$$

$$= a e^{at} \left[\frac{e^{-au}}{-a}\right]_0^t$$

$$= a e^{at}\left(-\frac{e^{-at}}{a} + \frac{1}{a}\right)$$

$$= (e^{at} - 1).$$

(66) $\bar{f}_1(s) = \frac{1}{s}$ & $\bar{f}_2(s) = \frac{1}{s^2+a^2}$

$f_1(t) = 1$ & $f_2(t) = \frac{1}{a}\sin at$

$$L^{-1}\left[\frac{1}{s(s^2+a^2)}\right] = \int_0^t \frac{1}{a}\sin au\,du$$

$$= \frac{1}{a}\left[\frac{-\cos au}{a}\right]_0^t$$

$$= \frac{1}{a^2}\left[1 - \cos at\right]$$

(67) $\bar{f}_1(s) = \frac{1}{s-2}$ & $\bar{f}_2(s) = \frac{1}{(s+2)^2}$

$f_1(t) = e^{2t}$ & $f_2(t) = e^{-2t}\cdot t$

$$L^{-1}\left[\frac{1}{(s-2)(s+2)^2}\right] = \int_0^t e^{2u} e^{-2(t-u)}(t-u)\,du$$

$$\int_{x_1}^{x_2} e^{at} \sin bt\, dt = \left[\frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt)\right]_{x_1}^{x_2}$$

$$= e^{-2t}\int_0^t (t-u)\, e^{4u}\, du$$

$$= e^{-2t}\left[(t-u)\left(\frac{e^{4u}}{4}\right) - (-1)\left(\frac{e^{4u}}{16}\right)\right]_0^t$$

$$= e^{-2t}\left[\frac{e^{4t}}{16} - \frac{t}{4} - \frac{1}{16}\right]$$

$$= \frac{1}{16}\left[e^{2t} - e^{-2t} - 4te^{-2t}\right]$$

68) $\bar{f}_1(s) = \frac{1}{s+1}$ & $\bar{f}_2(s) = \frac{1}{s^2+1}$

$f_1(t) = e^{-t}$ & $f_2(t) = \sin t$

$$L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right] = \int_0^t \sin u\, e^{-(t-u)}\, du$$

$$= e^{-t}\int_0^t e^{u} \sin u\, du$$

$$= e^{-t}\left[\frac{e^{u}}{}(\sin u - \cos u)\right]^t$$

$$= \frac{e^{-t}}{2}\left[e^{t}(\sin t - \cos t) + 1\right]$$

$$= \frac{1}{2}\left(\sin t - \cos t + e^{-t}\right)$$

69) $F_1(s) = \frac{s}{s^2+4}$ & $F_2(s) = \frac{s}{s^2+4}$

$f_1(t) = \cos 2t$ & $f_2(t) = \cos 2t$

$$L^{-1}\left[\frac{s^2}{(s^2+4)^2}\right] = \frac{1}{2}\int_0^t 2\cos 2u \cos(2t-2u)\,du$$

$$= \frac{1}{2}\int_0^t \left[\cos 2t + \cos(4u-2t)\right] du$$

$$\cancel{= \frac{1}{2}\left[\frac{\sin 2t}{2} + \sin\right.}$$

$$= \frac{1}{2}\left[u\cos 2t + \frac{\sin(4u-2t)}{4}\right]_0^t$$

$$= \frac{1}{2}\left[t\cos 2t + \cancel{\frac{\sin 2t}{4}} - \cancel{\frac{\sin(-2t)}{4}}\right]$$

$$= \frac{t}{2}\cos 2t$$

70) $\bar{f}_1(s) = \frac{1}{s^3}$ & $\bar{f}_2(s) = \frac{1}{s^2+1}$

$f_1(t) = \frac{t^2}{2}$ & $f_2(t) = \sin t$

$$L^{-1}\left[\frac{1}{s^3(s^2+1)}\right] = \int_0^t \frac{(t-u)^2}{2}\sin u\,du$$

$$= \frac{1}{2}\int_0^t (t-u)^2 \sin u\,du$$

$$= \frac{1}{2}\left[(t-u)^2(-\cos u) + 2(t-u)(-\sin u) - 2(\cos u)\right]_0^t$$

$$= \frac{1}{2}\left[-2\cos t + t^2 + 2\right]$$

$$= \frac{t^2}{2} - \cos t + 1$$

Type V.

(71) Let $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right] = f(t)$

$\therefore L[f(t)] = \log(s+a) - \log(s+b)$

$L[t f(t)] = (-1)^1 \frac{d}{ds}[\log(s+a) - \log(s+b)]$

$= -\frac{1}{s+a} + \frac{1}{s+b}$

$t f(t) = L^{-1}\left[\frac{1}{s+b} - \frac{1}{s+a}\right]$

$= e^{-bt} - e^{-at}$

$f(t) = \frac{e^{-bt} - e^{-at}}{t}$

(72). Let $L^{-1}[\cot^{-1}(s)] = f(t)$

$\therefore L[f(t)] = \cot^{-1} s$

$L[t f(t)] = (-1)^1 \frac{d}{ds} \cot^{-1} s$

$$= (-1)^1 \left[\frac{-1}{s^2+1}\right] = \frac{1}{s^2+1}$$

$$t f(t) = L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t.$$

$$f(t) = \frac{\sin t}{t}$$

Type VI

(73) Let $L^{-1}\left[\frac{2s+1}{(s^2+s+1)^2}\right] = f(t)$

$$\therefore L[f(t)] = \frac{2s+1}{(s^2+s+1)^2}$$

$$L\left[\frac{f(t)}{t}\right] = \int_s^\infty \frac{2s+1}{(s^2+s+1)^2}\, ds$$

Let $s^2+s+1 = u$ $\therefore (2s+1)\,ds = du$

s	s	∞
u	s^2+s+1	∞

$$= \int_{s^2+s+1}^{\infty} \frac{du}{u^2} = \left[-\frac{1}{u}\right]_{s^2+s+1}^{\infty}$$

$$= \frac{1}{s^2+s+1}$$

$$\therefore \frac{f(t)}{t} = L^{-1}\left[\frac{1}{s^2+s+1}\right]$$

$$= L^{-1}\left[\frac{1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

$$= \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

$$\therefore f(t) = \frac{2}{\sqrt{3}} t e^{-t/2} \sin\left(\frac{\sqrt{3}t}{2}\right)$$

(74) Let $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = f(t)$

$$L[f(t)] = \frac{s}{(s^2+a^2)^2}$$

$$L\left[\frac{f(t)}{t}\right] = \frac{1}{2}\int_s^{\infty} \frac{2s}{(s^2+a^2)^2}\, ds$$

$$= \frac{1}{2}\left[-\frac{1}{s^2+a^2}\right]_s^\infty$$

$$= \frac{1}{2}\left[\frac{1}{s^2+a^2}\right]$$

$$\frac{f(t)}{t} = \frac{1}{2a} L^{-1}\left[\frac{a}{s^2+a^2}\right]$$

$$= \frac{1}{2a} \sin at$$

$$f(t) = \frac{t}{2a} \sin at$$

Type VII

(75) Let $F(s) = \frac{s}{s^2+25}$

$$f(t) = L^{-1}\left[\frac{s}{s^2+25}\right] = \cos 5t$$

$$L^{-1}\left[e^{-\frac{4\pi s}{5}}\left(\frac{s}{s^2+25}\right)\right] = \cos 5\left(t-\frac{4\pi}{5}\right) \quad t \geq 4\pi/5$$

*

$$= 0, \quad t < 4\pi/5$$

(76) Let $\bar{f}(s) = \frac{1}{(s-2)^4}$

$f(t) = e^{2t}\frac{t^3}{6}$

$L^{-1}\left[e^{-3s}\left(\frac{1}{(s-2)^4}\right)\right] = e^{2(t-3)}\frac{(t-3)^3}{6}, t \geq 3$

$= 0 \quad , t < 3$

(77) $\bar{f}_1(s) = \frac{e^{3-2s}}{(s+4)^{5/2}} = \left[\frac{e^3}{(s+4)^{5/2}}\right] e^{-2s}$

$\bar{f}(s) = \frac{e^3}{(s+4)^{5/2}}$

$f(t) = e^3 \cdot e^{-4t} \cdot \frac{t^{\frac{5}{2}-1}}{\Gamma 5/2} = \frac{e^{3-4t}\, t^{3/2}}{\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}$

$= \frac{4e^{3-4t}\, t^{3/2}}{3\sqrt{\pi}}$

$$L^{-1}\left[e^{-2s}\left(\frac{e^{3}}{(s+4)^{5/2}}\right)\right]=\frac{4e^{3-4(t-2)}(t-2)^{3/2}}{3\sqrt{\pi}} \quad t\geq 2$$

$$=0 \quad , t<2$$

(78) $F(s)=\dfrac{8}{s^2+4}$

$f(t)=4\sin 2t$

$$L^{-1}\left[e^{-3s}\left(\frac{8}{s^2+4}\right)\right]=4\sin 2(t-3), \quad t\geq 3$$

$$=0, t<3$$

(79) $F(s)=\dfrac{1}{s^2+8s+25}=\dfrac{1}{(s+4)^2+9}$

$f(t)=\dfrac{e^{-4t}}{3}\sin 3t$

$$L^{-1}\left[\frac{e^{-2s}}{s^2+8s+25}\right]=\frac{e^{-4(t-2)}}{3}\sin 3(t-2); \quad t\geq 2$$

$$=0 \quad , t<2$$

80) $\bar{f}(s) = \dfrac{1}{(s+b)^{5/2}}$

$$f(t) = e^{-bt}\frac{t^{\frac{5}{2}-1}}{\Gamma\frac{5}{2}} = \frac{e^{-bt}\,t^{3/2}}{\frac{3}{2}\cdot\frac{1}{2}\sqrt{\pi}}$$

$$= \frac{4e^{-bt}\,t^{3/2}}{3\sqrt{\pi}}$$

$$L^{-1}\left[\frac{e^{-as}}{(s+b)^{5/2}}\right] = \frac{4e^{-b(t-a)}(t-a)^{3/2}}{3\sqrt{\pi}}, \; t \geq a$$

$$= 0 \quad , t < a.$$

81) $\bar{f}(s) = \dfrac{1-2\sqrt{s}+s}{s^4}$

$$f(t) = \frac{t^3}{6} - \frac{2t^{\frac{7}{2}-1}}{\Gamma\frac{7}{2}} + \frac{t^2}{2}$$

$$= \frac{t^3}{6} - \frac{2t^{5/2}}{\sqrt{\pi}\times\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}} + t^2/2$$

$$L^{-1}\left[e^{-s}\left(\frac{1-\sqrt{s}}{s^2}\right)^2\right] = \frac{(t-1)^3}{6} - \frac{16(t-1)^{5/2}}{15\sqrt{\pi}} + \frac{(t-1)^2}{2}$$

Type VIII :-

Pg 169 (82) & (Q83) Pg 179 184 last pg of bk. M15 (1) (2) (3)

Also give Pg (180) (Q2).

(Q82) Let $L[\sin at] = \dfrac{a}{s^2+a^2}$

$$L[t \sin at] = (-1)^1 \frac{d}{ds}\left[\frac{a}{s^2+a^2}\right]$$

$$= \frac{2as}{(s^2+a^2)^2}$$

$$L\left[\int_0^t t \sin at \, dt\right] = \frac{1}{s}\left[\frac{2as}{(s^2+a^2)^2}\right]$$

$$= \frac{2a}{(s^2+a^2)^2}.$$

$$\therefore \int_0^t t\sin at\,dt = L^{-1}\left[\frac{2a}{(s^2+a^2)^2}\right]$$

$$\therefore L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right] = \frac{1}{2a}\int_0^t t\sin at\,dt.$$

$$= \frac{1}{2a}\left[t\left(\frac{-\cos at}{a}\right) - 1\left(\frac{-\sin at}{a^2}\right)\right]_0^t$$

$$= \frac{1}{2a}\left[\frac{\sin at}{a^2} - \frac{t\cos at}{a}\right]$$

Q83) $L^{-1}\left[\frac{s^2}{(s^2+a^2)^2}\right] = f(t)$

$$L[f(t)] = \frac{s^2}{(s^2+a^2)^2}$$

$$L\left[\int_0^t f(t)\,dt\right] = \frac{1}{s}\left[\frac{s^2}{(s^2+a^2)^2}\right]$$

$$= \frac{s}{(s^2+a^2)^2}$$

$$\int_0^t f(t)\,dt = L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$$

** { from (Q 82)

$\therefore \int_0^t f(t)\,dt = \frac{1}{2a}[t \sin at]$

$f(t)\,dt = \frac{1}{2a} \frac{d}{dt}(t \sin at)$

$= \frac{1}{2a}[at \cos at + \sin at]$

Misc

Q1) Let $f_1(x) = x^{m-1}$

$f_2(x) = x^{n-1}$

$L[f_1(x)] = \frac{\overline{|m}}{s^m}$

$\bar{f}_1(s) = L[f_2(x)] = \frac{\overline{|n}}{s^n}$

$\bar{f}_2(s) = L[f_1(x)] \cdot L[f_2(x)] = \frac{\overline{|m}\,\overline{|n}}{s^{m+n}}$

By convolution,

$$\int_0^t f_1(x) f_2(t-x)\,dx = L^{-1}\left[\bar{f}_1(s)\bar{f}_2(s)\right]$$

$$\int_0^t x^{m-1}(t-x)^{n-1}\,dx = L^{-1}\left[\frac{\Gamma m\,\Gamma n}{s^{m+n}}\right]$$

$$= \Gamma m\,\Gamma n\left[\frac{t^{m+n-1}}{\Gamma m+n}\right]$$

Put $t=1$

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$

$$\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$$

(Q2) $\left[\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots\right]$

$$\therefore \tan^{-1}\left(\frac{a}{s}\right) = \frac{a}{s} - \frac{1}{3}\frac{a^3}{s^3} + \frac{1}{5}\frac{a^5}{s^5} \dots$$

$$L^{-1}\left[\tan^{-1}\left(\frac{a}{s}\right)\right] = L^{-1}\left[\frac{a}{s} - \frac{a^3}{3s^3} + \frac{a^5}{5s^5} \dots\right]$$

$$= a.1 - \frac{a^3}{3}\cdot\frac{t^2}{2!} + \frac{a^5}{5}\cdot\frac{t^4}{4!} - \dots$$

$$= \frac{1}{t}\left[at - \frac{a^3t^3}{3!} + \frac{a^5t^5}{5!} - \dots\right]$$

$$= \frac{1}{t}\sin(at).$$

(Q3) Hint behind.

Extra

(Q4) If $L[J_0(x)] = \frac{1}{\sqrt{s^2+1}}$ then

S.T. $\int_0^t J_0(x)\, J_0(t-x)\,dx = \sin t$

Let $\bar{f}_1(s) = \bar{f}_2(s) = L[J_0(x)] = \frac{1}{\sqrt{s^2+1}}$

$\cancel{L^{-1}[\bar{f}_1}$ $\bar{f}_1(s) \times \bar{f}_2(s) = \frac{1}{s^2+1}$

$L^{-1}\left[F_1(s)\,F_2(s)\right] = L^{-1}\left[\frac{1}{s^2+1}\right] = \sin t$

By convol. theo.

$\int_0^t J_0(x)\,J_0(t-x)\,dx = L^{-1}\left[F_1(s)\,F_2(s)\right]$

$= \sin t$