Navlakhi's

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AMIII / KUNAL NAVLAKHI

INVERSE LAPLACE TRANSFORM.

Formula:-

$L^{-1}\left[\frac{1}{s-a}\right]$	e^{at}
$L^{-1}\left[\frac{1}{s}\right]$	1
$L^{-1}\left[\frac{1}{s^{n+1}}\right]$	$\frac{t^n}{\lceil n+1}$
$L^{-1}\left[\frac{a}{s^2+a^2}\right]$	$\sin at$
$L^{-1}\left[\frac{s}{s^2+a^2}\right]$	$\cos at$
$L^{-1}\left[\frac{a}{s^2-a^2}\right]$	$\sinh at$
$L^{-1}\left[\frac{s}{s^2-a^2}\right]$	$\cosh at$
$L^{-1}\left[\bar{f}(s-a)\right]$	$e^{at}f(t)$
$L^{-1}\left[e^{-as}\bar{f}(s)\right]$	$F(t) = f(t-a)$; $t > a$ $= 0$; $t < a$
$L^{-1}\left[\bar{f}\left(\frac{s}{a}\right)\right]$	$af(at)$
$L^{-1}\left[s\bar{f}(s)\right]$	$f'(t)$; $f(0)=0$
$L^{-1}\left[(-1)^n \frac{d^n}{ds^n}\bar{f}(s)\right]$	$t^n f(t)$
$L^{-1}\left[\frac{d}{ds}\bar{f}(s)\right]$	$(-1)tf(t)$; when $n=1$
$L^{-1}\left[\frac{1}{s}\bar{f}(s)\right]$	$\int_0^t f(t)dt$
$L^{-1}\left[\int_s^\infty \bar{f}(s)ds\right]$	$\frac{1}{t}f(t)$
$L^{-1}\left[\bar{f}_1(s)\cdot\bar{f}_2(s)\right]$	$\int_0^t f_1(u)f_2(t-u)du$

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POWER

TYPE I :- Basic Formula :- AM III / KUNAL NAVLAKHI

(1) Find $L^{-1}\left[\frac{1}{s+4}\right]$ (2) Find $L^{-1}\left[\frac{2s+6}{s^2+4}\right]$

Find inverse Laplace Transform of the following :-

(3) $\frac{1}{2s-3}$ (4) $\frac{3s+5\sqrt{2}}{s^2+8}$ (5) $\frac{4s+15}{16s^2-25}$ (6) $\frac{3(s^2-1)^2}{2s^5}$ (7) $\frac{1}{s^{3/2}}$ (8) $\frac{s+1}{s^{4/3}}$

(9) $\left(\frac{1-\sqrt{s}}{s^2}\right)^2$ (10) $\frac{1}{s^2+9}$ (11) $\frac{5}{3s-1}$ (12) $\frac{6}{s^3}$ (13) $\frac{3}{s^4}$ (14) $\frac{7s}{s^2+4}$ (15) $\frac{4s}{s^2-16}$

(16) $\frac{3}{s^2-7}$ (17) $\frac{1}{s+4}$ (18) $\frac{2s+6}{s^2-4}$

TYPE II : Using Shifting Theorem :-

i.e. $L^{-1}[\bar{f}(s-a)] = e^{at} f(t)$ where $L^{-1}[\bar{f}(s)] = f(t)$

(19) $\frac{s+7}{s^2+2s+5}$ (20) $\frac{1}{(s-1)^5}$ (21) $\frac{1}{\sqrt{2s+3}}$ (22) $\frac{1}{(s+4)^{3/2}}$ (23) $\frac{4s+12}{s^2+8s+16}$

(24) $\frac{s+1}{s^2+s+1}$ (25) $\frac{3s+7}{s^2-2s-3}$ (26) $\frac{s-1}{s^2+2s}$ (27) $\frac{2}{(s-3)^5}$ (28) $\frac{3}{s^2-4s+13}$ (29) $\frac{2(s+1)}{s^2+2s+10}$

(30) $\frac{s}{(s-3)^5}$ (31) $\frac{1}{(s+2)^2}$ (32) $\frac{1}{(s^2+2s+2)}$ (33) $\frac{s+7}{s^2+2s+2}$ (34) $\frac{3s+1}{(s+1)^4}$

TYPE III :-

(35) $\frac{2s^2-6s+5}{s^3-6s^2+11s-6}$ (36) $\frac{3s+1}{(s-1)(s^2+1)}$ (37) $\frac{6s^3-21s^2+20s-7}{(s+1)(s-2)^3}$

(38) $\frac{s^2+2s-4}{(s^2+2s+5)(s^2+2s+2)}$ (39) $\frac{1}{s(s+1)(s+2)(s+3)}$ (40) $\frac{s^2+1}{s^3+3s^2+2s}$ (41) $\frac{s^2-3}{(s+2)(s-3)(s^2+2s+5)}$

(42) $\frac{s+29}{(s+4)(s^2+9)}$ (43) $\frac{2s^2-4}{(s+1)(s-2)(s-3)}$ (44) $\frac{s+2}{s^3(s-1)^2}$ (45) $\frac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}$

(46) $\frac{21s-33}{(s+1)(s-2)^3}$ (47) $\frac{s+2}{s^2(s+3)}$ (48) $\frac{1}{s^3+a^3}$ (49) $\frac{4s-5}{s^2-s-2}$ (50) $\frac{9s^2+4s-10}{s(s-1)(s+2)}$

(51) $\frac{3s^3+s^2+12s+2}{(s-3)(s+1)^3}$ (52) $\frac{5s^2+8s-1}{(s+3)(s^2+1)}$ (53) $\frac{7s+13}{s(s^2+4s+13)}$ (54) $\frac{s}{(s^2+1)(s^2+4)}$ (55) $\frac{11-3s}{s^2+2s-3}$

(56) $\frac{2s^2-9s-35}{(s+1)(s-2)(s+3)}$ (57) $\frac{2s+3}{(s-2)^2}$ (58) $\frac{5s^2-2s-19}{(s+3)(s-1)^2}$ (59) $\frac{3s^2+16s+15}{(s+3)^3}$ (60) $\frac{s^2+5s+13}{(s^2+2)(s+1)}$

(61) $\frac{26-s^2}{s(s^2+4s+13)}$

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TYPE IV :- Convolution Theorem :

$$L^{-1}\left[\bar{f}_1(s).\bar{f}_2(s)\right] = \int_0^t f_1(t) f_2(t-u)\,du$$

(62) $\frac{1}{s^2(s+1)^2}$ (63) $\frac{s}{(s^2+a^2)^2}$ (64) $\frac{1}{(s-2)^4(s+3)}$ (65) $\frac{a}{s(s-a)}$ (66) $\frac{1}{s(s^2+a^2)}$

(67) $\frac{1}{(s-2)(s+2)^2}$ (68) $\frac{1}{(s+1)(s^2+1)}$ (69) $\frac{s^2}{(s^2+4)^2}$ (70) $\frac{1}{s^3(s^2+1)}$

TYPE V :- $L^{-1}\left[\frac{d}{ds}\bar{f}(s)\right] = (-1)\,t\,f(t)$

(71) $\log\left(\frac{s+a}{s+b}\right)$ (72) $\cot^{-1}(s)$ (73)

TYPE VI :- $L^{-1}\left[\int_s^\infty \bar{f}(s)\,ds\right] = \frac{1}{t} L^{-1}\left[\bar{f}(s)\right]$

(73) $\frac{2s+1}{(s^2+s+1)^2}$ (74) $\frac{s}{(s^2+a^2)^2}$

TYPE VII :- $L^{-1}\left[e^{-as}\bar{f}(s)\right] = f(t-a)$; $t \geq a$
$= 0$; $t < a$

(75) $\frac{s\,e^{-\frac{4\pi s}{5}}}{s^2+25}$ (76) $\frac{e^{-3s}}{(s-2)^4}$ (77) $\frac{e^{3-2s}}{(s+4)^{5/2}}$ (78) $\frac{8e^{-3s}}{s^2+4}$

(79) $\frac{e^{-2s}}{s^2+8s+25}$ (80) $\frac{e^{-as}}{(s+b)^{5/2}}$ (81) $\left(\frac{1-\sqrt{s}}{s^2}\right)^2 \cdot e^{-s}$

TYPE VIII :- $L^{-1}\left[\frac{1}{s}\bar{f}(s)\right] = \int_0^t f(t)\,dt$

(82) $\frac{1}{(s^2+a^2)^2}$ (83) $\frac{s^2}{(s^2+a^2)^2}$

MISCELLANEOUS :-

(1) Prove that $\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma m+n}$ using convolution

(2) Using series expansion of $\tan^{-1}\left(\frac{a}{s}\right)$, show that
$L^{-1}\left[\tan^{-1}\left(\frac{a}{s}\right)\right] = \frac{\sin at}{t}$

(3) (i) $\frac{s+1}{(s^2+2s+2)^2}$ (ii) $\frac{1}{(s-2)^4(s+3)}$ (iii) $\frac{s+1}{s^2+s+1}$ (iv) $\frac{s}{s^4+s^2+1}$ (v) $\frac{(s+2)^2}{(s^2+4s+8)^2}$

ANSWERS

TYPE I :- (1) e^{-4t} (2) $2\cos 2t + 3\sin 2t$ (3) $\frac{e^{\frac{3t}{2}}}{2}$ (4) $3\cos(2\sqrt{2}t) + \frac{5}{2}\sin(2\sqrt{2}t)$

(5) $\frac{1}{4}\cosh\frac{5}{4}t + \frac{3}{4}\sinh\frac{5}{4}t$ (6) $\frac{3}{2} - \frac{3}{2}t^2 + \frac{t^4}{16}$ (7) $2\sqrt{\frac{t}{\pi}}$ (8) $\frac{1}{\Gamma\frac{1}{3}}\left(t^{-2/3} + 3t^{1/3}\right)$

(9) $\frac{t^3}{6} + \frac{t^2}{2} - \frac{16t^{5/2}}{15\sqrt{\pi}}$ ~~(10)~~ $\cancel{2e^{-4t}\sqrt{\frac{t}{\pi}}}$ (10) $\frac{1}{3}\sin 3t$ (11) $\frac{5}{3}\cdot e^{t/3}$ (12) $3t^2$ (13) $\frac{t^3}{2}$

(14) $7\cos 2t$ (15) $4\cosh 4t$ (16) $\frac{3}{\sqrt{7}}\sinh\sqrt{7}t$ (17) e^{-4t} (18) $2\cosh 2t + 3\sinh 2t$

TYPE II :- (19) $e^{-t}\cos 2t + 3e^{-t}\sin 2t$ (20) $\frac{e^{t}t^4}{24}$ (21) $\frac{e^{-\frac{3t}{2}}}{\sqrt{2\pi t}}$ (22) $2e^{-4t}\sqrt{\frac{t}{\pi}}$

(23) $4e^{-4t}(1-t)$ (24) $\frac{e^{-t/2}}{\sqrt{3}}\left(\sqrt{3}\cos\frac{\sqrt{3}t}{2} + \sin\frac{\sqrt{3}t}{2}\right)$ (25) $4e^{3t} - e^{-t}$

(26) $-\frac{1}{2} + \frac{3}{2}e^{-2t}$ (27) $\frac{e^{3t}\cdot t^4}{12}$ (28) $e^{2t}\sin 3t$ (29) $2e^{-t}\cos 3t$ (30) $e^{3t}\cdot t^3\left(\frac{1}{6} + \frac{t}{8}\right)$

(31) te^{-2t} (32) $e^{-t}\sin t$ (33) $e^{-t}(\cos t + 6\sin t)$ (34) $t^2e^{-t}\left(\frac{3}{2} - \frac{t}{3}\right)$

TYPE III :-

(35) $\frac{e^t}{2} - e^{2t} + \frac{5}{2}e^{3t}$ (36) $2e^t - 2\cos t + \sin t$ (37) $2e^{-t} + \left(4 + 3t - \frac{t^2}{2}\right)e^{2t}$

(38) $\frac{3}{2}e^{-t}\sin 2t - 2e^{-t}\sin t$ (39) $\frac{1}{6} - \frac{e^{-t}}{2} + \frac{e^{-2t}}{2} - \frac{e^{-3t}}{6}$ (40) $\frac{1}{2} - 2e^{-t} + \frac{5}{2}e^{-2t}$

(41) $\frac{3}{50}e^{3t} - \frac{1}{25}e^{-2t} - \frac{1}{50}e^{-t}(\cos 2t - 18\sin 2t)$ (42) $e^{-4t} - \cos 3t + \frac{5}{3}\sin 3t$

(43) $-\frac{e^{-t}}{6} - \frac{4}{3}e^{2t} + \frac{7}{2}e^{3t}$ (44) $(3t-8)e^t + t^2 + 5t + 8$ (45) $\frac{e^{-t}}{3}(\sin t + \sin 2t)$

(46) $2e^{-t} - 2e^{2t} + 6te^{2t} + \frac{3}{2}t^2e^{2t}$ (47) $\frac{2t}{3} + \frac{1}{9} - \frac{e^{-3t}}{9}$ (48) $\frac{1}{3a^2}\left\{e^{-at} - e^{\frac{at}{2}}\left(\cos\frac{\sqrt{3}at}{2} - \sqrt{3}\sin\frac{\sqrt{3}at}{2}\right)\right\}$ (49) $e^{2t} + 3e^{-t}$ (50) $5 + e^t + 3e^{-2t}$ (51) $2e^{3t} + e^{-t} - 4te^{-t} + \frac{3}{2}t^2e^{-t}$ (52) $2e^{-3t} + 3\cos t - \sin t$ (53) $1 - e^{-2t}\cos 3t + \frac{5}{3}e^{-2t}\sin 3t$

(54) $\frac{1}{3}(\cos t - \cos 2t)$ (55) $2e^t - 5e^{-3t}$ (56) $4e^{-t} - 3e^{2t} + e^{-3t}$ (57) $2e^{2t} + 7te^{2t}$

(58) $2e^{-3t} + 3e^t - 4e^t t$ (59) $e^{-3t}(3 - 2t - 3t^2)$ (60) $5e^{-t} + 2\cos\sqrt{2}t + \frac{3}{\sqrt{2}}\sin\sqrt{2}t$

(61) $2 - 3e^{-2t}\cos 3t - \frac{8}{3}e^{-2t}\sin 3t$

(62) $te^{-t}+2e^{-t}+t-2$ (63) $\frac{t}{2a}\sin at$ (64) $\frac{1}{6}\left(\frac{t^3}{5}-\frac{3t^2}{25}+\frac{6t}{125}-\frac{6}{625}\right)-\frac{e^{-5t}}{625}$ (65) $e^{at}-1$ (66) $\frac{1}{a^2}(1-\cos at)$ (67) $\frac{1}{16}(e^{2t}-e^{-2t}-4te^{-2t})$

(68) $\frac{1}{2}(\sin t-\cos t+e^{-t})$ (69) $\frac{t}{2}\cos 2t+\frac{1}{4}\sin 2t$ (70) $\frac{t^2}{2}+\cos t-1$

TYPE V: (71) $\frac{e^{-bt}-e^{-at}}{2}$ (72) $\frac{\sin t}{t}$ (73) $\frac{2}{\sqrt{3}}te^{-t/2}\sin\left(\frac{\sqrt{3}t}{2}\right)$

(74) $\frac{t}{2a}\sin at$ (75) $\cos 5\left(t-\frac{4\pi}{3}\right);\ t\geq\frac{4\pi}{3}=0;\ t<\frac{4\pi}{3}$

(76) $\frac{(t-3)^3}{6}\cdot e^{2(t-3)}$ (77) $e^3\left[\frac{4(t-2)^{3/2}\cdot e^{-4(t-2)}}{3\sqrt{\pi}}\right]$

(78) $4\sin 2(t-3)$ (79) $\frac{e^{-4(t-2)}}{3}\sin 3(t-2)$ (80) $\frac{4e^{-b(t-a)}}{3\sqrt{\pi}}\cdot(t-a)^{3/2}$

(81) $\frac{(t-1)^3}{6}+\frac{(t-1)^2}{2}-\frac{16}{15\sqrt{\pi}}(t-1)^{5/2}$

MISCELLANEOUS

3(ii) $\frac{e^{2t}}{30}\left[t^3-\frac{3t^2}{5}+\frac{6t}{25}-\frac{6}{125}+\frac{6}{125}e^{-5t}\right]$ (i) $\frac{e^{-t}}{2}t\sin t$

(iii) $e^{-t/2}\left[\cos\frac{\sqrt{3}}{2}t+\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right]$ (iv) $\frac{2}{\sqrt{3}}\sinh\frac{t}{2}\cdot\sin\frac{\sqrt{3}}{2}t$

(v) $\frac{te^{-2t}}{2}\cos 2t+\frac{e^{-2t}}{4}\sin 2t$