Relations & Digraph

Solutions

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Semester 3: Discrete Structures/Mathematics

RELATION & DIAGRAPHS
(SOLUTIONS)

Ex. 1. $A = \{1,2,3,4\}$ $B = \{a, b, c\}$

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c), (4,a), (4,b), (4,c)\}$

$B \times A = \{(a,1), (a,2), (a,3), (a,4), (b,1), (b,2), (b,3), (b,4), (c,1), (c,2), (c,3), (c,4)\}$

Ex 2. (x,y) signifies any point on the Real plane & all such (x,y) would signify the plane itself.

Ex 3. $S = \{m, f\}$
$E = \{e, h, j, g, p\}$

where m = male
f = female
e = elementary school
h = high school
j = junior college
g = graduate school
p = post graduate school.

$S \times E = \{(m,e), (m,h), (m,j), (m,g), (m,p), (f,e), (f,h), (f,j), (f,g), (f,p)\}$

Ex 4. $O = \{u, w, x\}$
$V = \{d, f\}$
$S = \{a, b\}$

When u = UNIX, w = Windows 98
x = Windows XP,
d = demo f = full version
a = software 1
b = software 2

$S \times V \times O = \{(a,d,u), (a,d,w), (a,d,x), (a,f,u), (a,f,w), (a,f,x), (b,d,u), (b,d,w), (b,d,x), (b,f,u), (b,f,w), (b,f,x)\}$

Ex 5. (a) $y = 4$
(b) $3x = 9$
$x = 3$

(c) $3y + 1 = 7$
$y = 2$
(d) $x = C{+}{+}$
$y = C$

Q.6 $A = \{a, b\}$ $B = \{1, 2, 3\}$

(a) $A \times B = \{(a,1), (a,2), (a,3),$
$(b,1), (b,2), (b,3)\}$

(b) $B \times A = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$

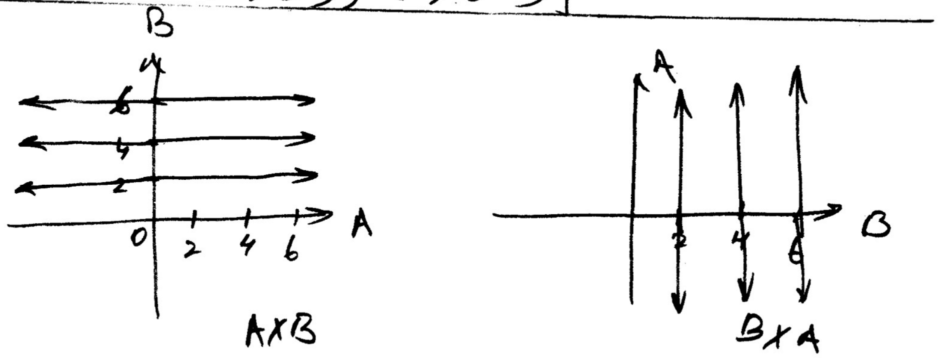
(c) $A \times A = \{(a,a), (a,b), (b,a), (b,b)\}$

(d) $B \times B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3),$
$(3,1), (3,2), (3,3)\}$

Q.7 $A = \{a, b\}$ $B = \{1, 2, 3\}$, $C = \{*, \#\}$

$A \times B \times C = \{(a,1,*), (a,1,\#),$
$(a,2,*), (a,2,\#),$
$(a,3,*), (a,3,\#),$
$(b,1,*), (b,1,\#),$
$(b,2,*), (b,2,\#),$
$(b,3,*), (b,3,\#)\}$

Ex 8.



Ex 9. Let $(x, y) \in A \times (B \cap C)$

$\therefore x \in A$ & $y \in B \cap C$

$\therefore x \in A$ & $y \in B$ & $y \in C$

$\therefore (x, y) \in A \times B$ $(x, y) \in A \times C$

$\therefore (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ — (1)

Ex 10. (a) Yes [Since all elements of A are there & there are no repeats]

(b) No [Since 4 repeats ie $A_1 \cap A_3 \neq \emptyset$]

(c) Yes

(d) No [Since all elements of A are not present ie $A_1 \cup A_2 \neq A$]

Ex 11.

$\{\{a\}, \{b\}, \{c\}, \{d\}\}$

$\{\{a, b\}, \{c\}, \{d\}\}$

$\{\{a, c\}, \{b\}, \{d\}\}$

$\{\{a, d\}, \{b\}, \{c\}\}$

$\{\{a\}, \{b, c\}, \{d\}\}$

$\{\{a\}, \{b, d\}, \{c\}\}$

$\{\{a\}, \{b\}, \{c, d\}\}$

$\{\{a, b\}, \{c, d\}\}$

$\{\{a, c\}, \{b, d\}\}$

$\{\{a, d\}, \{b, c\}\}$

$\{\{a, b, c\}, \{d\}\}$

$\{\{a, b, d\}, \{c\}\}$

$\{\{a, c, d\}, \{b\}\}$

$\{\{b, c, d\}, \{a\}\}$

$\{\{a, b, c, d\}\}$

Ex 12.

(a) $\{\{0, 6, 12, 18, \ldots\}, \{3, 9, 15, \ldots\}\}$

(b) $\{\{0, 3, 9, 15, 21, \ldots\}, \{6, 18, 30, \ldots\}, \{12, 24, 36, \ldots\}\}$

Ex 13. $R = \{\{1,2\}\{1,3\},\{1,4\},\{2,3\},\{2,4\}\{3,4\}\}$

Ex 14.

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$x^2 = 4\left(1 - \frac{y^2}{9}\right)$$

$$x = \sqrt{4 - \frac{4y^2}{9}}$$

$$\therefore \frac{4y^2}{9} \le 4$$

$$\therefore y^2 \le 9$$

$$\therefore -3 \le y \le 3$$

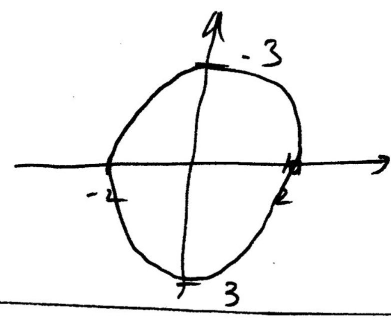
$$y^2 = 9\left(1 - \frac{x^2}{4}\right)$$

$$y = \sqrt{9 - \frac{9x^2}{4}}$$

$$\therefore \frac{9x^2}{4} \le 9$$

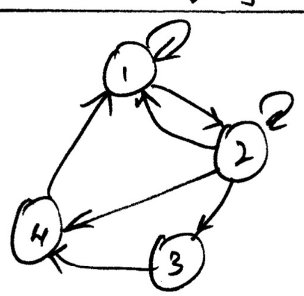
$$\therefore x^2 \le 4$$

$$\therefore -2 \le x \le 2$$



Ex 15. $R = \{(p,q), (p,r), (q,r), (q,s), (r,q), (r,s), (s,q)\}$

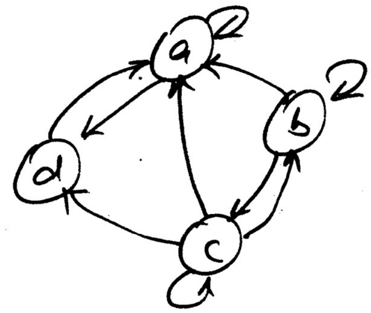
Ex 16.



Ex 17. $R = \{(a,a), (a,b), (a,c), (c,c), (d,b), (d,c)\}$

Ex.18

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



Vertex	a	b	c	d
Indegree	4	2	2	2
Outdegree	2	3	4	1

No. of 1's in column 'a'

No. of 1's in row 'a'

Ex 19.

$R = \{(1,4), (1,5), (4,1), (4,4), (4,5), (5,4), (5,5)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

Ex 20.

Domain $(R) = \{a, b, c\} \Longleftarrow$ x-coord

Range $(R) = \{1, 2, 3\} \Longleftarrow$ y-co-ord

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Ex 21.

$R = \{(1,1), (4,2), (9,3)\}$

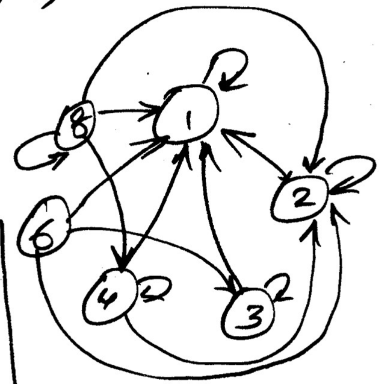
Domain $(R) = \{1, 4, 9\}$

Range $(R) = \{1, 2, 3\}$

Co-domain $(R) = \{1, 2, 3\} \Leftarrow$ set B

$$M_R = \begin{array}{c} \\ 1 \\ 4 \\ 6 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex 22. R = { (1,1), (2,1), (3,1), (4,1), (6,1), (8,1),
(2,2), (4,2), (6,2), (8,2),
(3,3), (6,3),
(4,4), (8,4), (6,6)
(8,8) }



$$M_R = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 6 & 8 \\ \hline 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 1 & 0 & 0 \\ 6 & 1 & 1 & 1 & 0 & 1 & 0 \\ 8 & 1 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Domain (R) = { 1, 2, 3, 4, 6, 8 }
Range (R) = { 1, 2, 3, 4, 6, 8 }

Vertex	1	2	3	4	6	8
In-degree	6	4	2	2	1	1
Out-degree	1	2	2	3	4	4

Ex 23. (a) $4 = 16^{1/2}$ $\quad 1/2 \notin Z$ $\quad \therefore (4,16) \notin R$

(b) $2 = 32^{1/4}$ $\quad 1/4 \notin Z$ $\quad \therefore (2,32) \notin R$

(c) $3 = 3^0$ $\quad 0 \in Z$ $\quad \therefore (3,3) \in R$

(d) $2 = 8^{1/3}$ $\quad 1/3 \notin Z$ $\quad \therefore (2,8) \notin R$

(e) $8 = 2^3$ $\quad 3 \in Z$ $\quad \therefore (8,2) \in R$

(f) $1 = 7^0$ $\quad 0 \in Z$ $\quad \therefore (1,7) \in R$

Ex 24. $a^2 + b^2 = 25$

$a^2 = 25 - b^2$

$a = \pm\sqrt{25 - b^2}$

$\therefore b^2 \le 25$

$\therefore -5 \le b \le 5$

$b^2 = 25 - a^2$

$b = \pm\sqrt{25 - a^2}$

$\therefore a^2 \le 25$

$\therefore -5 \le a \le 5$

Domain $(G) = \{a \mid -5 \le a \le 5 \wedge a \in R\}$

Range $(G) = \{b \mid -5 \le b \le 5 \wedge b \in R\}$

Ex 25. let $A = \{1, 2, 4\}$

$B = \{c, d\}$

$n(A) = 3$, $n(B) = 2$

$R = A \times B$

$\therefore n(R) = 3 \times 2 = 6$

$|R \times R| = 6 \times 6 = 36$ elements.

No. of relations on $R = 2^{|R \times R|}$

$= 2^{36}$

Ex 26. $R \cap (B \times B) = \{(a,a), (a,c), (b,c)\}$

Ex 27. $R(a) = \{a, c\} \Leftarrow$ all y's where $x = a$,

$R(b) = \{a, c\}$

$R(c) = \{a\}$

$R(A_1) = R(\{b, c\}) = R(b) \cup R(c)$

$= \{a, c\}$

all y's where $x = b$ or $x = c$

Ex 28 $R = \{ \ldots\ldots, (-1,0),(0,0), \ldots\ldots, (-1,1),(0,1),(1,1), \ldots\ldots \}$

$$R(A_1) = R(\{0,1,2\}) = R(0) \cup R(1) \cup R(2)$$
$$= \{\ldots\ldots, -1, 0, 1, 2\}$$
$$= \{y \mid y \le 2 \wedge y \in \mathbb{Z}\}$$

$$R(A_2) = R(\{7,8\}) = R(7) \cup R(8)$$
$$= \{\ldots\ldots, 5, 6, 7, 8\}$$
$$= \{y \mid y \le 8 \wedge y \in \mathbb{Z}\}$$

$$R(A_1 \cup A_2) = R(\{0,1,2,7,8\})$$
$$= \{\ldots\ldots, 5, 6, 7, 8\}$$
$$= \{y \mid y \le 8 \wedge y \in \mathbb{Z}\}$$

$$R(A_1 \cap A_2) = R(\{\}) = \cancel{\emptyset} = \{\}$$

Ex 29 $R(A_1 \cap A_2) = R(2) = \{a, b, d\}$

Ex 30. No. of diff. relations $= 2^{|A \times B|} = 2^{mn}$

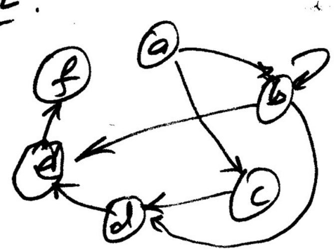
Ex 31. (i) $\pi : 1,2,3$ length $= 2$

(ii) $\pi : 2,4,1,2$ length $= 3$

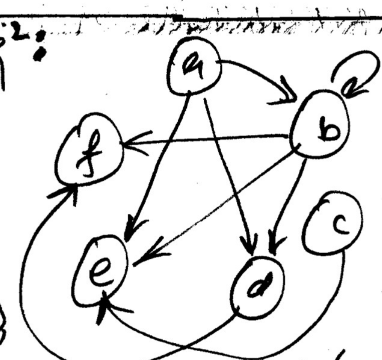
(iii) $\pi : 3,3$ length $= 1$

Ex 32.

R:



R^2:



$R^\infty = \{(a,b), (a,c), (a,d), (a,e), (a,f), (b,b), (b,d), (b,e), (b,f), (c,d), (c,e), (c,f), (d,e), (e,f)\}$

Ex 33. (1)
$\pi : 1,2$
$\pi : 1,3$
$\pi : 2,2$
$\pi : 2,3$
$\pi : 3,4$
$\pi : 3,5$
$\pi : 5,1$

(2)
$\pi : 1,2,2$
$\pi : 1,2,3$
$\pi : 1,3,4$
$\pi : 1,3,5$
$\pi : 2,2,2$
$\pi : 2,2,3$
$\pi : 2,3,4$
$\pi : 2,3,5$
$\pi : 3,5,1$
$\pi : 5,1,2$
$\pi : 5,1,3$

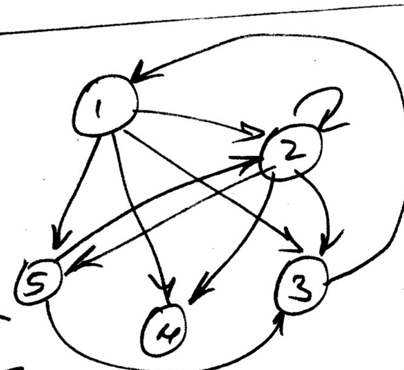
(3)
$\pi : 2,2,2$
$\pi : 2,2,3$
$\pi : 2,3,4$
$\pi : 2,3,5$

(4)
$\pi : 1,2,2,2$
$\pi : 1,2,2,3$
$\pi : 1,2,3,4$
$\pi : 1,2,3,5$
$\pi : 1,3,5,1$
$\pi : 1,3,5,4$
$\pi : 2,2,2,2$
$\pi : 2,2,2,3$
$\pi : 2,2,3,4$
$\pi : 2,2,3,5$
$\pi : 2,3,5,4$
$\pi : 2,3,5,1$
$\pi : 3,5,1,3$
$\pi : 3,5,1,2$
$\pi : 5,1,3,5$
$\pi : 5,1,2,2$
$\pi : 5,1,2,3$
$\pi : 5,1,3,4$

(5)
$\pi : 3,5,1,3$
$\pi : 3,5,1,2$

(6) $\pi : 1,3,5,1$

(7)



(8) $M_R^2 = M_R \odot M_R$

$$= \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \odot \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(9) $R^{\infty} = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(2,1),(2,2),(2,3),(2,4),(2,5),$
$(3,1),(3,2),(3,3),(3,4),(3,5),$
$(5,1),(5,2),(5,3),(5,4),(5,5)\}$

(10)
$$M_{R^{\infty}} = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Ex 34. (i) π_1: a, b, c
(ii) π_2: c, e, f, b, d

$\pi_2 \circ \pi_1$ = a, b, c, e, f, b, d

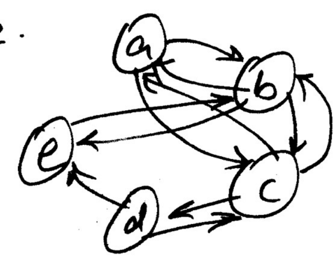
2nd 1st

Ex 35. (a) Not symmetric
Not asymmetric
Not antisymmetric

(b) Symmetric
Not asymmetric
Not antisymmetric

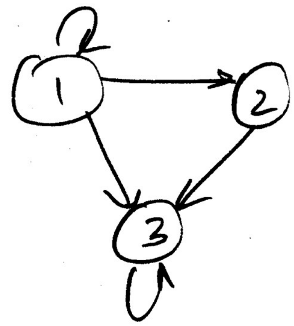
(c) Not symmetric
Asymmetric
Antisymmetric

Ex 36.



Symmetric
Not asymmetric
Not antisymmetric

Ex 37.



Yes it is transitive

[Better Technique is to use Warshall's algo].

Ex 38. Reflexive

Let $(a, a) \in R$

$\therefore a \le a + 2$

false

$\therefore (a, a) \notin R$

$\therefore$ not Reflexive

Symmetric

Let $(x, y) \in R$

$\therefore x \le y + 2$

$\therefore y \ge x - 2$

$\therefore$ its possible $y \nleq x + 2$

$\therefore$ Not symmetric.

Asymmetric

Let $(x, y) \in R$

$\therefore x \le y + 2$

$\therefore y \ge x - 2$ $\therefore y + 4 \ge x + 2$

it is possible $y \le x + 2$ $\therefore (y, x) \in R$ for some x & y

$\therefore$ Not asymmetric.

Antisymmetric. Let $(x, y) \in R$ & $x \ne y$

$\therefore x \le y + 2$ $\therefore y \ge x - 2$

$\therefore y + 4 \ge x + 2$ $\therefore (y, x) \in R$

$\therefore$ Its possible $x \ne y$ & $y \le x + 2$ for some x & y $\therefore$ Not Antisym.

Ex 39. Reflexive

Let $(a,a) \in R$

$\therefore$ $a+a$ is even ie $2a$ is even

Which is true for all a.

$\therefore$ Reflexive

Symmetric

Let $(x,y) \in R$ $\therefore$ $x+y$ is even

$\therefore$ $y+x$ even $\therefore$ $(y,x) \in R$

$\therefore$ Symmetric.

Asymmetric

$\because$ Reflexive $\therefore$ Not asymmetric

Antisymmetric

Let $(x,y) \in R$ & $x \neq y$

$\therefore$ $x+y$ is even

$\therefore$ $y+x$ is even

$\therefore$ $(y,x) \in R$

$\therefore$ Antisymmetric.

Ex 40.

Note R is like $\{ \dots ((a,b),(c,d)), \dots \}$

Reflexive :

Let $((a,b),(a,b)) \in R$

$\therefore$ $ab = ba$ which is true

$\therefore$ Reflexive

Symmetric

Let $((a,b),(c,d)) \in R$

$\therefore$ $ad = bc$ $\therefore$ $cb = da$

$\therefore \quad cb = da$

$\therefore ((c,d),(a,b)) \in R$

$\therefore$ Symmetric.

Asymmetric :

$\because$ Reflexive $\therefore$ Not asymmetric

Antisymmetric

Let $((a,b),(c,d)) \in R$ when $(a,b) \neq (c,d)$

$\therefore ad = bc$

$\therefore cb = da$

$\therefore ((c,d),(a,b)) \in R$

$\therefore$ Antisymmetric

Ex 41. $\because$ Transitive

$\therefore (a,b) \in R, (b,c) \in R \Rightarrow (a,c) \in R$

$\because$ Irreflexive $\therefore (a,a) \notin R$ for each a.

If $(a,b) \in R$ & $(b,a) \in R$ then

$\because$ transitive $\therefore (a,a) \in R$ but

$\because$ Irreflexive $\therefore (a,a) \notin R$

$\therefore (a,b) \in R \Rightarrow (b,a) \notin R$.

Ex 42. $A/R = \{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),$
$(c,a),(c,b),(c,c),(d,d)\}$

Ex 44

$R(1) = [1] = \{1, 2\}$

$R(3) = [3] = \{3, 4\}$

$A/R = \{\{1,2\}, \{3, 4\}\}$

Ex 45. Reflexive

Let $((a,b), (a,b)) \in R$ $\therefore \frac{a}{b} = \frac{a}{b}$

Which is always true

$\therefore$ Reflexive.

Symmetric

Let $((a,b), (c,d)) \in R$

$\frac{a}{b} = \frac{c}{d}$

$\therefore \frac{c}{d} = \frac{a}{b}$

$\therefore ((c,d), (a,b)) \in R$

$\therefore$ Symmetric

Transitive

Let $((a,b), (c,d)) \in R$ & $((c,d), (e,f)) \in R$

$\therefore \frac{a}{b} = \frac{c}{d}$ & $\frac{c}{d} = \frac{e}{f}$

$\therefore \frac{c}{d} = \frac{a}{b}$ & $\frac{c}{d} = \frac{e}{f}$

$\therefore \frac{a}{b} = \frac{e}{f}$

$\therefore ((a,b), (e,f)) \in R$

$\therefore$ R is transitive

$\therefore$ R is a equivalence Relation

$R = \{((1,2),(1,2)), ((1,3),(1,3)), ((1,4),(1,4)), ((1,1),(1,1)), ((1,5),(1,5)), ((1,1),(2,2)), ((1,2),(2,4)), \ldots\}$

$[(1,1)] = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$

$[(1,2)] = \{(1,2), (2,4)\}$

$[(1,3)] = \{(1,3)\}$

$[(1,4)] = \{(1,4)\}$

$[(1,5)] = \{(1,5)\}$

$[(2,1)] = \{(2,1), (4,2)\}$

$[(2,3)] = \{(2,3)\}$

$[(2,5)] = \{(2,5)\}$

$[(3,1)] = \{(3,1)\}$

$[(3,2)] = \{(3,2)\}$

$[(3,4)] = \{(3,4)\}$

$[(3,5)] = \{(3,5)\}$

$[(4,1)] = \{(4,1)\}$

$[(4,3)] = \{(4,3)\}$

$[(4,5)] = \{(4,5)\}$

$[(5,1)] = \{(5,1)\}$

$[(5,2)] = \{(5,2)\}$

$[(5,3)] = \{(5,3)\}$

$[(5,4)] = \{(5,4)\}$

Ex 46

(a) $\bar{R} = \{(1,b),(2,b),(3,a),(3,c),(4,a)\}$

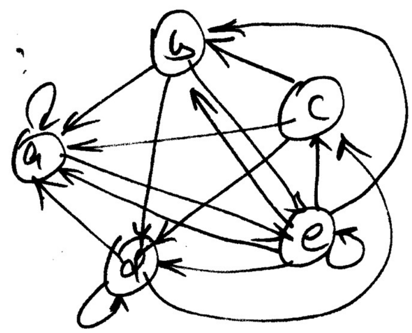
(b) $R \cap S = \{(2,c),(3,b),(4,b),(4,c)\}$

(c) $R \cup S = \{(1,a),(1,b),(1,c),(2,a),(2,c),$
$(3,a),(3,b),(4,b),(4,c)\}$

(d) $R^{-1} = \{(a,1),(b,1),(a,2),(c,2),(b,3),$
$(b,4),(c,4)\}$

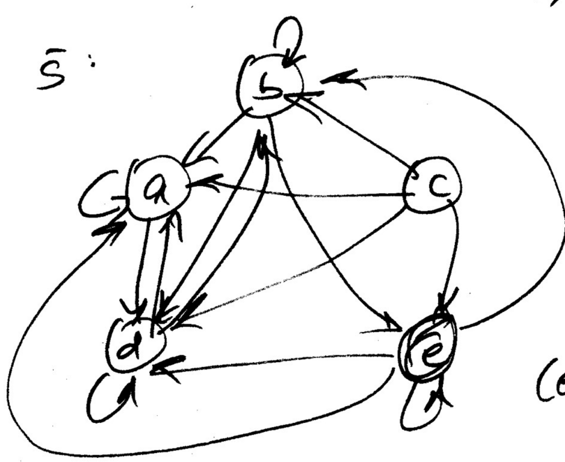
Ex 47.

$\bar{R}$:



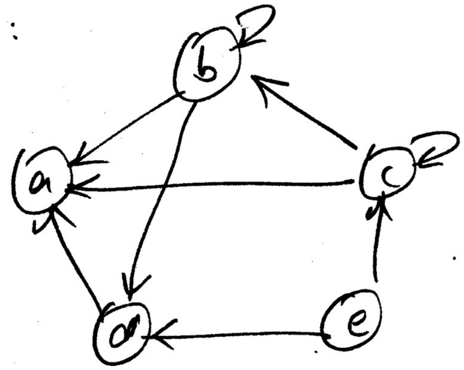
$\bar{R} = \{(a,a),(a,e),(b,a),(b,d),(b,e),$
$(c,a),(c,b),(c,d),(d,a),(d,c),(d,d),$
$(e,a),(e,b),(e,c),(e,d),(e,e)\}$

$\bar{S}$:



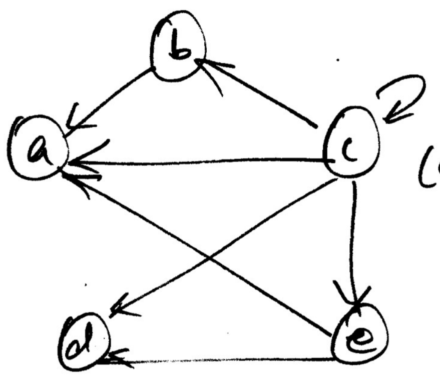
$\bar{S} = \{(a,a),(a,d),$
$(b,a),(b,b),(b,d),$
$(b,e),(c,a),(c,b),$
$(c,d),(c,e),(d,a),$
$(d,b),(d,d),(e,a),$
$(e,b),(e,d),(e,e)\}$

R^{-1}:



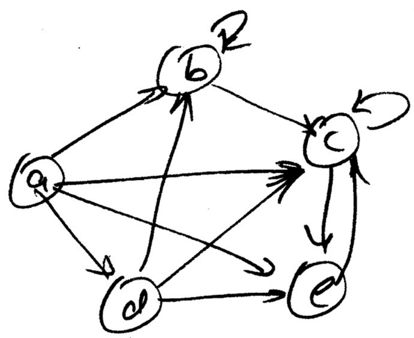
$R^{-1} = \{(b,b), (b,a), (b,d), (c,a), (c,b), (c,c), (d,a), (e,d), (e,c)\}$

S^{-1}:



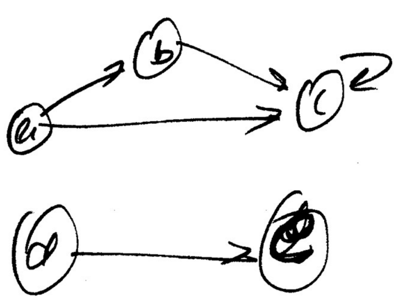
$S^{-1} = \{(b,a), (c,b), (c,c), (c,a), (c,d), (c,e), (e,a), (e,d)\}$

$R \cup S$:



$R \cup S = \{(a,b), (a,c), (a,d), (a,e), (b,b), (b,c), (c,c), (c,e), (d,b), (d,c), (d,e), (e,c)\}$

$R \cap S$:



$R \cap S = \{(a,b), (a,c), (b,c), (c,c), (d,e)\}$

Ex 48: $\therefore R \cap S = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

Note: R & S are equivalence then $R \cap S$ is also equivalence & $R \cup S$ may not be equivalence.

$[1] = \{1,2\}$ $[4] = \{4\}$ $[6] = \{6\}$

$[3] = \{3\}$ $[5] = \{5\}$

$\therefore A/R = \{\{1,2\}, \{3\}, \{4\}, \{5\}, \{6\}\}$

Ex 49. Ref. closure $= \{(1,1), (1,2), (2,2), (2,3), (1,3), (3,3), (3,4), (4,4)\}$

Symmetric closure $= \{(1,2), (2,1), (2,3), (3,2), (1,3), (3,1), (3,4), (4,3)\}$

Ex 50. $S \circ R = \{(a,d), (a,c), (a,a), (b,a), (c,c)\}$

$(a,a) \in R$ & $(a,d) \in S$

Ex 51. $M_{S \circ R} = M_R \circ M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \circ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$S \circ R = \{(1,1), (1,3), (2,1), (2,2), (2,3), (3,2), (3,3)\}$

Ex 52. Let $(x,y) \in (S \cup T) \circ R$

$\therefore (x,t) \in R$ & $(t,y) \in S \cup T$

$\therefore (x,t) \in R$ & $(t,y) \in S$ or $(t,y) \in T$

$\therefore (x,y) \in S \circ R$ or $(x,y) \in T \circ R$

$\therefore (x,y) \in (S \circ R) \cup (T \circ R)$

$\therefore (S \cup T) \circ R \subseteq (S \circ R) \cup (T \circ R)$ — ①

Now let $(x,y) \in (S \circ R) \cup (T \circ R)$

$\therefore (x,y) \in S \circ R$ or $(x,y) \in T \circ R$

$\therefore (x,t) \in R$ & $(t,y) \in S$ or $(x,t) \in R$ & $(t,y) \in T$

$\therefore (x,t) \in R$ & $(t,y) \in S$ or $(t,y) \in T$

$\therefore (x,t) \in R$ & $(t,y) \in S \cup T$

$\therefore (x,y) \in (S \cup T) \circ R$ —

$\therefore (S \circ R) \cup (T \circ R) \subseteq (S \cup T) \circ R$ ②

From ① & ②

$(S \cup T) \circ R = (S \circ R) \cup (T \circ R)$

Example 54:

$$W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

row = 2
col = 2

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

row = 1, 2
col = 1, 2, 3

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

row = 1, 2
col = 4

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

row = 1, 2, 3
col = -

$$W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$\therefore$ Transitive closure = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4)}

NOTE: If ans = quest then the quest was already transitive. This can be used to check if quest is transitive or not.

Ex 55. $M_{R \cup S} = M_R \vee M_S$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$\boxed{\text{Step 1}}$: Make it reflexive by ORing with unity matrix.

$\boxed{\text{Step 2}}$: Make symmetric by ORing with the transpose

$\boxed{\text{Step 3}}$: Making it transitive by Warshall algo.

Here $M_{R \cup S}$ is already Reflexive & Symmetric $\therefore$ Step 3 is only to be done.

$$W_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array}$$

row = 1, 2
col = 1, 2

$$W_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array}$$

row = 1, 2
col = 1, 2

$$W_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 & 1 & 0 \\ 4 & 0 & 0 & 1 & 1 & 1 \\ 5 & 0 & 0 & 0 & 1 & 1 \end{array}$$

rows = 3, 4
col = 3, 4

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

row = 3, 4, 5
col = 3, 4, 5

$$W_4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

row = 3, 4, 5
col = 3, 4, 5

$$W_5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$\therefore$ The final equivalence relation is

$= \{(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (3,5), (4,3), (4,4), (4,5), (5,3), (5,4), (5,5)\}$

$[1] = \{1, 2\}$

$[3] = \{3, 4, 5\}$

$A/R = \{\{1,2\}, \{3,4,5\}\}$