IMPORTANT PRACTISE QUESTIONS

Navlakhi's

Tel:23868356/23886023/9820246760 POWER

INVERSE LAPLACE TRANSFORMS

(Q1) $L^{-1}\left[\dfrac{s+2}{s^2+4s+7}\right]$ (Q2) $L^{-1}\left[\dfrac{2s+3}{s^2+2s+2}\right]$ (Q3) $L^{-1}\left[\dfrac{s}{(2s+1)^2}\right]$

(Q4) $L^{-1}\left[\dfrac{2s^2-1}{(s^2+1)(s^2+4)}\right]$ (Q5) $L^{-1}\left[\dfrac{s^2}{(s+a)^3}\right]$ (Q6) $L^{-1}\left[\dfrac{1}{s^2(s+1)}\right]$

(Q7) $L^{-1}\left[\dfrac{s^2+2}{s(s^2+4)}\right]$ (Q8) $L^{-1}\left[\dfrac{1}{s(s+1)^3}\right]$ (Q9) $L^{-1}\left[\dfrac{5s^2-15s-11}{(s+1)(s-2)^2}\right]$

(Q10) $L^{-1}\left[\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}\right]$ (Q11) $L^{-1}\left[\dfrac{s+4}{(s+1)(s^2-1)}\right]$ (Q12) $L^{-1}\left[\dfrac{1}{s^3(s-1)}\right]$

(Q13) $L^{-1}\left[\dfrac{s}{(s+1)^2(s^2+1)}\right]$ (Q14) $L^{-1}\left[\dfrac{1}{s^3(s^2+1)}\right]$ ~~(Q15)~~

(Q15) Find inverse of following using convolution theorem for (Q15) (i) –

(i) $\dfrac{s^2}{(s^2-a^2)^2}$ (ii) $\dfrac{s^2}{(s^2+1)(s^2+4)}$ (iii) $\dfrac{(s+2)^2}{(s^2+4s+8)^2}$ (iv) $\dfrac{1}{(s+3)(s^2+2s+2)}$

(v) $\dfrac{1}{(s^2+4s+13)^2}$ (vi) $\dfrac{s}{(s^2+1)(s^2+4)}$ (vii) $\dfrac{1}{s\sqrt{s+4}}$

(viii) $\dfrac{s^2+s}{(s^2+1)(s^2+2s+2)}$ (ix) $\dfrac{s+3}{(s^2+6s+13)^2}$ (x) $\dfrac{s}{(s^2-a^2)^2}$ (xi) $\dfrac{1}{(s^2+1)^3}$

(xii) $\dfrac{s+2}{s^2(s-1)^2}$ (xiii) $\dfrac{s+29}{(s+4)(s^2+9)}$

Q16) Using convolution theorem, prove that

(i) $L^{-1}\left[\dfrac{1}{s}\log\left(a+\dfrac{b}{s^2}\right)\right] = \displaystyle\int_0^t \frac{2}{u}\left[1-\cos\left(\frac{b}{a}\right)u\right]du$

(ii) $L^{-1}\left[\dfrac{1}{s}\log\left(\dfrac{s+a}{s+b}\right)\right] = \displaystyle\int_0^t \left(\frac{e^{-bu}-e^{-au}}{u}\right)du$

Find inverse of following:-

(Q17) $L^{-1}\left[2\tanh^{-1}s\right]$ (Q18) $L^{-1}\left[\log\sqrt{1+\frac{a^2}{s^2}}\right]$ (Q19) $L^{-1}\left[\log\left(\frac{s^2+a^2}{\sqrt{s+b}}\right)\right]$

(Q20) $L^{-1}\left[\log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right]$ (Q21) $L^{-1}\left[\tan^{-1}\left(\frac{2}{s^2}\right)\right]$ (Q22) $L^{-1}\left[\tan^{-1}\left(\frac{2}{s}\right)\right]$

(Q23) $L^{-1}\left[\tan^{-1}\left(\frac{s+a}{b}\right)\right]$ (Q24) $L^{-1}\left[\log\left(\frac{s^2-4}{(s-3)^2}\right)\right]$ (Q25) $\log\sqrt{1-\frac{a^2}{s^2}}$

(Q26) $L^{-1}\left[\log\left(1+\frac{a}{s}\right)\right]$ (Q27) $L^{-1}\left[\tan^{-1}\left(\frac{s}{2}\right)\right]$ (Q28) $L^{-1}\left[\frac{e^{-s}(s+1)}{s^2+s+1}\right]$

(Q29) $L^{-1}\left[\frac{e^{-4s}}{\sqrt{2s+7}}\right]$ (Q30) $L^{-1}\left[\frac{s e^{-2s}}{s^2+2s+2}\right]$ (Q31) $L^{-1}\left[\frac{e^{-\pi s/2}+e^{-3\pi s/2}}{s^2+1}\right]$

(Q32) $L^{-1}\left[\frac{1}{s}\cos\frac{1}{s}\right]$ (Q33) $L^{-1}\left[\frac{1}{s+e^{-s}}\right]$ (Q34) $L^{-1}\left[\frac{2s}{s^4+4}\right]$

(Q35) Solve $\frac{dx}{dt}+y=\sin t$ and $\frac{dy}{dt}+x=\cos t$ where $x=0,\ y=2$ at $t=0$

(Q36) Solve $(D^2-3D+2)y=4e^{2t}$ with $y(0)=-3$ and $y'(0)=5$

(Q37) Solve $\frac{d^2y}{dt^2}+4y=f(t)$ with conditions $y(0)=0$ $y'(0)=1$ and $f(t) = 1$ when $0<t<1$
$= 0$ when $t>1$

ANSWERS

(1) $e^{-2t}\cos\sqrt{3}t$ (2) $e^{-t}(2\cos t+\sin t)$ (3) $\frac{e^{-t/2}(t-4)}{16}$

(4) $-\sin t+\frac{3}{2}\sin 2t$ (5) $e^{-at}(1-2at+\frac{1}{2}a^2t^2)$ (6) $t-1+e^{-t}$

(7) $\cos^2 t$ (8) $1-e^{-t}(1+t+t^2/2)$ (9) $e^{-t}+4e^{2t}-7te^{2t}$

(10) $e^{-t}/3\,[\sin t+\sin 2t]$ (11) $\frac{5}{4}e^{t}-\frac{5}{4}e^{-t}+\frac{3}{2}te^{-t}$ (12) $1-t+\frac{t^2}{2}-e^{-t}$

(13) $1/2\,[\sin t-te^{-t}]$ (14) $\cos t-1+t^2/2$ (15) i) $1/2\,[\sinh at+at\cosh at]$

(ii) $1/3\,[2\sin 2t-\sin t]$

(iii) Use FST & then convol (iii) $e^{-2t}/4\,[\sin 2t+2t\cos 2t]$

(iv) FST & then convol $\frac{1}{5}[e^{-3t}+e^{-t}(2\sin t-\cos t)]$

(v) $e^{-2t}/18\,[(\sin 3t)/3-t\cos 3t]$ (vi) $1/3\,[\cos t-\cos 2t]$ (vii) $\frac{1}{2}\,\mathrm{erf}(2\sqrt{t})$

(viii) $1/10\,[2\sin(e^{-t}+1)-6\cos t(e^{-t}-1)]$ (ix) $\frac{1}{4}e^{-3t}t\sin 2t$

(x) $\frac{1}{2a}[at\cosh at+\sinh at]$ (xi) $1/8\,[(3-t^2)\sin t-3t\cos t]$

(xii) $e^{t}(3t-5)+2t+5$ (xiii) $e^{-4t}+5/3\sin 3t-\cos 3t$

(17) $(e^{t}-e^{-t})/t$ (18) $(1-\cos at)/t$ (19) $\frac{1}{t}\left[\frac{e^{bt}}{2}-2\cos at\right]$ (20) $\frac{\cos bt-\cos at}{t/2}$

(21) $2\sin t\sinh t$ (22) $(\sin 2t)/t$ (23) $(e^{-at}\sin bt)/t$ (24) $\frac{2}{t}(e^{3t}-\cosh 2t)$

(25) $\frac{1-\cosh at}{t}$ (26) $\frac{1-e^{-at}}{t}$ (27) $-\frac{\sin 2t}{t}$ (28) $e^{-(t-1)/2}$ *

$\left[\cos\left(\frac{\sqrt{3}(t-1)}{2}\right)+\frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}(t-1)}{2}\right)\right]$ (29) $\frac{e^{-7(t-4)/2}}{\sqrt{2\pi(t-4)}}$

(30) $e^{-(t-2)}[\cos(t-2)-\sin(t-2)]$ (31) $\sin(t-\pi/2)+\sin(t-3\pi/2)$

(32) $\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{[(2n)!]^2}$ (33) $\sum_{n=0}^{\infty}\frac{(-1)^n(t-n)^n}{n!}$ (34) $\sin t\sinh t$

(35) $x=-2\sinh t$, $y=\sin t+2\cosh t$ (36) $y=-7e^{t}+4e^{2t}+4te^{2t}$

(37) $y=\frac{1}{2}\sin 2t+\frac{1-\cos 2t}{4}-\frac{1-\cos(t-1)}{4}$