NAVLAKHI'S

Education @ its BEST

FOURIER SERIES

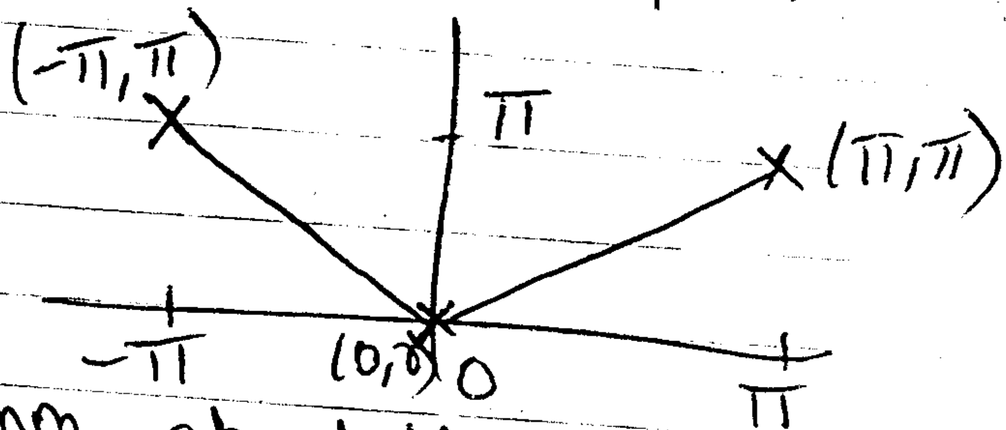
A function can be even or odd if 0 is the mid-pt of given interval.

If $f(x)$ is defined in $(-l, l)$ the 0 is the mid-pt. & if
$f(-x) = f(x)$ then $f(x)$ is even
$f(-x) = -f(x)$ " " " odd

$f(x) = x^2$ in $(0, 3)$ is neither even nor odd $\because$ 0 is not the mid-pt. of interval.
$f(x) = x^2$ in $(-3, 3)$ $\because$ 0 is mid-pt.
$f(-x) = (-x)^2 = x^2$
$f(-x) = f(x)$ then $f(x)$ is even funct.

$$f(x) = -x \quad ; \quad -\pi < x < 0$$
$$= x \quad ; \quad 0 < x < \pi$$

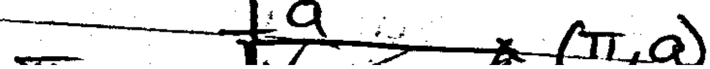
When $x = -\pi$; $f(x) = +\pi$; $(-\pi$
$x = 0$; $f(x) = 0$; $(0,$
$x = 0$; $f(x) = 0$; $(0,$
$x = \pi$; $f(x) = \pi$; $(\pi,$



Symm. about y-axis ∴ Even func

) $f(x) = -a$; $-\pi < x < 0$
$= a$; $0 < x < \pi$

When $x = -\pi$; $f(x) = -a$; $(-\pi, -a$
$x = 0$; $f(x) = -a$; $(0, -a$
$x = 0$; $f(x) = a$; $(0, a)$
$x = \pi$; $f(x) = a$; (π, a)



$\therefore$ Symm. about origin i.e. Odd funct.

) Zero is mid-pt. of given interval

a) $f(x) = x^4$

$f(-x) = x^4$

$\therefore f(-x) = f(x) \therefore$ funct. is even

b) $f(x) = \tan 3x$

$f(-x) = -\tan 3x$

$f(-x) = -f(x) \therefore$ funct. is odd

) $f(t) = 2e^{3t}$

$f(-t) = 2e^{-3t}$

Neither even nor odd.

) $f(x) = \sin^2 x$

$f(-x) = \sin^2 x$

$\because f(-x) = f(x)$ $\therefore$ funct. is even.

a) $f(\theta) = \theta$; $-\pi < \theta < 0$

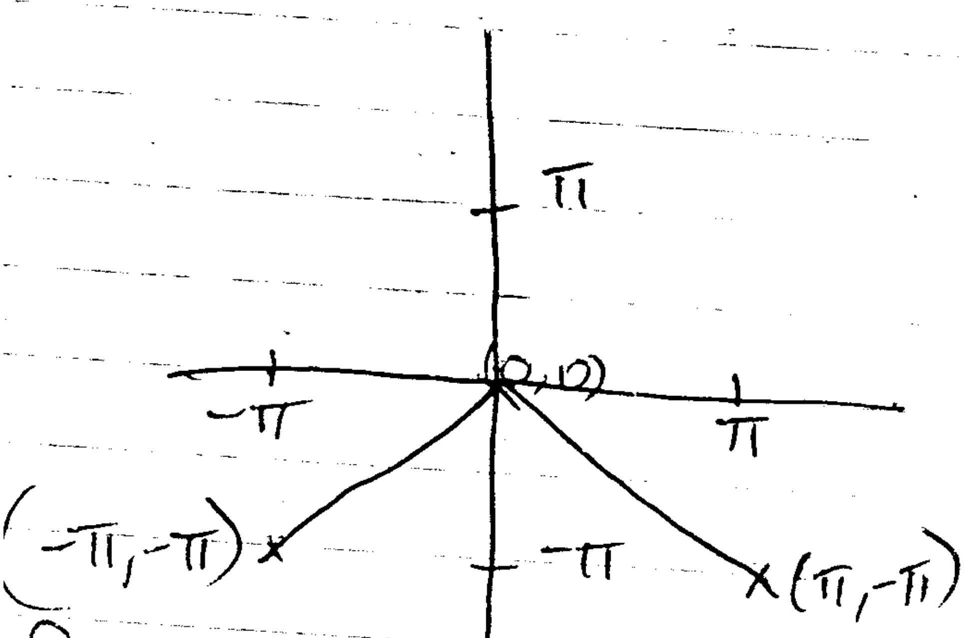
$= -\theta$; $0 < \theta < \pi$

When $\theta = -\pi$; $f(\theta) = -\pi$; $(-\pi, -\pi)$

$\theta = 0$; $f(\theta) = 0$; $(0, 0)$

$\theta = 0$; $f(\theta) = 0$; $(0, 0)$

$\theta = \pi$; $f(\theta) = -\pi$; $(\pi, -\pi)$

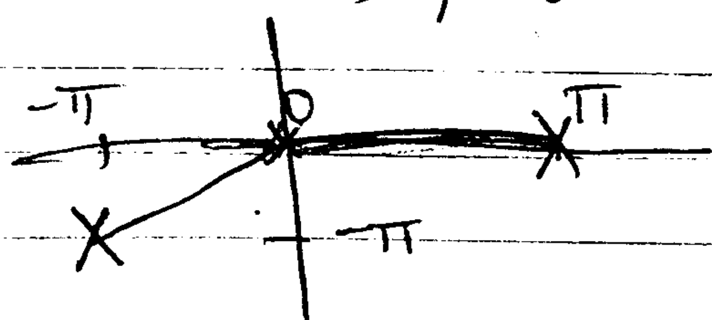


Symm. about y-axis

$\therefore$ Even

b) $f(x) = x \quad ; -\pi < x < 0$
$\quad = 0 \quad ; 0 < x < \pi$

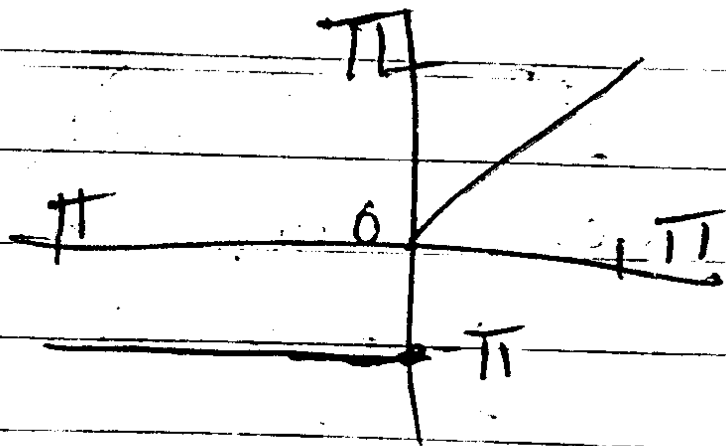
When $x = -\pi$, $f(x) = -\pi$; $(-\pi, -\pi)$
$x = 0$, $f(x) = 0$; $(0, 0)$
$x = 0$, $f(x) = 0$; $(0, 0)$
$x = \pi$; $f(x) = 0$, $(\pi, 0)$



Neither even nor odd.

Ɛ:- If a graph of funct. has no jumps or breaks it is continuous eg. sine, cosine wave

If $f(x) = -\pi \quad , -\pi < x < 0$
$\quad = x \quad , 0 < x < \pi$



For discont. funct. where a jump occurs from $-\pi$ to 0 then

$$f(0) = \frac{f(0^-) + f(0^+)}{2} \quad \{\text{By Dirichlet Cond}$$

$$= \frac{-\pi + 0}{2} = \frac{-\pi}{2}$$

$\cos 0 = 1$, $\sin 0 = 0$

$\cos n\pi = (-1)^n$, $\sin n\pi = 0$

$\cos 2n\pi = 1$, $\sin 2n\pi = 0$

$\cos \pi/2 = 0$, $\sin \pi/2 = 1$

Q2) $f(x) = e^{kx} \quad (0, 2\pi)$

$f(x)$ is defined in $(0, 2\pi)$ f.S. is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx \quad \text{——— (I)}$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{kx}\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{kx}}{k}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi k} - e^{0}}{k}\right]$$

$$= \frac{1}{\pi k}\left(e^{2\pi k} - 1\right)$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{kx}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{kx}}{k^2+n^2}(k\cos nx + n\sin nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi k}}{k^2+n^2}(k\cos 2\pi n) - \frac{e^0}{k^2+n^2}(k\cos 0)\right]$$

$$= \frac{1}{\pi(k^2+n^2)}\left[ke^{2\pi k} - k\right]$$

$$= \frac{k}{\pi(k^2+n^2)}\left[e^{2\pi k} - 1\right]$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} e^{kx}\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{e^{kx}}{k^2+n^2}(k\sin nx - n\cos nx)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{2\pi k}}{k^2+n^2}(-n\cos 2\pi n) - \frac{e^0}{k^2+n^2}(-n\cos 0)\right]$$

$$= \frac{n}{\pi(k^2+n^2)}\left[-e^{2\pi k} + 1\right] = \frac{-n}{\pi(k^2+n^2)}\left(e^{2\pi k} - 1\right)$$

Subs. in eq (I) we get,

$$f(x) = \frac{1}{2\pi k}\left(e^{2\pi k} - 1\right) + \sum_{n=1}^{\infty}\left[\frac{k}{\pi(k^2+n^2)}\left(e^{2\pi k} - 1\right)\cos nx - \frac{n}{\pi(k^2+n^2)}\left(e^{2\pi k} - 1\right)\sin nx\right]$$

$$= \frac{\left(e^{2\pi k} - 1\right)}{\pi}\left[\frac{1}{2k} + \sum_{n=1}^{\infty}\frac{k\cos nx - n\sin nx}{k^2+n^2}\right]$$

3) $f(x) = \cosh ax$ in $(-\pi, \pi)$

$$= \frac{e^{ax} + e^{-ax}}{2}$$

$$f(-x) = \frac{e^{-ax} + e^{ax}}{2} = f(x)$$

$\because$ 0 is midpt. of given int & $f(-x) = f(x)$ $\therefore$ $f(x)$ is Even

$\therefore$ F.S. is given by

Subs. in eq (I) we get,

$$f(x) = \frac{1}{2\pi k}(e^{2\pi k}-1) + \sum_{n=1}^{\infty}\left[\frac{k}{\pi(k^2+n^2)}(e^{2\pi k}-1)\cos nx - \frac{n}{\pi(k^2+n^2)}(e^{2\pi k}-1)\sin nx\right]$$

$$= \frac{(e^{2\pi k}-1)}{\pi}\left[\frac{1}{2k} + \sum_{n=1}^{\infty}\frac{k\cos nx - n\sin nx}{k^2+n^2}\right]$$

3) $f(x) = \cosh ax$ in $(-\pi, \pi)$

$$= \frac{e^{ax}+e^{-ax}}{2}$$

$$f(-x) = \frac{e^{-ax}+e^{ax}}{2} = f(x)$$

$\because$ 0 is midpt. of given int & $f(-x) = f(x)$ $\therefore$ $f(x)$ is Even

$\therefore$ F.S. is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \longrightarrow (I)$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cosh ax\,dx$$

$$= \frac{2}{\pi}\left[\frac{\sinh ax}{a}\right]_0^{\pi}$$

$$= \frac{2}{\pi a}\left[\sinh a\pi - \sinh a0\right]$$

$$= \frac{2}{\pi a}\left[\sinh a\pi\right]$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x) \cos nx\,dx$$

$$= \frac{\cancel{2}}{\pi}\int_0^{\pi}\left(\frac{e^{ax} + e^{-ax}}{\cancel{2}}\right) \cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} e^{ax}\cos nx + \frac{1}{\pi}\int_{0}^{\pi} e^{-ax}\cos nx\,dx$$

$$\frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\cos nx + n\sin nx)\right]_{-\pi}^{0} +$$

$$\frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}(-a\cos nx + n\sin nx)\right]_{0}^{\pi}$$

$$\frac{1}{\pi}\left[\frac{e^{0}}{a^2+n^2}(a\cos 0) - \frac{e^{-a\pi}}{a^2+n^2}(a\cos n\pi)\right]$$

$$+\frac{1}{\pi}\left[\frac{e^{-a\pi}}{a^2+n^2}(-a\cos n\pi) - \frac{e^{0}}{a^2+n^2}(-a\cos 0)\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{a\pi}}{a^2+n^2}(-1)^n a - \cancel{\frac{a}{a^2+n^2}} - \frac{e^{-a\pi}}{a^2+n^2}(-1)^n a + \cancel{\frac{a}{a^2+n^2}}\right]$$

$$= \frac{(-1)^n a}{(a^2+n^2)\pi}\left[\frac{e^{a\pi} - e^{-a\pi}}{2}\right]\times 2$$

$$= \frac{2a(-1)^n}{\pi(a^2+n^2)}(\sinh a\pi)$$

Subs. in eq. (I)

$$f(x) = \frac{1}{\pi a}\sinh a\pi + \sum_{n=1}^{\infty}\frac{2a(-1)^n\sinh a\pi}{\pi(a^2+n^2)}\cos nx$$

$$= \frac{\sinh a\pi}{\pi}\left[\frac{1}{a} + \sum_{n=1}^{\infty}\frac{2a(-1)^n\cos nx}{a^2+n^2}\right]$$

Q4) $f(x) = \sinh ax$ in $(-\pi, \pi)$

$$= \frac{e^{ax}-e^{-ax}}{2}$$

$$f(-x) = \frac{e^{-ax}-e^{ax}}{2} = -\left(\frac{e^{ax}-e^{-ax}}{2}\right)$$

$$= -f(x)$$

$\because$ 0 is midpt of given int & $f(-x) = -f(x)$ $\therefore$ $f(x)$ is odd.

f.S is given by

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{—(I)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{\cancel{2}}{\pi}\int_0^{\pi}\left(\frac{e^{ax}-e^{-ax}}{\cancel{2}}\right)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} e^{ax}\sin nx\,dx - \int_0^{\pi} e^{-ax}\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}(a\sin nx - n\cos nx) - \frac{e^{-ax}}{a^2+n^2}(-a\sin nx - n\cos nx)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{e^{a\pi}}{a^2+n^2}(-n\cos n\pi) + \frac{e^{-a\pi}}{a^2+n^2}(n\cos n\pi) - \frac{e^0}{a^2+n^2}(-n\cos 0) + \frac{e^0}{a^2+n^2}(-n\cos 0)\right]$$

$$= \frac{1}{\pi}\left[\frac{e^{a\pi}}{a^2+n^2}(-1)^n(-n) + \frac{e^{-a\pi}}{a^2+n^2}(-1)^n(n) + \cancel{\frac{n}{a^2+n^2}} - \cancel{\frac{n}{a^2+n^2}}\right]$$

$$= \frac{2n(-1)^n}{\pi(a^2+n^2)}\left[\frac{e^{-a\pi} - e^{a\pi}}{2}\right] = \frac{-2n(-1)^n}{\pi(a^2+n^2)}\left[\sinh a\pi\right]$$

$$= \frac{2n(-1)^{n+1} \sinh a\pi}{\pi(a^2+n^2)}$$

Subs. in eq (I)

$$f(x) = \sum_{n=1}^{\infty} \frac{2n(-1)^{n+1}}{\pi(a^2+n^2)} \sinh a\pi \sin nx$$

$$= \frac{2\sinh a\pi}{\pi} \sum_{n=1}^{\infty} \frac{n(-1)^{n+1} \sin nx}{a^2+n^2}$$

Q5) $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}$ in $(-\pi, \pi)$

$$f(-x) = \frac{\pi^2}{12} - \frac{(-x)^2}{4} = f(x)$$

$\because$ O is midpt. of given int. &

$f(-x) = f(x)$ $\therefore$ $f(x)$ is even

F.S. is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\left(\frac{\pi^2}{12} - \frac{x^2}{4}\right)dx$$

$$= \frac{2}{\pi}\left[\frac{\pi^2 x}{12} - \frac{x^3}{12}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{\pi^3}{12} - \frac{\pi^3}{12}\right] = 0$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\underbrace{\left(\frac{\pi^2}{12} - \frac{x^2}{4}\right)}_{u}\underbrace{\cos nx}_{v}\,dx$$

$$\frac{2}{\pi}\left[\left(\frac{\pi^2}{12} - \frac{x^2}{4}\right)\left(\frac{\sin nx}{n}\right) - \left(\frac{-2x}{4}\right)\left(-\frac{\cos nx}{n^2}\right) + \left(\frac{-1}{2}\right)\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\left(\frac{\pi^2}{12}-\frac{x^2}{4}\right)\left(\frac{\sin nx}{n}\right) - \frac{x\cos nx}{2n^2} + \frac{\sin nx}{2n^3}\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[0 - \frac{\pi\cos n\pi}{2n^2} + 0 - 0 + \frac{0\cos 0}{2n^2} - 0\right]$$

$$= \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{-\pi(-1)^n}{\cancel{2}n^2}\right] = \frac{(-1)^{n+1}}{n^2}$$

Subs. in (I)

$$f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\cos nx$$

$$\frac{\pi^2}{12} - \frac{x^2}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}\cos nx$$

Put $x=0$ on both sides,

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$$

$$\frac{\pi^2}{12} = \frac{(-1)^2}{1^2} + \frac{(-1)^3}{2^2} + \frac{(-1)^4}{3^2} + \cdots$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$$

) $f(x) = x + \frac{x^2}{4}$ in $(-\pi, \pi)$

Let $f(x) = f_1(x) + f_2(x)$ — (I)

$f_1(x) = x$

$f_1(-x) = -x = -f_1(x)$

$\therefore$ 0 is midpt. of given int & $f_1(-x) = -f_1(x) \therefore f_1(x)$ is odd

F.S. is given by

$$f_1(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{— (II)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f_1(x) \sin nx \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \underset{u}{x} \, \underset{v}{\sin nx} \, dx$$

$$= \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - (1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] = \frac{-2(-1)^n}{n}$$

$$= \frac{2(-1)^{n+1}}{n} \quad \text{Subs. in (II)}$$

$$f_1(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$= 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$$

$$f_2(x) = \frac{x^2}{4}$$

$$f_2(-x) = \frac{x^2}{4} = f_2(x)$$

$\therefore$ 0 is mid-pt of give int. & $f_2(-x) = f_2(x)$ $\therefore$ $f_2(x)$ is even

F.S. is given by

$$f_2(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{—— (III)}$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f_2(x)\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{x^2}{4_2}\, dx = \frac{1}{2\pi}\left[\frac{x^3}{3}\right]_0^{\pi}$$

$$= \frac{\pi^3}{6\pi} = \frac{\pi^2}{6}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f_2(x)\cos nx\, dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \frac{x^2}{4} \cdot \cos nx\, dx$$

$$= \frac{1}{2\pi}\int_0^{\pi} \underline{\underline{x^2}}_{u}\ \underline{\underline{\cos nx}}_{v}\, dx$$

$$= \frac{1}{2\pi}\left[x^2\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_0^\pi$$

$$= \frac{1}{2\pi}\left[\frac{2\pi\cos n\pi}{n^2}\right] = \frac{(-1)^n}{n^2}$$

Subs. in (II)

$$f_2(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx$$

Subs. in (I)

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nx + 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}\sin nx$$

7) $f(x) = x\sin x$ in $(0, 2\pi)$

{0 is not midpt ∴ neither even nor odd}

F.S. is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$\longrightarrow$ CF

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} \underset{u}{x}\,\underset{v}{\sin x}\,dx$$

$$\frac{1}{\pi}\left[x(-\cos x) - (1)(-\sin x)\right]_0^{2\pi}$$

$$\frac{1}{\pi}\left[-x\cos x + \cancel{\sin x}\right]_0^{2\pi}$$

$$\frac{1}{\cancel{\pi}}\left[-2\pi\cos 2\pi\right] = -2$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin x\cos nx\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\,(2\cos nx \sin x)\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x\left[\sin(nx+x) - \sin(nx-x)\right]dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} \underset{u}{x}\,\underset{v}{\sin(n+1)x}\,dx - \int_0^{2\pi} \underset{u}{x}\,\underset{v}{\sin(n-1)x}\,dx\right]$$

$$= \frac{1}{2\pi}\left\{\left[x\left(\frac{\cos(n+1)x}{-(n+1)}\right) - (1)\left(\frac{\sin(n+1)x}{-(n+1)^2}\right)\right]_0^{2\pi}\right.$$

$$\left. - \left[x\left(\frac{\cos(n-1)x}{-(n-1)}\right) - (1)\left(\frac{\sin(n-1)x}{-(n-1)^2}\right)\right]_0^{2\pi}\right\}$$

$$\frac{1}{2\pi}\left\{\left[-\frac{x\cos(n+1)x}{(n+1)} + \cancel{\frac{\sin(n+1)x}{(n+1)^2}}\right]_0^{2\pi}\right.$$

$$\left. - \left[-\frac{x\cos(n-1)x}{(n-1)} + \cancel{\frac{\sin(n-1)x}{(n-1)^2}}\right]_0^{2\pi}\right\}$$

$$\frac{1}{2\pi}\left\{-\frac{2\pi\cos(n+1)2\pi}{(n+1)} + \frac{2\pi\cos(n-1)2\pi}{n-1}\right\}$$

$$= \frac{\cancel{2\pi}}{\cancel{2\pi}}\left[\frac{-\cos(2\pi n+2\pi)}{(n+1)} + \frac{\cos(2\pi n-2\pi)}{(n-1)}\right]$$

$$= \left[\frac{-(\cos 2\pi n \cos 2\pi - \sin 2\pi n \sin 2\pi)}{(n+1)} + \frac{(\cos 2\pi n \cos 2\pi + \sin 2\pi n \sin 2\pi)}{(n-1)}\right]$$

$$= \left[\frac{-1}{(n+1)} + \frac{1}{n-1}\right]$$

$$= \left[\frac{-1}{n+1} + \frac{1}{n-1}\right]$$

$$= \left[\frac{-\cancel{n}+1+\cancel{n}+1}{n^2-1}\right]$$

$$a_n = \frac{+2}{n^2-1}$$

a_n is not defined for $n^2-1=0$

$\therefore n=1$

$$a_1 = \frac{1}{2\pi}\int_0^{2\pi} 2x \sin x \cos x \, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{x}{u}\left(\frac{\sin 2x}{v}\right) dx$$

$$= \frac{1}{2\pi}\left[x\left(-\frac{\cos 2x}{2}\right) - (1)\left(-\frac{\sin 2x}{4}\right)\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-\frac{x\cos 2x}{2} + \frac{\sin 2x}{4}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi}\left[-\frac{2\pi\cos 4\pi}{2}\right] = -\frac{1}{2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin nx\, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x(\sin x \sin nx)\, dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} x(2\sin nx \sin x)\, dx$$

$$= \frac{1}{2\pi}\left[\int_0^{2\pi} x\cos(nx-x)\, dx - \int_0^{2\pi} x\cos(nx+x)\, dx\right]$$

$$= \frac{1}{2\pi}\left\{\int_0^{2\pi} x\cos(n-1)x\,dx - \int_0^{2\pi} x\cos(n+1)x\,dx\right.$$

$$= \frac{1}{2\pi}\left\{\left[x\left(\frac{\sin(n-1)x}{(n-1)}\right) - \left(-\frac{\cos(n-1)x}{(n-1)^2}\right)\right]_0^{2\pi}\right.$$

$$\left. - \left[x\left(\frac{\sin(n+1)x}{(n+1)}\right) - \left(-\frac{\cos(n+1)x}{(n+1)^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{x\sin(n-1)x}{(n-1)} + \frac{\cos(n-1)x}{(n-1)^2}\right]_0^{2\pi}\right.$$

$$\left. - \left[\frac{x\sin(n+1)x}{(n+1)} + \frac{\cos(n+1)x}{(n+1)^2}\right]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{\cos(n-1)2\pi}{(n-1)^2} - \frac{1}{(n-1)^2} - \frac{\cos(n+1)2\pi}{(n+1)^2} + \frac{1}{(n+1)^2}\right.$$

$$= \frac{1}{2\pi}\left\{\frac{\cos(2\pi n-2\pi)}{(n-1)^2} - \frac{\cos(2\pi n+2\pi)}{(n+1)^2} - \frac{1}{(n-1)^2} + \frac{1}{(n+1)^2}\right\}$$

$$\left\{\begin{aligned}\cos(2\pi n-2\pi) &= \cos 2\pi n\cos 2\pi + \sin 2\pi n\sin 2\pi \\ &= 1 \\ \cos(2\pi n+2\pi) &= 1\end{aligned}\right\}$$

$$b_n = \frac{1}{2\pi}\left\{\frac{1}{(n-1)^2} - \frac{1}{(n+1)^2} - \frac{1}{(n-1)^2} + \frac{1}{(n+1)^2}\right\}$$

$$= \frac{0}{(n^2-1)^2} = 0$$

b_n is not def. for $n = 1$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} x\sin x \sin x\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin^2 x\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \frac{1}{2\pi}\left\{\int_0^{2\pi} x\,dx - \int_0^{2\pi} x\cos 2x\,dx\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{x^2}{2}\right]_0^{2\pi} - \left[(x)\left(\frac{\sin 2x}{2}\right) - (1)\left(\frac{-\cos 2x}{4}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{4\pi^2}{2} - \left[\frac{\cos 4\pi}{4} - \frac{\cos 0}{4}\right]\right\}$$

$$= \frac{1}{2\pi}\left\{2\pi^2 - \left[\frac{\pi}{4} - \frac{\pi}{4}\right]\right\}$$

$$= \pi$$

From eq. (I)

$$f(x) = \frac{a_0}{2} + a_1\cos x + \sum_{n=2}^{\infty} a_n \cos nx + b_1 \sin x + \sum_{n=2}^{\infty} b_n \sin nx$$

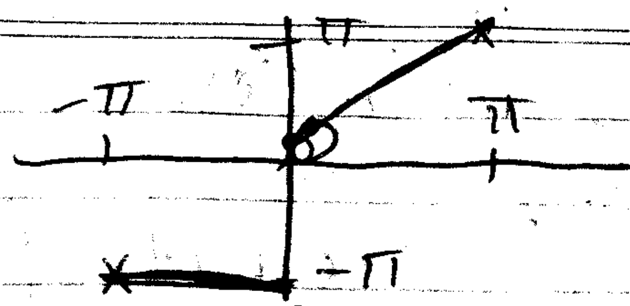
$$= -1 - \frac{1}{2}\cos x + \sum_{n=2}^{\infty} \frac{2}{(n^2-1)}\cos nx + \pi \sin x + \sum_{n=2}^{\infty} 0 \cdot \sin nx$$

$$= -1 - \frac{1}{2}\cos x + \pi \sin x + 2\sum_{n=2}^{\infty} \frac{\cos nx}{(n^2-1)}$$

Navlakhi's

FOR THE TRUE ENGINEER

8)



$f(x)$ is neither even nor odd

F.S. is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx) \qquad \text{---} (I)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left\{\left[-\pi x\right]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\pi^2 + \frac{\pi^2}{2}\right\} = \frac{1}{\cancel{\pi}}\left(\frac{-\pi^{\cancel{2}}}{2}\right) = \frac{-\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \cos nx \, dx + \int_{0}^{\pi} x \cos nx \, dx\right]$$

$$= \frac{1}{\pi}\left\{\left[-\pi\left(\frac{\sin nx}{n}\right)\right]_{-\pi}^{0} + \left[x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{-\cos nx}{n^2}\right)\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{\cos n\pi}{n^2} - \frac{1}{n^2}\right\} = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \sin nx \, dx + \int_{0}^{\pi} x \sin nx \, dx\right]$$

$$= \frac{1}{\pi}\left\{-\pi\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0} + \left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\pi\left(\frac{-1}{n} + \frac{(-1)^n}{n}\right) + \left[\frac{-\pi(-1)^n}{n}\right]\right\}$$

$$= \frac{1}{\pi}\left\{\frac{\pi}{n} - \frac{\pi(-1)^n}{n} - \frac{\pi(-1)^n}{n}\right\}$$

$$= \frac{1}{n} - \frac{2(-1)^n}{n} = \frac{1-2(-1)^n}{n}$$

Subs. in (I)

$$\Rightarrow f(x) = \frac{-\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{\pi n^2}\cos nx + \frac{1-2(-1)^n}{n}\sin nx\right]$$

Put $x=0$ on both sides;

$$f(0) = \frac{-\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{(-1)^n-1}{\pi n^2}\right]$$

By Dirichlet's condn

$$f(0) = \frac{f(0^-)+f(0^+)}{2} = \frac{-\pi+0}{2} = \frac{-\pi}{2}$$

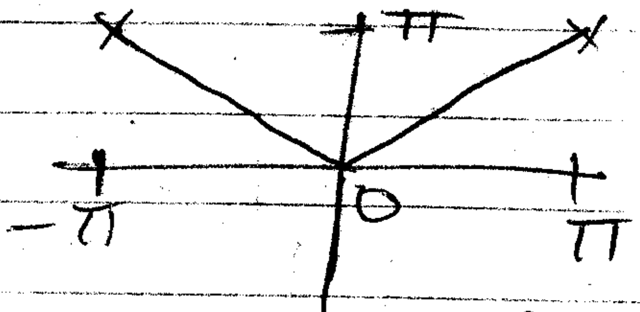
$$\frac{-\pi}{2} = \frac{-\pi}{4} + \sum_{n=1}^{\infty}\frac{(-1)^n-1}{\pi n^2}$$

$$\frac{-\pi}{4} = \sum_{n=1}^{\infty}\frac{(-1)^n-1}{\pi n^2}$$

$$\frac{-\pi}{4} = \frac{-2}{\pi} - \frac{2}{\pi 3^2} - \frac{2}{\pi 5^2} - \cdots$$

$$\therefore \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

9)



Symm. about Y-axis

$\therefore$ f(x) is even funct.

F.S is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ — (I)

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= \frac{\pi^2}{\pi} = \pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \underbrace{x}_{u}\,\underbrace{\cos nx}_{v}\,dx$$

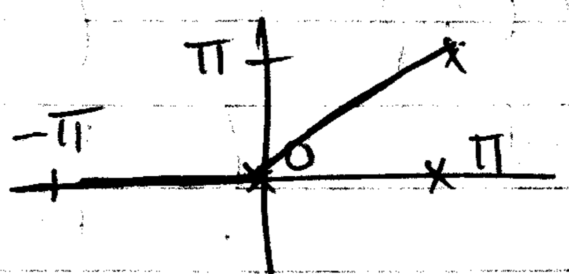
$$= \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

Subs. in (I)

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\left[(-1)^n - 1\right]}{n^2}\cos nx$$

10)



f(x) is neither even nor odd

f.s. is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right) \quad \text{(1)}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} x\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} = \frac{\pi^2}{2\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\cos nx\,dx + \int_{0}^{\pi} \underset{u}{x}\,\underset{v}{\cos nx}\,dx\right]$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - (1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{(-1)^n}{n^2} - \frac{1}{n^2}\right] = \frac{1}{\pi n^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \underset{u}{x}\,\underset{v}{\sin nx}\,dx\right]$$

$$b_n = \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi(-1)^n}{n}\right] = \frac{(-1)^{n+1}}{n}$$

Subs. in eq (I)

$$\therefore f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{\pi n^2}\cos nx + \frac{(-1)^{n+1}}{n}\sin nx\right]$$

ii) When $x = -\pi$; $f(x) = 0$; $(-\pi, 0)$

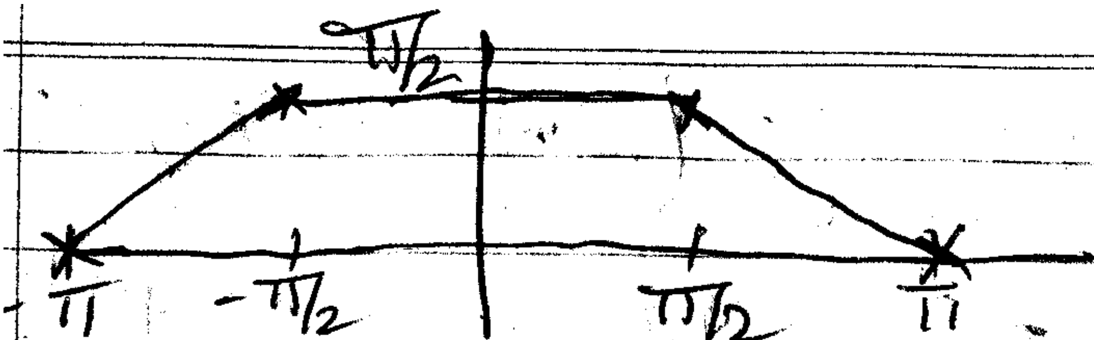
$x = -\pi/2$; $f(x) = \pi/2$; $(-\pi/2, \pi/2)$

$x = -\pi/2$; $f(x) = \pi/2$; $(-\pi/2, \pi/2)$

$x = \pi/2$; $f(x) = \pi/2$; $(\pi/2, \pi/2)$

$x = \pi/2$; $f(x) = \pi/2$; $(\pi/2, \pi/2)$

$x = \pi$; $f(x) = 0$; $(\pi, 0)$



Symm. about Y-axis ∴ f(x) is even, F.S. is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \quad \text{—(1)}$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \frac{\pi}{2}\,dx + \int_{\pi/2}^{\pi} (\pi - x)\,dx\right]$$

$$= \frac{2}{\pi}\left\{\left[\frac{\pi}{2}x\right]_0^{\pi/2} + \left[\pi x - \frac{x^2}{2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{\pi^2}{4} + \pi^2 - \frac{\pi^2}{2} - \frac{\pi^2}{2} + \frac{\pi^2}{8}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{3\pi^2}{8}\right\} = \frac{3\pi}{4}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\left\{\int_0^{\pi/2}\frac{\pi}{2}\cos nx\,dx + \int_{\pi/2}^{\pi}\underbrace{(\pi - x)}_{u}\underbrace{\cos nx\,dx}_{v}\right\}$$

$$= \frac{2}{\pi}\left\{\left[\frac{\pi}{2}\left(\frac{\sin nx}{n}\right)\right]_0^{\pi/2} + \left[(\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(\frac{-\cos nx}{n^2}\right)\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\cancel{\frac{\pi}{2n}\sin\left(\frac{n\pi}{2}\right)} - \frac{(-1)^n}{n^2} - \cancel{\frac{\pi}{2n}\sin\left(\frac{n\pi}{2}\right)} + \frac{\cos\left(\frac{n\pi}{2}\right)}{n^2}\right\}$$

$$= \frac{2}{\pi n^2}\left\{\cos\frac{n\pi}{2} - (-1)^n\right\}$$

Subs. in (I)

$$f(x) = \frac{3\pi}{8} + \sum_{n=1}^{\infty}\frac{2}{\pi n^2}\left\{\cos\frac{n\pi}{2} - (-1)^n\right\}\cos nx$$

PE 41 :-

$x = \pi + z \quad \therefore z = x - \pi$

Old Limits : $x = 0$; $x = 2\pi$

$z = -\pi$; $z = \pi$

$f(x)$ is transformed to $\phi(z)$

$\phi(z)$ is def. in $(-\pi, \pi)$

$\therefore$ 0 is midpt. of int.

$\therefore \phi(z)$ can be verified to be even or odd

$$f(x) = \left(\frac{\pi - x}{2}\right)^2 \text{ in } (0, 2\pi)$$

Let $x = \pi + z$

$\therefore z = x - \pi$

when $x = 0$; $z = -\pi$

$x = 2\pi$; $z = \pi$

$$\phi(z) = \left(\frac{-z}{2}\right)^2 \text{ in } (-\pi, \pi)$$

$$\phi(z) = z^2/4$$

$$\phi(-z) = z^2/4 = \phi(z)$$

$\because$ 0 is mid-pt of given int. &
$\phi(-z) = \phi(z)$ $\therefore$ $\phi(z)$ is even
F.S. is $\phi(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nz$ — (I)

$a_0 = \frac{2}{\pi}\int_0^{\pi} \phi(z)\,dz$

$= \frac{2}{\pi}\int_0^{\pi} \left(\frac{z^2}{4}\right) dz$

$= \frac{1}{2\pi}\left[\frac{z^3}{3}\right]_0^{\pi} = \frac{\pi^3}{6\pi} = \frac{\pi^2}{6}$

$a_n = \frac{2}{\pi}\int_0^{\pi} \phi(z)\cos nz\,dz$

$= \frac{2}{4\pi}\int_0^{\pi} \underset{u}{z^2} \cdot \underset{v}{\cos nz}\,dz$

$= \frac{1}{2\pi}\left[\cancel{z^2\left(\frac{\sin nz}{n}\right)} - (2z)\left(\frac{-\cos nz}{n^2}\right) + \cancel{2\left(\frac{-\sin nz}{n^3}\right)} \right]_0^{\pi}$

$$a_n = \frac{1}{2\pi n^2}\left[2z\cos nz\right]_0^{\pi}$$

$$= \frac{1}{2\pi n^2}\left[2\pi(-1)^n\right] = \frac{(-1)^n}{n^2}$$

Subs. in (I)

$$\phi(z) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos nz$$

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos n(x-\pi)$$

$$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\left[\cos nx\cos n\pi + \cancel{\sin nx\sin n\pi}\right]$$

$$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\left[(-1)^n\cos nx\right]$$

$$= \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2}\cos nx$$

$$\left(\frac{\pi-x}{2}\right)^2 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$$

(i) Put $x = 0$

$$\frac{\pi^2}{4} = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \quad \text{---} (II)$$

ii) Put $x = \pi$

$$0 = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$-\frac{\pi^2}{12} = \frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12} \quad \text{---} (III)$$

iii) Add (II) & (III)

$$\frac{\pi^2}{4} = 2\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots\right)$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

13) $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$

$= \sqrt{2\sin^2 x/2}$

$= \sqrt{2}(\sin x/2)$

Let $x = \pi + z$

$\therefore z = x - \pi$

When $x = 0$; $z = -\pi$

$x = 2\pi$; $z = \pi$

$\phi(z) = \sqrt{2} \sin\left(\frac{\pi+z}{2}\right)$ in $(-\pi, \pi)$

$= \sqrt{2} \sin\left(\frac{\pi}{2} + \frac{z}{2}\right) = \sqrt{2}\cos\left(\frac{z}{2}\right)$

$\phi(-z) = \sqrt{2}\cos\left(\frac{z}{2}\right) = \phi(z)$

$\because$ 0 is mid-pt. of int &

$\phi(-z) = \phi(z)$ $\therefore$ $\phi(z)$ is even

F.S. is $\phi(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nz$ — (I)

$a_0 = \frac{2}{\pi}\int_0^{\pi} \phi(z)\,dz$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \sqrt{2}\cos\left(\frac{z}{2}\right) dz$$

$$= \frac{2\sqrt{2}}{\pi}\left[\frac{\sin(z/2)}{1/2}\right]_0^{\pi}$$

$$= \frac{4\sqrt{2}}{\pi}\left[\sin\frac{\pi}{2}\right] = \frac{4\sqrt{2}}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \phi(z)\cos nz\, dz$$

$$= \frac{2\sqrt{2}}{\pi}\int_0^{\pi} \cos(nz)\cos\left(\frac{z}{2}\right) dz$$

$$= \frac{\sqrt{2}}{\pi}\left[\int_0^{\pi}\cos\left(n+\frac{1}{2}\right)z\,dz + \int_0^{\pi}\cos\left(n-\frac{1}{2}\right)z\,dz\right]$$

$$= \frac{\sqrt{2}}{\pi}\left\{\left[\frac{\sin(n+\frac{1}{2})z}{(n+1/2)}\right]_0^{\pi} + \left[\frac{\sin(n-1/2)z}{(n-1/2)}\right]_0^{\pi}\right\}$$

$$= \frac{\sqrt{2}}{\pi}\left\{\frac{\sin(n+1/2)\pi}{n+1/2} + \frac{\sin(n-1/2)\pi}{n-1/2}\right\}$$

$$= \frac{\sqrt{2}}{\pi}\left\{\frac{\sin(\pi/2+n\pi)}{n+1/2} - \frac{\sin(\pi/2-n\pi)}{n-1/2}\right\}$$

$$= \frac{\sqrt{2}}{\pi}\left\{\frac{\cos n\pi}{n+1/2} - \frac{\cos n\pi}{n-1/2}\right\}$$

$$= \frac{\sqrt{2}(-1)^n}{\pi}\left\{\frac{\cancel{n}-1/2 - \cancel{n}-1/2}{n^2-1/4}\right\}$$

$$= \frac{\sqrt{2}(-1)^n}{\pi}\left\{\frac{-4}{4n^2-1}\right\}$$

$$= \frac{-4\sqrt{2}(-1)^n}{\pi(4n^2-1)}$$

Subs. in eqn (I)

$$\phi(z) = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(4n^2-1)}\cos nz$$

$$f(x) = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(4n^2-1)}\cos n(x-\pi)$$

$$\{\cos n(x-\pi) = \cos(nx-n\pi) = (-1)^n\cos nx\}$$

$$f(x) = \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{2n}\cos nx}{(4n^2-1)}$$

$$= \frac{2\sqrt{2}}{\pi} - \frac{4\sqrt{2}}{\pi}\sum_{n=1}^{\infty}\frac{\cos nx}{4n^2-1}$$

14) $f(x) = \frac{x}{12}(\pi - x)(2\pi - x)$; in $(0, 2\pi)$

Let $x = \pi + z$

$\therefore z = x - \pi$

When $x = 0$; $z = -\pi$

$x = 2\pi$; $z = \pi$

$\phi(z) = \frac{(\pi + z)}{12}(-z)(\pi - z)$ in $(-\pi, \pi)$

$= \frac{-z(\pi^2 - z^2)}{12} = \frac{z^3 - \pi^2 z}{12}$

$\phi(-z) = \frac{-z^3 + \pi^2 z}{12} = -\phi(z)$

$\therefore$ 0 is mid-pt. of int &

$\phi(-z) = -\phi(z)$ $\therefore \phi(z)$ is odd

F.S. is $\phi(z) = \sum_{n=1}^{\infty} b_n \sin nz$ —— (I)

$b_n = \frac{2}{\pi}\int_0^{\pi} \phi(z) \sin nz \, dz$

$$b_n = \frac{2}{\pi}\int_0^{\pi}\left(\frac{z^3-\pi^2 z}{12}\right)\sin nz\,dz$$

$$= \frac{1}{6\pi}\int_0^{\pi}\underbrace{(z^3-\pi^2 z)}_{u}\underbrace{\sin nz}_{v}\,dz$$

$$\frac{1}{6\pi}\Big[(z^3-\pi^2 z)\left(\frac{-\cos nz}{n}\right) - \cancel{(3z^2-\pi^2)\left(\frac{-\sin nz}{n^2}\right)}^{0}$$

$$+(6z)\left(\frac{\cos nz}{n^3}\right) - \cancel{(6)\left(\frac{\sin nz}{n^4}\right)}^{0}\Big]_0^{\pi}$$

$$\frac{1}{6\pi}\left[0+\frac{6\pi(-1)^n}{n^3}\right] = \frac{(-1)^n}{n^3}$$

Subs. in (I)

$$\phi(z) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}.\sin nz$$

$$f(x) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}\sin n(x-\pi)$$

$$\{\sin(nx-n\pi) = (-1)^n \sin nx\}$$

$$f(x) = \sum_{n=1}^{\infty}\frac{(-1)^n(-1)^n \sin nx}{n^3} = \sum_{n=1}^{\infty}\frac{\sin nx}{n^3}$$

) Let $x = \pi + z$

$\therefore z = x - \pi$

When $x = 0$; $z = -\pi$

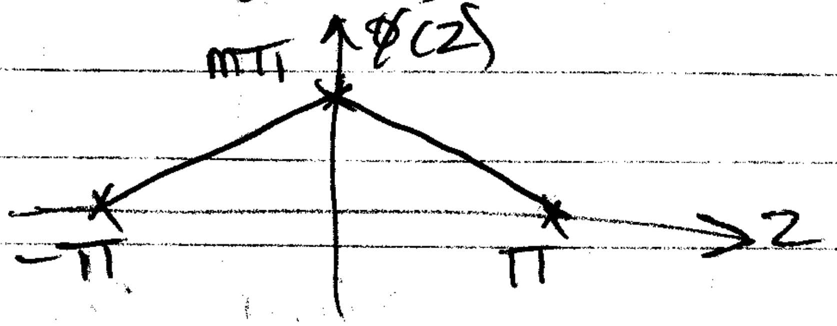
$x = \pi$; $z = 0$

$x = 2\pi$; $z = \pi$

$\phi(z) = m(\pi + z)$; $-\pi < z < 0$

$= -m(\pi + z) + 2m\pi$; $0 < z < \pi$

$= m(\pi - z)$



When $z = -\pi$; $\phi(z) = 0$; $(-\pi, 0)$

$z = 0$; $\phi(z) = m\pi$; $(0, m\pi)$

$z = 0$; $\phi(z) = m\pi$; $(0, m\pi)$

$z = \pi$; $\phi(z) = 0$; $(\pi, 0)$

$\therefore$ Symm about Y-axis

$\therefore \phi(z)$ is even

$$\phi(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nz \qquad \text{---(I)}$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} \phi(z)\,dz$$

$$= \frac{2m}{\pi}\int_0^{\pi} (\pi - z)\,dz$$

$$= \frac{2m}{\pi}\left[\pi z - \frac{z^2}{2}\right]_0^{\pi}$$

$$= \frac{2m}{\pi}\left[\pi^2 - \frac{\pi^2}{2}\right] = \frac{2m}{\pi}\left(\frac{\pi^2}{2}\right) = m\pi$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} \phi(z)\cos nz\,dz$$

$$= \frac{2m}{\pi}\int_0^{\pi} \underbrace{(\pi - z)}_{u}\,\underbrace{\cos nz}_{v}\,dz$$

$$= \frac{2m}{\pi}\left[(\pi - z)\left(\frac{\sin nz}{n}\right) - (-1)\left(-\frac{\cos nz}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{2m}{\pi}\left[-\frac{(-1)^n}{n^2} + \frac{1}{n^2}\right] = \frac{2m}{n^2\pi}\left[1 - (-1)^n\right]$$

Subs. in eqn (I)

$$\phi(z) = \frac{m\pi}{2} + \frac{2m}{\pi} \sum_{n=1}^{\infty} \frac{[1-(-1)^n]}{n^2} \cos nz$$

$$f(x) = \frac{m\pi}{2} + \frac{2m}{\pi} \sum_{n=1}^{\infty} \frac{[1-(-1)^n]}{n^2} \cos n(x-\pi)$$

$$\{\cos(nx - n\pi) = (-1)^n \cos nx\}$$

$$f(x) = \frac{m\pi}{2} + \frac{2m}{\pi} \sum_{n=1}^{\infty} \frac{[1-(-1)^n]}{n^2} (-1)^n \cos nx$$

$$= \frac{m\pi}{2} + \frac{2m}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2} \cos nx$$

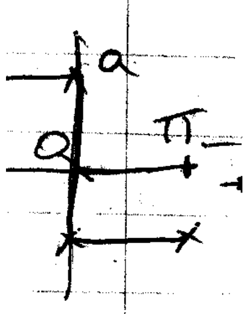
Q16) Let $x = \pi + z$ $\therefore z = x - \pi$

When $x = 0$; $z = -\pi$

$x = \pi$; $z = 0$

$x = 2\pi$; $z = \pi$

$$\phi(z) = a \quad ; \quad -\pi < z < 0$$
$$= -a \quad ; \quad 0 < z < \pi$$



Symm. about origin
$\therefore \phi(z)$ is odd

$$\phi(z) = \sum_{n=1}^{\infty} b_n \sin nz \quad \text{—— (I)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} \phi(z) \sin nz \, dz$$

$$= \frac{-2a}{\pi}\int_0^{\pi} \sin nz \, dz$$

$$= \frac{-2a}{\pi}\left[-\frac{\cos nz}{n}\right]_0^{\pi}$$

$$= \frac{+2a}{\pi n}\left[(-1)^n - 1\right] \cancel{= \frac{2a}{n\pi}\left[1 - (-1)^n\right]}$$

Subs. in (I)

$$\phi(z) = \sum_{n=1}^{\infty} \frac{2a}{n\pi}[1+(-1)^n] \sin nz$$

$$= \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{[1+(-1)^n]}{n} \sin nz$$

$$f(x) = \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{[1+(-1)^n]}{n} \sin n(x-\pi)$$

$\{\sin(nx - n\pi) = (-1)^n \sin nx\}$

$$f(x) = \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{[1+(-1)^n]}{n} (-1)^n \sin nx$$

$$= \frac{2a}{\pi} \sum_{n=1}^{\infty} \frac{[(-1)^n+1]}{n} \sin nx$$

TYPE III :-

17) $f(x) = x^2$ in $(-l, l)$

$f(-x) = x^2 = f(x)$

0 is mid-pt. of given int &

$f(-x) = f(x) \therefore f(x)$ is even

F.S. ie $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$ —(5)

A**

$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

$= \frac{2}{l}\int_0^l \frac{x^2}{4}\cos\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{l}\left[x^2\left(\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - (2x)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right) + (2)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^3}\right)\right]_0^l$

$\frac{2}{l}\left[\frac{2l^3\cos n\pi}{n^2\pi^2}\right] = \frac{4l^2}{n^2\pi^2}(-1)^n$

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$$a_0 = \frac{2}{l}\int_0^l x^2\,dx = \frac{2}{l}\left[\frac{x^3}{3}\right]_0^l$$

$$= \frac{2}{l}\left(\frac{l^3}{3}\right) = \frac{2l^2}{3}$$

Subs. in (I)

$$f(x) = \frac{l^2}{3} + \sum_{n=1}^{\infty}\frac{4l^2}{\pi^2 n^2}(-1)^n\cos\left(\frac{n\pi x}{l}\right)$$

$$x^2 = \frac{l^2}{3} + \frac{4l^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos\left(\frac{n\pi x}{l}\right)$$

Put $x=0$

$$-\frac{\cancel{l^2}}{3} = \frac{4\cancel{l^2}}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$$

$$-\frac{\pi^2}{12} = \frac{-1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \cdots$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$$

18) $f(x) = 2x - x^2$ in $[0,3)$

$\equiv (0, 2l)$

$2l = 3 \therefore l = 3/2$

{0 is not mid-pt}

F.S. is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\,dx$$

$$= \frac{2}{3}\left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{x^3}{3}\right]_0^3$$

$$= \frac{2}{3}[9-9] = 0$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\int_0^3 \underbrace{(2x - x^2)}_{u}\,\underbrace{\cos\left(\frac{2n\pi x}{3}\right)dx}_{v}$$

$$= \frac{2}{3}\left[\cancel{(2x - x^2)\left(\frac{\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)}\right)} - (2-2x)\left(\frac{-\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^2}\right) + \cancel{(-2)\left(\frac{-\sin\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^3}\right)} \right]_0^3$$

$$= \frac{2}{3}\left[\frac{-4\cos(2n\pi)}{\frac{4n^2\pi^2}{9}} - \frac{2 \times 1}{\frac{4n^2\pi^2}{9}} \right]$$

$$= \frac{2 \times 9}{12n^2\pi^2}\left[-4-2\right] = -\frac{12 \times 9}{12n^2\pi^2} = \frac{-9}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\int_0^3 (2x - x^2)\sin\left(\frac{2n\pi x}{3}\right)dx$$

$$b_n = \frac{2}{3}\left[(2x-x^2)\left(\frac{-\cos(2n\pi x/3)}{\left(\frac{2n\pi}{3}\right)}\right)\right.$$

$$\left. -(2-2x)\left(\frac{-\sin(2n\pi x/3)}{\left(\frac{2n\pi}{3}\right)^2}\right) + (-2)\left(\frac{\cos\left(\frac{2n\pi x}{3}\right)}{\left(\frac{2n\pi}{3}\right)^3}\right)\right]_0^3$$

$$= \frac{2}{3}\left[\left(\frac{(-3)(-1)\times 3}{2n\pi} - \frac{2(1)\times 27}{8n^3\pi^3}\right) + \left(\frac{2\times 1\times 27}{8n^3\pi^3}\right)\right]$$

$$= \frac{\cancel{2}}{3}\left[\frac{9}{\cancel{2}n\pi} - \frac{\cancel{54}}{\cancel{8}n^3\pi^3} + \frac{\cancel{54}}{\cancel{8}n^3\pi^3}\right]$$

$$= \frac{3}{n\pi}$$

Subs. in (I)

$$f(x) = \sum_{n=1}^{\infty}\left[\frac{-9}{n^2\pi^2}\cos\left(\frac{2n\pi x}{3}\right) + \frac{3}{n\pi}\sin\left(\frac{2n\pi x}{3}\right)\right]$$

Q19) $f(x) = x - x^2$

$= f_1(x) - f_2(x)$ —— (I)

~~$2\left[x\left(\frac{\sin n\pi x}{n\pi}\right)-(1)\left(\frac{\cos n\pi x}{n^2\pi^2}\right)\right]_0^1 = 2\left[(-1)^n \ldots\right]$~~

$$2\left[x\left(-\frac{\cos n\pi x}{n\pi}\right)-(1)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_0^1 = 2\left[\frac{-(-1)^n}{n\pi}\right]$$

$f_1(x) = x$ in $(-1,1) \equiv (-l,l)$

$f_1(-x) = -x = -f_1(x)$

$\therefore$ 0 is mid-pt. of int. &

$f_1(-x) = -f_1(x)$ $\therefore f_1(x)$ is odd

F.S. is $f_1(x) = \sum^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ —— (II)

$$b_n = \frac{2}{l}\int_0^l f_1(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{2}{1}\int_0^1 x \sin n\pi x \, dx = \cancel{2\left[\frac{x^2}{2}\right]_0^1 = 1}$$

Subs. in (II)

$$\therefore f_1(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\sin(n\pi x)\right]\frac{(-1)^{n+1}}{n}$$

$f_2(x) = x^2$ in $(-1,1) \equiv (-l,l)$

$f_2(-x) = x^2 = f_2(x)$

0 is mid-pt. of int. &

$f_2(-x) = f_2(x)$ $\therefore f_2(x)$ is even

F.S. of $f_2(x)$ is $f_2(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$ — (III)

$$a_0 = \frac{2}{l}\int_0^l f_2(x)\,dx = 2\int_0^1 x^2\,dx$$

$$= 2\left[\frac{x^3}{3}\right]_0^1 = \frac{2}{3}$$

$$a_n = \frac{2}{l}\int_0^l f_2(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= 2\int_0^1 x^2 \cos(n\pi x)\,dx$$

$$= 2\left[\cancel{x^2\left(\frac{\sin n\pi x}{n\pi}\right)} - (2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + \cancel{(2)\left(\frac{-\sin n\pi x}{n^3\pi^3}\right)}\right]_0^1$$

$$= 2\left[\frac{2\cos n\pi}{n^2\pi^2}\right] = \frac{4(-1)^n}{n^2\pi^2}$$

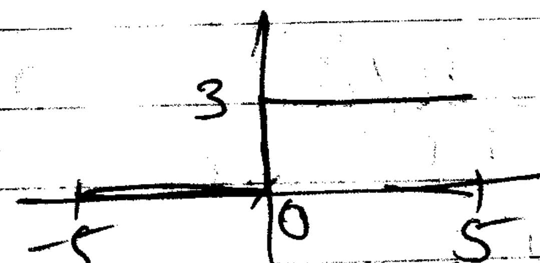
Subs. in (III)

$$f_2(x) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)$$

Subs. in (I)

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi x - \frac{1}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos n\pi x$$

(Q20)



f(x) is neither even nor odd
F.S. is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{l} + b_n\sin\frac{n\pi x}{l}\right) \quad \text{---(I)}$$

$l = 5$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx = \frac{1}{5}\int_{-5}^{5} f(x)\,dx$$

$$= \frac{1}{5}\left[\int_{-5}^{0} 0\,dx + \int_{0}^{5} 3\,dx\right] = \frac{1}{5}\left[3\times 5\right]$$

$$a_0 = 3$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{5}\left[\int_{-5}^{5} f(x)\cos\left(\frac{n\pi x}{5}\right)dx\right]$$

$$= \frac{1}{5}\left[\int_{-5}^{0} 0 + \int_{0}^{5} 3\cos\left(\frac{n\pi x}{5}\right)dx\right]$$

$$= \frac{3}{5}\left[\frac{\sin(n\pi x/5)}{\frac{n\pi}{5}}\right]_0^5 = 0$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{1}{5}\left[\int_{-5}^{5} f(x)\sin\left(\frac{n\pi x}{5}\right)dx\right]$$

$$= \frac{1}{5}\left[\int_{-5}^{0} 0 + \int_{0}^{5} 3\sin\left(\frac{n\pi x}{5}\right)dx\right]$$

$$= \frac{3}{5}\left[\frac{-\cos(n\pi x/5)}{n\pi/5}\right]_0^5$$

$$= \frac{3}{\cancel{5}}\left[\frac{-(-1)^n+1}{n\pi/\cancel{5}}\right] = \frac{3}{n\pi}\left[1-(-1)^n\right]$$

Subs. in (I)

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \left\{ \frac{3}{n\pi} [1-(-1)^n] \right\} \sin\left(\frac{n\pi x}{5}\right)$$

$$= \frac{3}{2} + \frac{3}{\pi} \sum_{n=1}^{\infty} \frac{[1-(-1)^n]}{n} \sin\left(\frac{n\pi x}{5}\right)$$

(Q21) $f(x) = \sin ax$ in $(-l, l)$

$f(-x) = -\sin ax = -f(x)$

$\therefore$ 0 is mid-pt. & $f(-x) = -f(x)$

$\therefore f(x)$ is odd & F.S. is

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right) \quad \text{—— (I)}$$

$$b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right)$$

$$= \frac{2}{l} \int_0^l \sin\left(\frac{n\pi x}{l}\right) \sin ax \, dx$$

$$= \frac{1}{l}\left[\int_0^l \cos\left(\frac{n\pi}{l} - a\right)x \, dx - \int_0^l \cos\left(\frac{n\pi}{l} + a\right)x \, dx\right]$$

$$b_n = \frac{1}{l}\left\{\left[\frac{\sin\left(\frac{n\pi}{l}-a\right)x}{\left(\frac{n\pi}{l}-a\right)}\right]_{-l}^{l} - \left[\frac{\sin\left(\frac{n\pi}{l}+a\right)x}{\left(\frac{n\pi}{l}+a\right)}\right]_{-l}^{l}\right\}$$

$$b_n = \frac{1}{l}\left\{\frac{\sin\left(\frac{n\pi}{l}-a\right)l}{\left(\frac{n\pi}{l}-a\right)} + \cancel{\frac{\sin\left(\frac{n\pi}{l}-a\right)l}{\left(\frac{n\pi}{l}-a\right)}} - \left[\frac{\sin\left(\frac{n\pi}{l}+a\right)l}{\left(\frac{n\pi}{l}+a\right)} + \cancel{\frac{\sin\left(\frac{n\pi}{l}+a\right)l}{\left(\frac{n\pi}{l}+a\right)}}\right]\right\}$$

$$b_n = \frac{2}{l}\left\{\frac{\sin\left(\frac{n\pi}{l}-a\right)l}{\frac{n\pi}{l}-a} - \frac{\sin\left(\frac{n\pi}{l}+a\right)l}{\frac{n\pi}{l}+a}\right\}$$

$$= \frac{2l}{\cancel{l}}\left\{\frac{\sin(n\pi - al)}{(n\pi - al)} - \frac{\sin(n\pi + al)}{(n\pi + al)}\right\}$$

$$= 2\left\{\frac{-(-1)^n \sin al}{(n\pi - al)} - \frac{(-1)^n \sin al}{(n\pi + al)}\right\}$$

$$= -2(-1)^n \sin al\left\{\frac{1}{n\pi - al} + \frac{1}{n\pi + al}\right\}$$

$$\therefore b_n = 2(-1)^{n+1} \sin al \left\{ \frac{2n\pi}{n^2\pi^2 - a^2l^2} \right\}$$

$$= 2(-1)^{n+1} \sin al \left(\frac{n\pi}{n^2\pi^2 - a^2l^2} \right)$$

Subs. in (I)

$$f(x) = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1} n\pi \sin al}{n^2\pi^2 - a^2l^2} \times \sin\left(\frac{n\pi x}{l}\right)$$

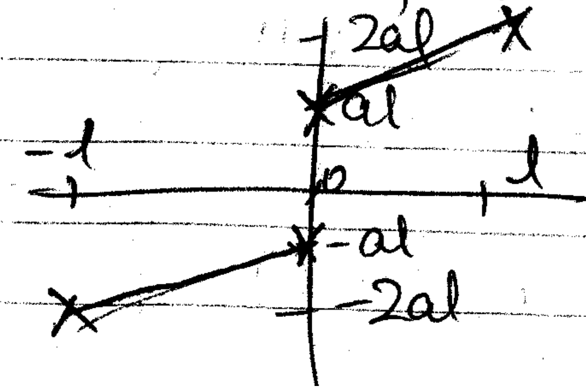
$$= 2\pi \sin al \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot n}{n^2\pi^2 - a^2l^2} \sin\left(\frac{n\pi x}{l}\right)$$

Q22) When $x = -l$, $f(x) = -2al$; $(-l, -2al)$

$x = 0$, $f(x) = -al$; $(0, -al)$

$x = 0$, $f(x) = al$, $(0, al)$

$x = l$, $f(x) = 2al$; $(l, 2a)$



$f(x)$ is symm. about origin

$\therefore f(x)$ is odd

F.S. is $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$ (I)

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2a}{l}\int_0^l \underbrace{(l+x)}_{u}\underbrace{\sin\left(\frac{n\pi x}{l}\right)}_{v}dx$$

$$= \frac{2a}{l}\left[(l+x)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)}\right) - (1)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\left(\frac{n\pi}{l}\right)^2}\right)\right]_0^l$$

$$= \frac{2a}{l}\left[2l\left(\frac{-\cos(n\pi)}{\left(\frac{n\pi}{l}\right)}\right) + l\left(\frac{\cos 0}{\frac{n\pi}{l}}\right)\right]$$

$$= \frac{2a}{l}\left[\frac{-2l^2}{n\pi}(-1)^n + \frac{l^2}{n\pi}\right]$$

$$= \frac{2al}{n\pi}\left[1 - 2(-1)^n\right]$$

Subs. in eq (I)

$$f(x) = \sum_{n=1}^{\infty} \frac{2al}{n\pi}\left[1-2(-1)^n\right] \sin\left(\frac{n\pi x}{l}\right)$$

Q23) Limits are $(0,2) \equiv (0, 2\pi)$

Let $x = 1+z$ $(\equiv x = \pi + z)$

$\therefore z = x - 1$

When $x = 0$; $z = -1$

$x = 2$; $z = 1$

$x = 1$; $z = 0$

$\phi(z) = \pi(1+z)$; $-1 \le z \le 0$

$= 0$; $z = 0$

$= \pi(z-1)$; $0 < z \le 1$

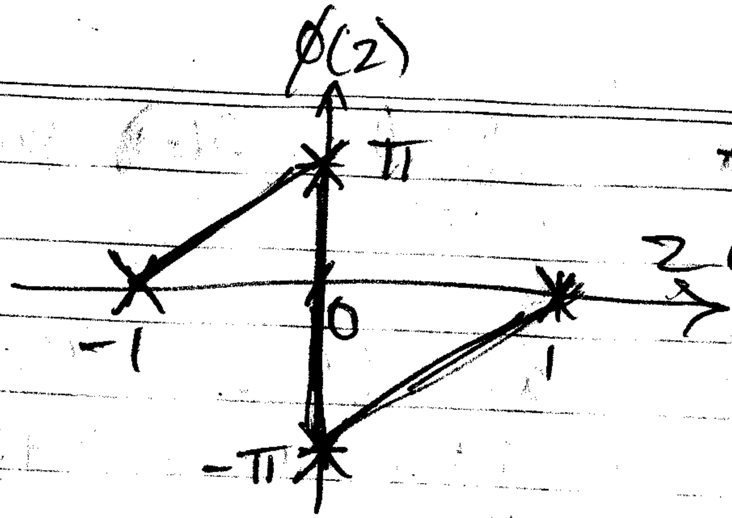
When $z = -1$; $\phi(z) = 0$; $(-1, 0)$

$z = 0$; $\phi(z) = \pi$; $(0, \pi)$

$z = 0$; $\phi(z) = 0$; $(0, 0)$

$z = 0$; $\phi(z) = -\pi$; $(0, -\pi)$

$z = 1$; $\phi(z) = 0$; $(1, 0)$



Symm. about origin $\therefore$ $\phi(z)$ is odd.

{NOTE: If gp. would have not been symm. about origin or y-axis then we would find F.S. of $f(x)$ directly & not of $\phi(z)$}

F.S. is

$$\phi(z) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi z}{l} \quad \{l = 1\}$$

$$b_n = \frac{2}{l}\int_0^l \phi(z)\sin\left(\frac{n\pi z}{l}\right)dz$$

$$= 2\pi\int_0^1 \underset{u}{(z-1)}\ \underset{v}{\sin(n\pi z)}\,dz \quad \{\because l=1\}$$

$$= 2\pi\left[(z-1)\left(\frac{-\cos(n\pi z)}{(n\pi)}\right) - (1)\cancel{\left(\frac{-\sin(n\pi z)}{n^2\pi^2}\right)}\right]_0^1$$

$$= 2\pi\left[\frac{-\cos 0}{n\pi}\right] = -\frac{2}{n}$$

Subs. in eq. (I)

$$\therefore \phi(z) = \sum_{n=1}^{\infty} -\frac{2}{n}\sin(n\pi z)$$

$$f(x) = \sum_{n=1}^{\infty} -\frac{2}{n}\sin[n\pi(x-1)]$$

$$= \sum_{n=1}^{\infty} -\frac{2}{n}\sin(n\pi x - n\pi)$$

$$f(x) = \sum_{n=1}^{\infty} -\frac{2}{n}(-1)^n \sin n\pi x$$

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the way its supposed to be

(Q24) $f(x) = 0 \quad ; -3 < x < -1$

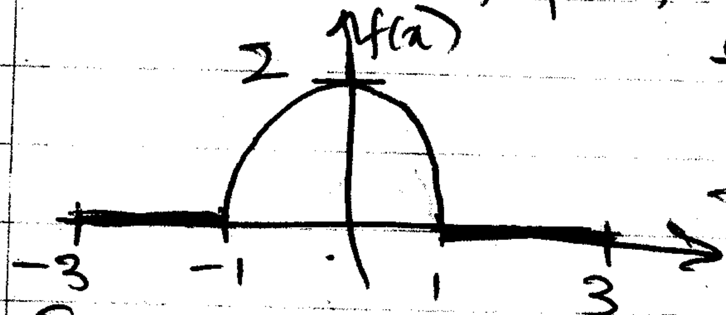
$\quad = 1 + \cos \pi x \quad ; -1 < x < 1$

$\quad = 0 \quad ; 1 < x < 3$

When $x = -1 \; ; \; f(x) = 0$

$x = 1 \; ; \; f(x) = 0$

$x = 0 \; ; \; f(x) = 2$



Symm. about y-axis

$\therefore$ f(x) Even

$l = 3$

F.S. is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) \longrightarrow (I)$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{3}\int_0^3 f(x)\,dx$$

$$= \frac{2}{3}\left[\int_0^1 (1+\cos\pi x)\,dx + \int_1^3 0\,dx\right]$$

$$= \frac{2}{3}\left[x + \cancel{\frac{\sin \pi x}{\pi}} \right]_0^1 = \frac{2}{3}$$

$$a_n = \frac{2}{3}\int_0^1 f(x).\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2}{3}\left[\int_0^3 f(x)\cos\left(\frac{n\pi x}{3}\right)dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 \left[(1+\cos\pi x)\cos\frac{n\pi x}{3}\right]dx + \int_1^3 0\,dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 \cos\frac{n\pi x}{3}dx + \int_0^1 \cos\frac{n\pi x}{3}.\cos\pi x\,dx\right]$$

$$= \frac{2}{3}\left\{\left[\frac{\sin\frac{n\pi x}{3}}{\frac{n\pi}{3}}\right]_0^1 + \frac{1}{2}\int_0^1\left(\cos\left(\frac{n\pi}{3}+\pi\right)x + \cos\left(\frac{n\pi}{3}-\pi\right)x\right)dx\right\}$$

$$= \frac{2}{3}\left\{\frac{3}{n\pi}\sin\left(\frac{n\pi}{3}\right) + \frac{1}{2}\left[\frac{\sin\left(\frac{n\pi}{3}+\pi\right)x}{\frac{n\pi}{3}+\pi} + \frac{\sin\left(\frac{n\pi}{3}-\pi\right)x}{\frac{n\pi}{3}-\pi}\right]_0^1\right\}$$

$$= \frac{2}{3}\left\{\frac{3}{n\pi}\sin\left(\frac{n\pi}{3}\right) + \frac{1}{2}\left[\frac{\sin\left(\pi+\frac{n\pi}{3}\right)}{\pi+\frac{n\pi}{3}} + \frac{\sin\left(\frac{n\pi}{3}-\pi\right)}{\frac{n\pi}{3}-\pi}\right]\right.$$

$$\left\{ \because \sin\left(\pi+\frac{n\pi}{3}\right) = -1\sin\left(\frac{n\pi}{3}\right)\right.$$

$$\left.\sin\left(\frac{n\pi}{3}-\pi\right) = -1\sin\frac{n\pi}{3}\right\}$$

$$\therefore a_n = \frac{2}{3}\left\{\frac{3}{n\pi}\sin\left(\frac{n\pi}{3}\right) - \frac{1}{2}\sin\frac{n\pi}{3}\left[\frac{1}{\pi+\frac{n\pi}{3}} + \frac{1}{\frac{n\pi}{3}-\pi}\right]\right\}$$

$$= \frac{2}{3}\left\{\frac{3}{n\pi}\sin\left(\frac{n\pi}{3}\right) - \cancel{\frac{1}{2}}\sin\left(\frac{n\pi}{3}\right)\left[\frac{\frac{\cancel{2}n\pi}{3}}{\frac{n^2\pi^2}{9}-\pi^2}\right]\right\}$$

$$= \frac{2}{3}\sin\left(\frac{n\pi}{3}\right)\left\{\frac{3}{n\pi} - \frac{3n\cancel{\pi}}{n^2\pi^2-9\pi^2}\right\}$$

$$= \frac{2}{\pi}\sin\left(\frac{n\pi}{3}\right)\left\{\frac{1}{n} - \frac{n}{n^2-9}\right\}$$

$$= \frac{2}{\pi}\sin\left(\frac{n\pi}{3}\right)\left\{\frac{-9}{n(n^2-9)}\right\}$$

$$= \frac{-18}{n\pi(n^2-9)}\sin\left(\frac{n\pi}{3}\right)$$

a_n is not def. when $n^2-9=0$

$\therefore n=3$

$$a_3 = \frac{2}{3}\left[\int_0^1 (1+\cos\pi x)\cos(\pi x)\,dx\right]$$

$$= \frac{2}{3}\left[\int_0^1 (\cos\pi x + \cos^2\pi x)\,dx\right]$$

$$= \frac{2}{3}\left\{\int_0^1 \left[\cos\pi x + \left(\frac{1+\cos 2\pi x}{2}\right)\right]dx\right\}$$

$$= \frac{2}{3}\left\{\int_0^1 \cos\pi x\,dx + \frac{1}{2}\int_0^1 dx + \frac{1}{2}\int_0^1 \cos 2\pi x\,dx\right\}$$

$$= \frac{2}{3}\left\{\cancel{\left[\frac{\sin\pi x}{\pi}\right]_0^1} + \frac{1}{2}[x]_0^1 + \frac{1}{2}\cancel{\left[\frac{\sin 2\pi x}{2\pi}\right]_0^1}\right\}$$

$$= \frac{2}{3}\left\{\frac{1}{2}\right\} = \frac{1}{3}$$

Subs. in eq (I)

$$f(x) = \frac{1}{3} + \sum_{n=1}^{2} \frac{-18}{n\pi(n^2-9)}\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi x}{3}\right)$$

$$+ \frac{1}{3}\cos\left(\frac{3\pi x}{3}\right) + \sum_{n=4}^{\infty} \frac{-18}{n\pi(n^2-9)}\sin\left(\frac{n\pi}{3}\right)\cos\left(\frac{n\pi x}{3}\right)$$

$$f(x) = \frac{1}{3} - \frac{18}{\pi}\sum_{n=1}^{2} \frac{\sin\frac{n\pi}{3}}{n(n^2-9)} \cos\left(\frac{n\pi x}{3}\right)$$

$$+ \frac{\cos \pi x}{3} - \frac{18}{\pi}\sum_{n=4}^{\infty} \frac{\sin\frac{n\pi}{3}}{n(n^2-9)} \cos\left(\frac{n\pi x}{3}\right)$$

(Q25) When $x = -2$; $f(x) = 0$; $(-2, 0)$

$x = -1$; $f(x) = 0$; $(-1, 0)$

$x = -1$; $f(x) = 0$; $(-1, 0)$

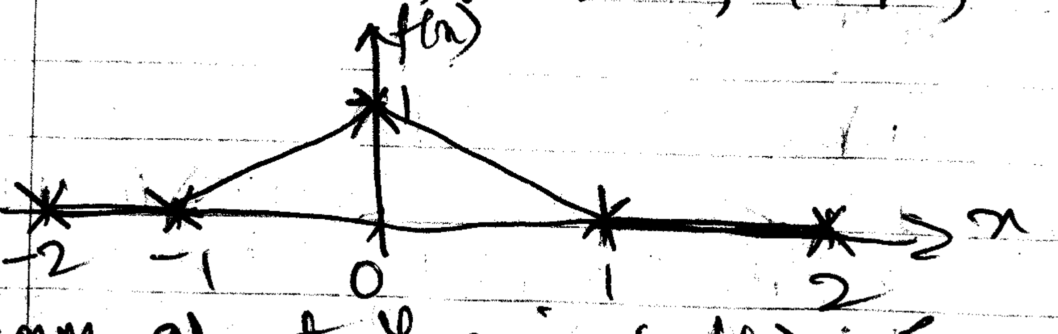
$x = 0$; $f(x) = 1$; $(0, 1)$

$x = 0$; $f(x) = 1$; $(0, 1)$

$x = 1$; $f(x) = 0$; $(1, 0)$

$x = 1$; $f(x) = 0$; $(1, 0)$

$x = 2$; $f(x) = 0$; $(2, 0)$



Symm. about Y-axis $\therefore$ f(x) is Even.

F.S. is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$

$l = 2$ ——— (I)

$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$= \int_0^2 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx$

$= \int_0^1 (1-x)\,dx + \int_1^2 0\,dx$

$= \left[x - \frac{x^2}{2}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$

$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

$= \int_0^2 f(x)\cos\left(\frac{n\pi x}{2}\right)dx$

$= \int_0^1 (1-x)\cos\left(\frac{n\pi x}{2}\right)dx + \int_1^2 0\,dx$

$= \left[(1-x)\left(\frac{\sin(n\pi x/2)}{n\pi/2}\right) - (-1)\left(\frac{-\cos(n\pi x/2)}{n^2\pi^2/4}\right)\right]_0^1$

$$= -\left[\frac{\cos(n\pi x/2)}{n^2\pi^2/4}\right]_0^1$$

$$= \frac{-4\cos(n\pi/2)}{n^2\pi^2} + \frac{4}{n^2\pi^2}$$

$$= \frac{4}{n^2\pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]$$

$$= \frac{8}{n^2\pi^2}\sin^2\left(\frac{n\pi}{4}\right)$$

Subs. in eq (I)

$$f(x) = \frac{1}{4} + \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin^2\left(\frac{n\pi}{4}\right)\cos\left(\frac{n\pi x}{2}\right)$$

TYPE IV :-

Q26) H.R.S.S is $f(x) = \sum_{n=0}^{\infty} b_n \sin nx$ —— (I)

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\left\{\int_0^{\alpha}\frac{x}{2}\sin nx\,dx + \int_{\alpha}^{\pi-\alpha}\frac{\alpha}{2}\sin nx\,dx + \int_{\pi-\alpha}^{\pi}\left(\frac{\pi-x}{2}\right)\sin nx\,dx\right.$$

$$= \frac{2}{\pi}\left\{\frac{1}{2}\int_0^{\alpha} x \sin nx \, dx + \frac{\alpha}{2}\int_{\alpha}^{\pi-\alpha} \sin nx \, dx + \frac{1}{2}\int_{\pi-\alpha}^{\pi} (\pi - x)\sin nx \, dx\right\}$$

$$= \frac{2}{\pi}\left\{\frac{1}{2}\left[x\left(\frac{-\cos nx}{n}\right) - (1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\alpha} + \frac{\alpha}{2}\left[\frac{-\cos nx}{n}\right]_{\alpha}^{\pi-\alpha}\right.$$

$$\left. + \frac{1}{2}\left[(\pi - x)\left(\frac{-\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_{\pi-\alpha}^{\pi}\right\}$$

$$= \frac{2}{\pi}\left\{\frac{1}{2}\left[\frac{-\alpha \cos n\alpha}{n} + \frac{\sin n\alpha}{n^2}\right] + \frac{\alpha}{2}\left[\frac{-\cos n(\pi-\alpha)}{n} + \frac{\cos n\alpha}{n}\right]\right.$$

$$\left. + \frac{1}{2}\left[\frac{\alpha \cos[n(\pi-\alpha)]}{n} + \frac{\sin[n(\pi-\alpha)]}{n^2}\right]\right\}$$

$$\left\{\because \cos(n\pi - n\alpha) = (-1)^n \cos n\alpha\right.$$

$$\left. \sin(n\pi - n\alpha) = -(-1)^n \sin n\alpha\right\}$$

$$= \frac{2}{\pi}\left\{\cancel{\frac{-\alpha \cos n\alpha}{2n}} + \frac{\sin n\alpha}{2n^2} - \cancel{\frac{(-1)^n \alpha \cos n\alpha}{2n}} + \cancel{\frac{\alpha \cos n\alpha}{2n}}\right.$$

$$\left. + \cancel{\frac{\alpha(-1)^n \cos n\alpha}{2n}} - \frac{(-1)^n \sin n\alpha}{2n^2}\right\}$$

$$= \frac{\sin n\alpha}{\pi n^2}\left[1 - (-1)^n\right]$$

Subs. in (I)

$$\therefore f(x) = \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{\sin n\alpha}{n^2}\left[1-(-1)^n\right]\sin nx$$

Q27) H.R.C.S is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ —— (I)

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x\,dx = \frac{2}{\pi}\left[-\cos x\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[1+1\right] = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \sin x \cos nx\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} (2\cos nx \sin x)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \left[\sin(n+1)x - \sin(n-1)x\right] dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos(n+1)x}{(n+1)} + \frac{\cos(n-1)x}{(n-1)}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-\cos(n\pi+\pi)}{n+1} + \frac{\cos(n\pi-\pi)}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{1}{\pi}\left[\frac{-(-1)^n(-1)}{n+1} + \frac{(-1)^n(-1)}{n-1} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{1}{\pi}\left[(-1)^{n+1}\left(-\frac{1}{n+1} + \frac{1}{n-1}\right) - \left(-\frac{1}{n+1} + \frac{1}{n-1}\right)\right]$$

$$= \frac{1}{\pi}\left\{\left[(-1)^{n+1} - 1\right]\left[\frac{1}{n-1} - \frac{1}{n+1}\right]\right\}$$

$$= \frac{1}{\pi}\left\{\left[(-1)^{n+1} - 1\right]\left[\frac{2}{n^2-1}\right]\right\}$$

$$= \frac{2}{\pi(n^2-1)}\left[(-1)^{n+1} - 1\right]$$

a_n is not def. for $n^2-1=0$

$\therefore n=1$

$$a_1 = \frac{2}{\pi}\int_0^{\pi} \sin x \cos x \, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin 2x \, dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos 2x}{2}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{1}{2} + \frac{1}{2}\right] = 0$$

Subs in (I)

$$f(x) = \frac{2}{\pi} + 0 + \sum_{n=2}^{\infty} \frac{2[(-1)^{n+1} - 1]}{\pi(n^2-1)} \cos nx$$

$$\therefore \sin x = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty} \frac{[(-1)^{n+1} - 1]}{(n^2-1)} \cos nx$$

Put $x = \pi/2$ on both sides

$$1 = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty} \frac{[(-1)^{n+1} - 1]}{(n^2-1)} \cos\left(\frac{n\pi}{2}\right)$$

$$1 - \frac{2}{\pi} = \frac{2}{\pi}\sum_{n=2}^{\infty} \frac{[(-1)^{n+1} - 1]}{n^2-1} \cos\left(\frac{n\pi}{2}\right)$$

$$\frac{\pi-2}{\pi} = \cancel{1} + \frac{2}{\pi}\left[\frac{+2}{3} - \frac{2}{15} + \frac{2}{35} - - -\right]$$

$$\frac{\pi-2}{\pi} \cancel{-1} = -\frac{2\times 2}{\pi}\left[-\frac{1}{3} + \frac{1}{15} - \frac{1}{35} + - - -\right]$$

$$\frac{\pi-2}{\pi} = -\frac{2}{\pi}\left[-\frac{1}{1\times 3} + \frac{1}{3\times 5} - \frac{1}{5\times 7} + - - -\right]$$

$$\frac{2-\pi}{\pi} = \frac{2}{\pi}\left[\left(\frac{1}{1} + \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) - \left(\frac{1}{5} - \frac{1}{7}\right) + - - \right]$$

$$1 - \frac{\pi}{2} = \left[-1 + \frac{2}{3} - \frac{2}{5} + - - - -\right]$$

$$\frac{\pi}{2} = 1 + 1 - \frac{2}{3} + \frac{2}{5} - - - - -$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - - - - - -$$

Q28) {Since exp. of f(x) has only sine terms

∴ we find H.R.S.S.}

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{—(I)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\left[\int_0^{\pi/2} \underset{u}{mx}\,\underset{v}{\sin nx} + \int_{\pi/2}^{\pi} m\underset{u}{(\pi-x)}\,\underset{v}{\sin nx}\right]$$

$$= \frac{2m}{\pi}\left\{\left[(x)\left(-\frac{\cos nx}{n}\right)-(1)\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi/2} + \left[(\pi-x)\left(-\frac{\cos nx}{n}\right)-(-1)\left(-\frac{\sin nx}{n^2}\right)\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{2m}{\pi}\left\{\cancel{-\frac{\pi}{2n}\cos\left(\frac{n\pi}{2}\right)} + \frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right) + \cancel{\frac{\pi}{2n}\cos\left(\frac{n\pi}{2}\right)} + \frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\right\}$$

$$= \frac{4m}{\pi n^2}\sin\left(\frac{n\pi}{2}\right)$$

Subs. in (I)

$$f(x) = \sum_{n=1}^{\infty}\frac{4m}{\pi n^2}\sin\left(\frac{n\pi}{2}\right)\sin nx$$

$$= \frac{4m}{\pi}\left[\frac{1}{1^2}\sin\frac{\pi}{2}\sin x + 0 + \frac{1}{3^2}\sin\frac{3\pi}{2}\sin 3x + \cdots\right]$$

$$= \frac{4m}{\pi}\left[\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \cdots\right]$$

(Q29) $f(x) = x^2$ in $(0, \pi)$

{Since $f(x)$ has only sine term we find H.R.S.S.}

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \quad \text{---(I)}$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2 \sin nx\,dx$$

$$= \frac{2}{\pi}\left[(x^2)\left(\frac{-\cos nx}{n}\right) - (2x)\left(\frac{-\sin nx}{n^2}\right) + (2)\left(\frac{\cos nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\pi^2\left(\frac{-\cos n\pi}{n}\right) + \frac{2\cos n\pi}{n^3} - \frac{2\cos 0}{n^3}\right]$$

$$= \frac{2}{\pi}\left[-(-1)^n\frac{\pi^2}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3}\right]$$

Subs. in (I)

$$\therefore x^2 = \frac{2}{\pi}\sum_{n=1}^{\infty}\left[-\frac{(-1)^n\pi^2}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3}\right]\sin nx$$

$$= \frac{2}{\pi}\left[(\pi^2 - 2 - 2)\sin x + \left(-\frac{\pi^2}{2} + \cancel{\frac{2}{2^3}} - \cancel{\frac{2}{2^3}}\right)\sin 2x + \left(\frac{\pi^2}{3} - \frac{2}{3^3} - \frac{2}{3^3}\right)\sin 3x + \dots\right]$$

$$\therefore x^2 = \frac{2}{\pi}\left[\left(\frac{\pi^2}{1} - \frac{4}{1^3}\right)\sin x - \left(\frac{\pi^2}{2}\right)\sin 2x + \left(\frac{\pi^2}{3} - \frac{4}{3^3}\right)\sin 3x \dots\right]$$

(Q30) H.R.C.S. of $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$ —— (I)

$l = 1$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$a_0 = 2\int_0^1 e^x dx = 2[e^x]_0^1 = 2(e-1)$

$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{1}\int_0^1 e^x \cdot \cos(n\pi x)\,dx$

$= 2\left[\frac{e^x}{1^2+n^2\pi^2}\left(1\cdot\cos n\pi x + n\pi \sin n\pi x\right)\right]_0^1$

$= 2\left[\frac{e}{1+n^2\pi^2}\left((-1)^n\right) - \frac{e^0}{1+n^2\pi^2}(1)\right]$

$= \frac{2}{1+n^2\pi^2}\left[e(-1)^n - 1\right]$

Subs in (I)

$f(x) = (e-1) + 2\sum_{n=1}^{\infty} \frac{[e(-1)^n - 1]}{1+n^2\pi^2}\cos n\pi x$

(Q31) H.R.C.S is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{l}$ ——— (I)

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\left[\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\,dx\right]$$

$$= \frac{2k}{l}\left\{\left[\frac{x^2}{2}\right]_0^{l/2} + \left[lx - \frac{x^2}{2}\right]_{l/2}^{l}\right\}$$

$$= \frac{2k}{l}\left\{\frac{l^2}{8} + \cancel{l^2} - \cancel{\frac{l^2}{2}} - \cancel{\frac{l^2}{2}} + \frac{l^2}{8}\right\}$$

$$= \frac{2k}{l}\left\{\frac{2l^2}{8}\right\} = \frac{kl}{2}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$= \frac{2k}{l}\left\{\int_0^{l/2} x\cos\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} (l-x)\cos\frac{n\pi x}{l}\,dx\right\}$$

$$= \frac{2k}{l}\left\{\left[(x)\left(\frac{\sin(n\pi x/l)}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{l/2}\right.$$

$$+\left[(l-x)\left(\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right)-(-1)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_{l/2}^{l}$$

$$a_n=\frac{2k}{l}\left\{\left[\cancel{\frac{l}{2}\left(\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{l}}\right)}+\frac{\cos\frac{n\pi}{2}}{\frac{n^2\pi^2}{l^2}}-\frac{\cos 0}{\frac{n^2\pi^2}{l^2}}\right]\right.$$

$$\left.+\left[-\frac{\cos(n\pi)}{\frac{n^2\pi^2}{l^2}}\cancel{-\frac{l}{2}\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{l}}}+\frac{\cos\frac{n\pi}{2}}{\frac{n^2\pi^2}{l^2}}\right]\right\}$$

$$=\frac{2k}{l}\left\{\frac{1}{\frac{n^2\pi^2}{l^2}}\left(2\cos\frac{n\pi}{2}-1-(-1)^n\right)\right\}$$

$$=\frac{2kl}{n^2\pi^2}\left\{2\cos\left(\frac{n\pi}{2}\right)-1-(-1)^n\right\}$$

$$n\}$$

Subs in (I)

$$f(x)=\frac{kl}{4}+\frac{2kl}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left\{2\cos\frac{n\pi}{2}-1-(-1)^n\right\}\cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$= \frac{2}{l}\left[\int_0^{l/2} kx\,dx + \int_{l/2}^{l} k(l-x)\,dx\right]$$

$$\frac{2k}{l}\left\{\left[\frac{x^2}{2}\right]_0^{l/2} + \left[lx - \frac{x^2}{2}\right]_{l/2}^{l}\right\}$$

$$\frac{2k}{l}\left\{\frac{l^2}{8} + \cancel{l^2} - \cancel{\frac{l^2}{2}} - \frac{l^2}{2} + \frac{l^2}{8}\right\}$$

$$\frac{2k}{l}\left\{\frac{2l^2}{8_2}\right\} = \frac{kl}{2}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$\frac{2}{l}\left\{\int_0^{l/2} \frac{x}{4}\cos\left(\frac{n\pi x}{l}\right)dx + \int_{l/2}^{l} \frac{(l-x)}{4}\cos\frac{n\pi x}{2}\right.$$

$$\frac{2k}{l}\left\{\left[(x)\left(\frac{\sin(n\pi x/l)}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^{l/2}\right.$$

$$+\left[(l-x)\left(\frac{\sin\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right)-(-1)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_{l/2}^{l}$$

$$a_n=\frac{2k}{l}\left\{\left[\cancel{\frac{l}{2}\left(\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{l}}\right)}+\frac{\cos\frac{n\pi}{2}}{\frac{n^2\pi^2}{l^2}}-\frac{\cos 0}{\frac{n^2\pi^2}{l^2}}\right]\right.$$

$$\left.+\left[-\frac{\cos(n\pi)}{\frac{n^2\pi^2}{l^2}}\cancel{-\frac{l}{2}\frac{\sin\frac{n\pi}{2}}{\frac{n\pi}{l}}}+\frac{\cos\frac{n\pi}{2}}{\frac{n^2\pi^2}{l^2}}\right]\right\}$$

$$=\frac{2k}{l}\left\{\frac{1}{\frac{n^2\pi^2}{l^2}}\left(2\cos\frac{n\pi}{2}-1-(-1)^n\right)\right\}$$

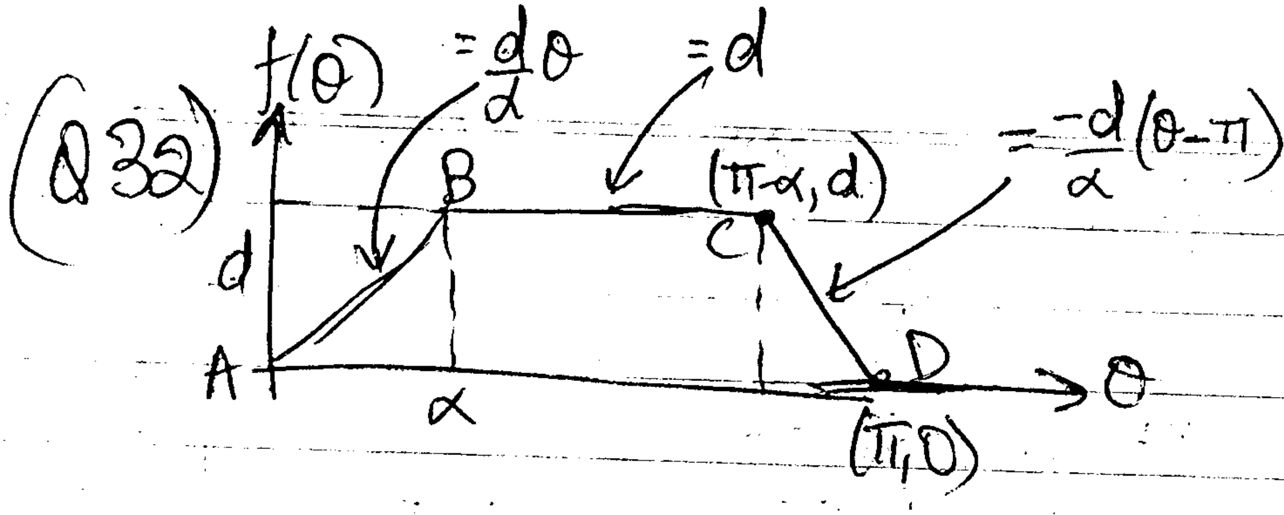
$$=\frac{2kl}{n^2\pi^2}\left\{2\cos\left(\frac{n\pi}{2}\right)-1-(-1)^n\right\}$$

dn

Subs in (I)

$$f(x)=\frac{kl}{4}+\frac{2kl}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left\{2\cos\frac{n\pi}{2}-1-(-1)^n\right\}\cos\left(\frac{n\pi x}{l}\right)$$

(Q32)



AB:- $f(\theta) = \frac{d}{\alpha}\theta \quad ; \quad 0<\theta<\alpha$

BC:- $= d \quad ; \quad \alpha<\theta<\pi-\alpha$

CD:- $= -\frac{d}{\alpha}(\theta-\pi) \quad ; \quad \pi-\alpha<\theta<\pi$

H.R.S.S. is $f(\theta) = \sum_{n=1}^{\infty} b_n \sin n\theta$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin n\theta \, d\theta$$

$$= \frac{2d}{\pi}\left[\frac{1}{\alpha}\int_0^{\alpha}\theta\sin n\theta\,d\theta + \int_{\alpha}^{\pi-\alpha}\sin n\theta\,d\theta - \frac{1}{\alpha}\int_{\pi-\alpha}^{\pi}(\theta-\pi)\sin n\theta\,d\theta\right]$$

$$b_n = \frac{2d}{\pi}\left\{ \frac{1}{\alpha}\left[\theta\left(-\frac{\cos n\theta}{n}\right) - (1)\left(-\frac{\sin n\theta}{n^2}\right)\right]_0^{\alpha} + \left[-\frac{\cos n\theta}{n}\right]_{\alpha}^{\pi-\alpha} - \frac{1}{\alpha}\left[(\theta-\pi)\left(-\frac{\cos n\theta}{n}\right) - (1)\left(-\frac{\sin n\theta}{n^2}\right)\right]\right.$$

$$= \frac{2d}{\pi}\left\{ \frac{1}{\alpha}\left[-\frac{\alpha\cos n\alpha}{n} + \frac{\sin n\alpha}{n^2}\right] - \left[\frac{\cos(n\pi - n\alpha)}{n}\right.\right.$$

$$\left. - \frac{1}{\alpha}\left[\alpha\left(-\frac{\cos n(\pi-\alpha)}{n}\right) - \frac{\sin(n\pi - n\alpha)}{n^2}\right]\right\}$$

$$= \frac{2d}{\pi}\left\{\cancel{-\frac{\cos n\alpha}{n}} + \frac{\sin n\alpha}{\alpha n^2} - \cancel{\left[\frac{(-1)^n\cos n\alpha}{n}\right] + \cos}\right.$$

$$\left. \cancel{+\frac{(-1)^n\cos n\alpha}{n}} - \frac{(-1)^n \sin n\alpha}{n^2\alpha}\right\}$$

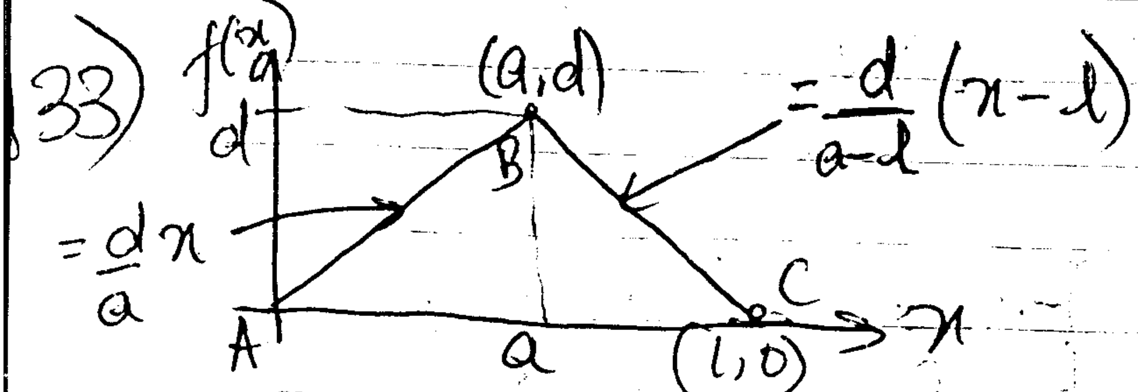
$$= \frac{2d}{\pi}\left\{\frac{\sin n\alpha}{\alpha n^2} - \frac{(-1)^n \sin n\alpha}{\alpha n^2}\right\}$$

$$= \frac{2d \sin n\alpha}{n^2\alpha\pi}\left\{1 - (-1)^n\right\}$$

Subs. in (I)

$$f(\theta) = \frac{2d}{\pi\alpha}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin n\alpha\left[1-(-1)^n\right]\sin n\theta$$

33)



B: $f(x) = \frac{dx}{a}$; $0 < x < a$

C: $= \frac{d}{a-l}(x-l)$; $a < x < l$

H.R.S.S is $f(x) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}$ (σ)

$$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$$

$$\frac{2}{l}\left\{\int_0^a \frac{dx}{a}\sin\left(\frac{n\pi x}{l}\right)dx + \int_a^l \frac{d}{(a-l)}(x-l)\sin\left(\frac{n\pi x}{l}\right)dx\right.$$

$$\left\{\frac{d}{a}\int_0^a x\sin\left(\frac{n\pi x}{l}\right)dx + \frac{d}{a-l}\int_a^l (x-l)\sin\left(\frac{n\pi x}{l}\right)dx\right.$$

$$= \frac{2}{l}\left\{ \frac{d}{a}\left[x\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_0^a \right.$$

$$\left. + \frac{d}{a-l}\left[(x-l)\left(\frac{-\cos\left(\frac{n\pi x}{l}\right)}{\frac{n\pi}{l}}\right) - (1)\left(\frac{-\sin\left(\frac{n\pi x}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right)\right]_a^l \right.$$

$$= \frac{2}{l}\left\{ \frac{d}{a}\left[\frac{-a\cos\left(\frac{n\pi a}{l}\right)}{\frac{n\pi}{l}} + \frac{\sin\left(\frac{n\pi a}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right]\right.$$

$$\left. + \frac{d}{a-l}\left[(a-l)\left(\frac{\cos\left(\frac{n\pi a}{l}\right)}{\frac{n\pi}{l}}\right) - \frac{\sin\left(\frac{n\pi a}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right]\right\}$$

$$= \frac{2}{l}\left\{ \cancel{\frac{-d\cos\left(\frac{n\pi a}{l}\right)}{\frac{n\pi}{l}}} + \frac{d}{a}\,\frac{\sin\left(\frac{n\pi a}{l}\right)}{\frac{n^2\pi^2}{l}}\right.$$

$$\left. + \cancel{\frac{d\cos\left(\frac{n\pi a}{l}\right)}{\frac{n\pi}{l}}} - \frac{d}{(a-l)}\,\frac{\sin\left(\frac{n\pi a}{l}\right)}{\frac{n^2\pi^2}{l^2}}\right]$$

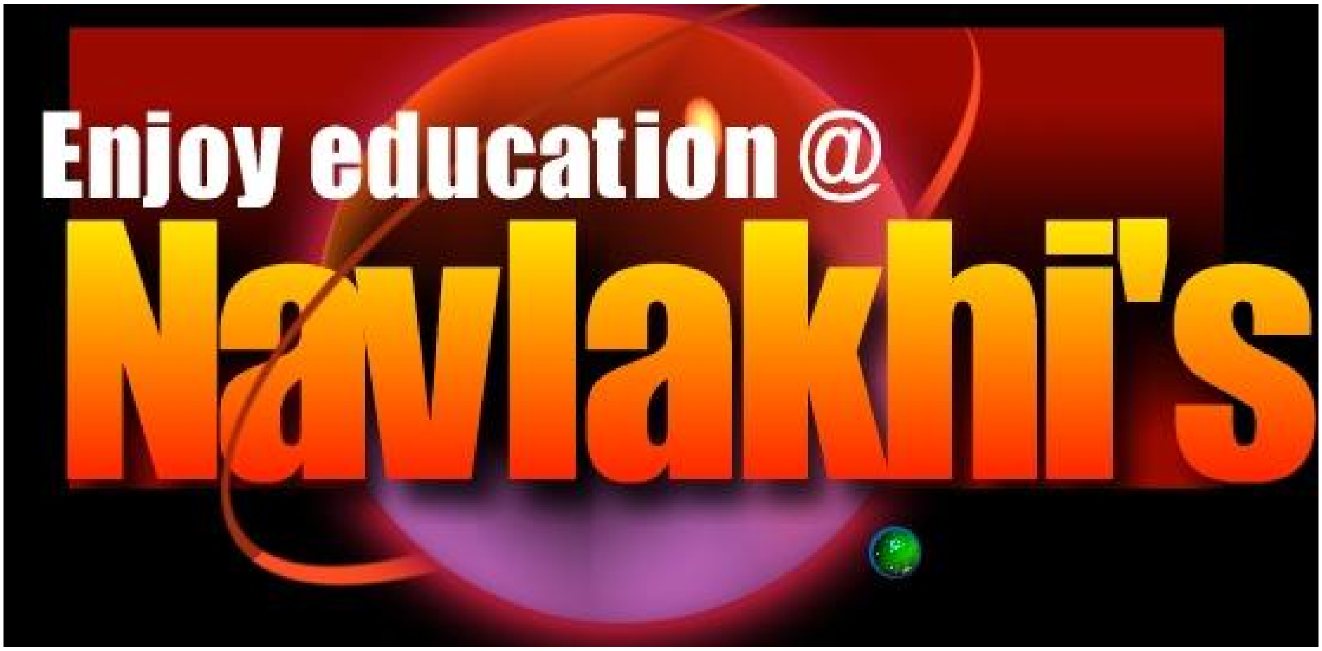
$$= \frac{2}{l} \frac{\sin\left(\frac{n\pi a}{l}\right)}{\frac{n^2\pi^2}{l^2}} \left[\frac{d}{a} - \frac{d}{a-l} \right]$$

$$= \frac{2ld}{n^2\pi^2} \sin\left(\frac{n\pi a}{l}\right) \left[\frac{-l}{a(a-l)} \right]$$

$$= \frac{2dl^2}{\pi^2 n^2 a(l-a)} \cdot \sin\left(\frac{n\pi a}{l}\right)$$

Subs in (I)

$$f(x) = \sum_{n=1}^{\infty} \frac{2dl^2}{\pi^2 n^2 a(l-a)} \sin\left(\frac{n\pi a}{l}\right) \sin\left(\frac{n\pi x}{l}\right)$$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Q1 $f(x) = e^{ax}$ in $(-\pi, \pi)$

C.F.S. is $f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$ — (i)

$$C_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx}\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{ax} . e^{-inx}\, dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{x(a-in)}\, dx$$

$$= \frac{1}{2\pi}\left[\frac{e^{x(a-in)}}{(a-in)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi(a-in)}\left[e^{\pi(a-in)} - e^{-\pi(a-in)}\right]$$

$$= \frac{1}{2\pi(a-in)}\left[e^{\pi a} . e^{-i\pi n} - e^{-\pi a} . e^{i\pi n}\right]$$

$$\left\{\begin{array}{l} e^{-i\pi n} = \cos n\pi - i\sin n\pi = (-1)^n \\ e^{in\pi} = \cos n\pi + i\sin n\pi = (-1)^n \end{array}\right.$$

$$C_n = \frac{(-1)^n}{2\pi(a-in)}\left[e^{a\pi} - e^{-a\pi}\right]$$

$$= \frac{(-1)^n}{\cancel{2}\pi(a-in)}\left[\cancel{2}\sinh a\pi\right] \times \left(\frac{a+in}{a+in}\right)$$

$$= \frac{(-1)^n(a+in)}{\pi(a^2+n^2)} \sinh a\pi$$

Subs. in eq (I)

$$\therefore f(x) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n(a+in)}{\pi(a^2+n^2)} \sinh a\pi . e^{inx}$$

$$e^{ax} = \frac{\sinh a\pi}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n(a+in)}{(a^2+n^2)} e^{inx} \quad \text{—— (II)}$$

(i). $\cos \alpha x = \dfrac{e^{i\alpha x} + e^{-i\alpha x}}{2}$ —— (III)

From (II) { put $a = i\alpha$ }

$$e^{i\alpha x} = \frac{\sinh(i\alpha\pi)}{\pi} \sum_{n=-\infty}^{\infty} \frac{(-1)^n(i\alpha+in)}{n^2-\alpha^2} e^{inx}$$

$$e^{i\alpha x} = \frac{i\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}\frac{i(-1)^n(\alpha+n)}{(n^2-\alpha^2)}e^{inx}$$

$$= -\frac{\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{(n+\alpha)}{n^2-\alpha^2}e^{inx} \quad \text{———(IV)}$$

Put $a = -i\alpha$ in (II)

$$e^{-i\alpha x} = \frac{\sinh(-i\alpha\pi)}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{(-i\alpha+in)}{(n^2-\alpha^2)}e^{inx}$$

$$= -\frac{\sinh(i\alpha\pi)}{\pi}\sum_{n=-\infty}^{\infty}i(-1)^n\frac{(n-\alpha)}{n^2-\alpha^2}e^{inx}$$

$$= -\frac{i^2\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{(n-\alpha)}{n^2-\alpha^2}e^{inx}$$

$$= \frac{\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{(n-\alpha)}{n^2-\alpha^2}e^{inx} \quad \text{—(V)}$$

Put (IV) & (V) in (III)

$$\therefore \cos\alpha x = \frac{1}{2}\left\{\frac{\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n e^{inx}\left(\frac{-n-\alpha}{n^2-\alpha^2}+\frac{n-\alpha}{n^2-\alpha^2}\right)\right.$$

$$= \frac{1}{\cancel{2}}\left\{\frac{\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n e^{inx}\left[\frac{-\cancel{2}\alpha}{n^2-\alpha^2}\right]\right\}$$

$$\cos\alpha x = \frac{\sin\alpha\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n e^{inx}\left(\frac{\alpha}{\alpha^2-n^2}\right)$$

(ii) Put $x=0$ in eq (II)

$$\therefore 1 = \frac{\sinh a\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\left(\frac{a}{a^2+n^2}+\frac{in}{a^2+n^2}\right)$$

Eq. real parts.

$$1 = \frac{\sinh a\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\cdot\frac{a}{a^2+n^2}$$

$$\therefore \frac{\pi}{a\sinh a\pi} = \sum_{n=-\infty}^{\infty}\frac{(-1)^n}{a^2+n^2}$$

NOTE: Parseval's Result:-

If $f(x)$ is def. in $(-\pi,\pi)$ the f.s is

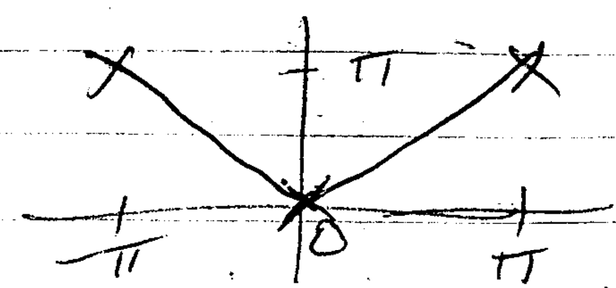
$$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin$$

then by parsevals result

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}[f(x)]^2\,dx = \frac{a_0^2}{4}+\frac{1}{2}\sum_{n=1}^{\infty}(a_n^2+$$

where a_0, a_n & b_n are fourie
co-eff. & LHS is power of [f(

2) When $x = -\pi$; $f(x) = \pi$; $(-\pi, \pi$
$x = 0$; $f(x) = 0$; $(0,0)$
$x = 0$; $f(x) = 0$; $(0,0)$
$x = \pi$; $f(x) = \pi$; $(\pi, \pi$



Symm ab
of axi
∴ $f(x)$ is

F.S is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n$
— (

) $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)\,dx$

$= \frac{2}{\pi}\int_0^{\pi} x\,dx = \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} =$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$\frac{2}{\pi}\left[(x)\left(\frac{\sin nx}{n}\right) - (1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$\frac{2}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi} = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

Subs. in (I)

$$f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[(-1)^n - 1\right]\cos nx$$

$$b_n = 0$$

$$\text{Power} = \frac{1}{2\pi}\int_{-\pi}^{\pi} g(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}[f(x)]^2\,dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\,dx$$

By Parseval result

$$\text{Power} = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

$$= \frac{\pi^2}{4} + \frac{4}{2\pi^2}\sum_{n=1}^{\infty} \frac{\left[(-1)^n - 1\right]^2}{n^4}$$

$$\therefore \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{\pi^2}{4} + \frac{4}{2\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^{2n} - 2(-1)^n + 1}{n^4}$$

$$\frac{1}{2\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{\pi^2}{4} + \frac{4}{2\pi^2}\sum_{n=1}^{\infty} \frac{2 - 2(-1)^n}{n^4}$$

$$\frac{1}{2\pi}\left[\frac{\pi^3}{3} + \frac{\pi^3}{3}\right] = \frac{\pi^2}{4} + \frac{8}{2\pi^2}\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^4}$$

$$\frac{\pi^2}{3} - \frac{\pi^2}{4} = \frac{8}{2\pi^2}\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^4}$$

$$\frac{\pi^2}{12} = \frac{8}{2\pi^2}\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^4}$$

$$\frac{\pi^4}{48} = \sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^4}$$

$$\frac{\pi^4}{48} = \frac{2}{1^4} + \frac{2}{3^4} + \frac{2}{5^4} + \cdots$$

$$\therefore \frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$$

TYPE II : FOURIER INTEGRAL

i) Fourier (Complex) integral of f(x) is

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(u)e^{i\lambda u}\,du\right]e^{-i\lambda x}\,d\lambda$$

ii) If f(x) is odd then Fourier Sine int of f(x) is –

$$f(x) = \frac{2}{\pi}\int_{0}^{\infty}\left[\int_{0}^{\infty} f(u)\sin\lambda u\,du\right]\sin\lambda x\,d\lambda$$

iii) If f(x) is even then Fourier Cosine int of f(x)

$$f(x) = \frac{2}{\pi}\int_{0}^{\infty}\left[\int_{0}^{\infty} f(u)\cos\lambda u\,du\right]\cos\lambda x\,d\lambda$$

3) F. Sine Int. is

$$f(x) = \frac{2}{\pi}\int_0^\infty \left[\int_0^\infty f(u)\sin\lambda u\,du\right]\sin\lambda x\,d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \left[\int_0^\infty e^{-\beta u}\sin\lambda u\,du\right]\sin\lambda x\,d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \left[\frac{e^{-\beta u}}{\beta^2+\lambda^2}(-\beta\sin\lambda u - \lambda\cos\lambda u)\right]_0^\infty \sin\lambda x\,d\lambda$$

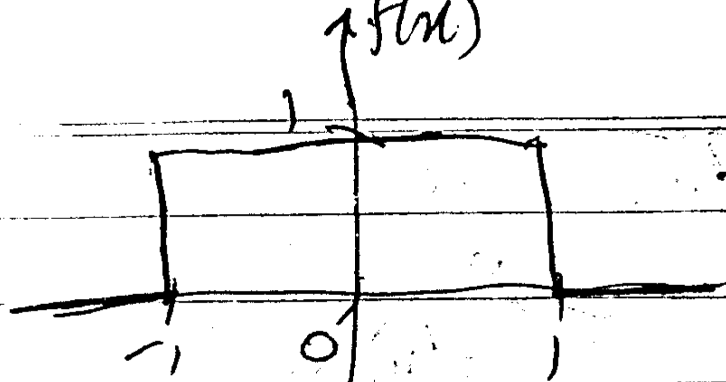
$$= \frac{2}{\pi}\int_0^\infty \left[\frac{1}{\beta^2+\lambda^2}(\lambda\cos 0)\right]\sin\lambda x\,d\lambda$$

i.e. $\therefore f(x) = \frac{2}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x}{\beta^2+\lambda^2}\,d\lambda$

$\therefore e^{-\beta x} = \frac{2}{\pi}\int_0^\infty \frac{\lambda\sin\lambda x}{\beta^2+\lambda^2}\,d\lambda$

$\therefore \frac{\pi}{2}e^{-\beta x} = \int_0^\infty \frac{\lambda\sin\lambda x}{\beta^2+\lambda^2}\,d\lambda$

1)



$f(x)$ is symm. about y-axis

$\therefore f(x)$ is even

$\therefore$ F. Cos Int is

$$f(x) = \frac{2}{\pi}\int_0^\infty \left[\int_0^\infty f(u)\cos \lambda u \, du\right]\cos \lambda x$$

$$= \frac{2}{\pi}\int_0^\infty \left[\int_0^1 1.\cos \lambda u \, du + \int_1^\infty 0.du\right]\cos \lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \left[\frac{\sin \lambda u}{\lambda}\right]_0^1 . \cos \lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \frac{\sin \lambda}{\lambda}\cos \lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} \, d\lambda$$

$$\frac{\pi}{2} = \int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} \, d\lambda; \text{ if } |x| \le 1$$

$$= 0 \quad ; \text{ if } |x| > 1$$

$\therefore$ At $|x|=1$ there is a discontinuity

Int is $= \frac{f(1^-)+f(1^+)}{2} = \left(\frac{\pi}{2}+0\right)/2$

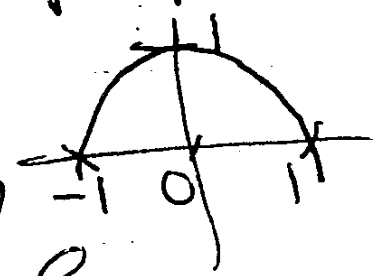
$= \pi/4$

5) $f(x) = 1-x^2$ (∩ shaped parab.)

When $x=0$; $f(x)=1$

$x=1$; $f(x)=0$

$x=-1$; $f(x)=0$



Symm. about 'y-axis' $\therefore$ Even

f. Cos. Int. is

$$f(x) = \frac{2}{\pi}\int_0^{\infty}\left[\int_0^{\infty} f(u)\cos\lambda u\,du\right]\cos\lambda x\,d\lambda$$

$$= \frac{2}{\pi}\int_0^{\infty}\left[\int_0^{1}(1-u^2)\cos\lambda u\,du\right]\cos\lambda x\,d\lambda$$

$$= \frac{2}{\pi}\int_0^{\infty}\left[(1-u^2)\left(\frac{\sin\lambda u}{\lambda}\right)-(-2u)\left(\frac{-\cos\lambda u}{\lambda^2}\right)\right.$$

$$\left.+(-2)\left(-\frac{\sin\lambda u}{\lambda^3}\right)\right]_0^1 \cos\lambda x\,d\lambda$$

$$f(x) = \frac{2}{\pi}\int_0^\infty \left[\frac{-2\cos\lambda}{\lambda^2} + \frac{2\sin\lambda}{\lambda^3}\right]\cos\lambda x \, d\lambda$$

$$f(x) = \frac{4}{\pi}\int_0^\infty \left(\frac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\right)\cos\lambda x \, d\lambda$$

Q6) F. Sine Int is

$$f(x) = \frac{2}{\pi}\int_0^\infty \left[\int_0^\infty f(u)\sin\lambda u \, du\right]\sin\lambda x \, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \left[\int_0^\infty \frac{e^{-au}}{u}\sin\lambda u \, du\right]\sin\lambda x \, d\lambda \quad \text{---(I)}$$

Let $I(\lambda) = \int_0^\infty \frac{e^{-au}}{u}\sin\lambda u \, du$ ---(II)

By DUIS: $I'(\lambda) = \int_0^\infty \frac{\partial}{\partial\lambda}\left(\frac{e^{-au}}{u}\sin\lambda u\right) du$

$$= \int_0^\infty \frac{e^{-au}}{u} \cdot u\cos\lambda u \, du$$

$$= \int_0^\infty e^{-au}\cos\lambda u \, du$$

$$= \left[\frac{e^{-au}}{a^2+\tau^2}\left(-a\cos\tau u+\tau\sin\tau u\right)\right]_0^{\infty}$$

$$= \left[\frac{1}{a^2+\tau^2}(a\cos 0)\right] = \frac{a}{a^2+\tau^2}$$

$$\therefore I(\tau) = a\int \frac{d\tau}{a^2+\tau^2} = a\cdot\frac{1}{a}\tan^{-1}\left(\frac{\tau}{a}\right)$$

$$I(\tau) = \tan^{-1}\left(\frac{\tau}{a}\right)+c \quad \text{——— (II)}$$

Put $\tau = 0$; $I(0) = c$ in (II)

$\tau = 0$ in (II) $\therefore I(0) = 0$

$\therefore c = 0$

$$\therefore I(\tau) = \tan^{-1}\left(\frac{\tau}{a}\right)$$

Subs. $I(\tau)$ in (I)

$$\therefore f(x) = \frac{2}{\pi}\int_0^{\infty}\tan^{-1}\left(\frac{\tau}{a}\right)\cdot\sin\tau x$$

TYPE III : ORTHOGONAL & ORTHONORMAL FUNCTIONS :-

Two signals $x_1(t)$ & $x_2(t)$ are orthogonal over closed interval $[t_1, t_2]$ if

$$\int_{t_1}^{t_2} x_n(t) \cdot x_m(t)\, dt = 0 ; m \neq n$$
$$= E_n ; m = n$$

If $x_1(t)$ & $x_2(t)$ are complex then the signals are orthogonal over closed interval $[t_1, t_2]$ if

$$\int_{t_1}^{t_2} x_1(t)\, \overline{x_2}(t)\, dt = 0 ; m \neq n$$
$$= E_n ; m = n$$

$$\int_{t_1}^{t} \overline{x_1}(t) \cdot x_2(t)\, dt = 0 ; m \neq n$$
$$= E_n ; m = n$$

where $\overline{x_1}(t)$ is complex conjugate of $x_1(t)$
$\overline{x_2}(t)$ " " " " $x_2(t)$

7) Show that the foll. signals are orthogonal over [0, 1] if
$f(t) = 1$; $x(t) = \sqrt{3}(1-2t)$

s) For orthogonal signal

$$\int_{t_1}^{t_2} x(t)\, f(t)\, dt = 0$$

$$\int_0^1 1 \times \sqrt{3}(1-2t)\,dt = \sqrt{3}\int_0^1 (1-2t)\,dt$$

$$= \sqrt{3}\left[t - \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^1 = \sqrt{3}\,[1-1] = 0$$

$\therefore$ $x(t)$ & $f(t)$ are orthogonal.

8) Show that the signal set $\{1, \cos\omega_0 t, \cos 2\omega_0 t \ldots \cos n\omega_0 t, \sin\omega_0 t, \sin 2\omega_0 t \ldots \sin n\omega_0 t\}$ are orthogonal over an interval $T_0 = \frac{2\pi}{\omega_0}$

8) ii) Consider orthogonality of cosine waves

Let $x_n(t) = \cos n\omega_0 t$; $x_m(t) = \cos m\omega_0 t$

$$I_1 = \int_t^{t+T_0} x_n(t) \cdot x_m(t)\, dt$$

$$= \int_t^{t+T_0} \cos n\omega_0 t \cdot \cos m\omega_0 t\, dt \quad \text{—(I)}$$

$$= \frac{1}{2}\left[\int_t^{t+T_0} \cos(n+m)\omega_0 t\, dt + \int_t^{t+T_0} \cos(n-m)\omega_0 t\, dt\right]$$

$$= \frac{1}{2}\left[\frac{\sin(n+m)\omega_0 t}{(n+m)\omega_0}\right]_t^{t+T_0} + \frac{1}{2}\left[\frac{\sin(n-m)\omega_0 t}{(n-m)\omega_0}\right]_t^{t+T_0}$$

If $n \neq m$; for integer values of m & n for one time period

$I_1 = 0$

If $n = m$; ~~$\cos(n-m)\omega_0 t = 1$~~

$$\int_t^{t+T_0} x_n(t)\, x_n(t)\, dt \quad \{\text{From (I)}\}$$

$$= \left[\int_t^{t+T_0} \cos^2 n\omega_0 t\, dt\right]$$

$$= \frac{1}{2}\left[\int_t^{t+T_0} 1 + \cos 2n\omega_0 t\right] dt$$

$$= \frac{1}{2}\left[T_0 + 0\right] \quad \left\{\text{For integer value of } n \;\; \cos 2n\omega_0 t = 0\right\}$$

$$= T_0/2$$

$$\therefore \int_t^{t+T_0} \cos n\omega_0 t \cos m\omega_0 t \, dt = \begin{cases} 0 \; ; & n \neq m \\ T_0/2 \; ; & n = m \end{cases}$$

ii) Consider orthog. of sine waves

Let $x_n(t) = \sin n\omega_0 t$; $x_m(t) = \sin m\omega_0 t$

$$I_2 = \int_t^{t+T_0} x_n(t)\, x_m(t)\, dt$$

$$= \int_t^{t+T_0} \sin n\omega_0 t \sin m\omega_0 t \, dt \quad \text{— (ii)}$$

$$= \frac{1}{2}\left[\int_t^{t+T_0} \cos (n-m)\omega_0 t \, dt - \int_t^{t+T_0} \cos (n+m)\omega_0 t \, dt\right]$$

$$= \frac{1}{2}\left[\frac{\sin (n-m)\omega_0 t}{(n-m)\omega_0}\right]_t^{t+T_0} - \frac{1}{2}\left[\frac{\sin (n+m)\omega_0 t}{(n+m)\omega_0}\right]$$

If $n \neq m$; for int. values of m & n for one time period $\therefore I_2 = 0$

If $n = m$ $\int_t^{t+T_0} x_n^2(t)\,dt$ {from (II)}

$$= \left[\int_t^{t+T_0} \sin^2 n\omega_0 t\,dt\right]$$

$$= \left[\int_t^{t+T_0} \left[\frac{1-\cos 2n\omega_0 t}{2}\right]\right] = \frac{T_0}{2}$$

$$\therefore \int_t^{t+T_0} \sin n\omega_0 t \sin m\omega_0 t\,dt = \begin{cases} 0 & ; n \neq m \\ T_0/2 & ; n = m \end{cases}$$

ii) Cons. orthog. of $\sin n\omega_0 t$ & $\cos m\omega_0 t$

Let $x_n(t) = \sin n\omega_0 t$; $x_m(t) = \cos m\omega_0 t$

$$I_3 = \int_t^{t+T_0} x_n(t) \cdot x_m(t)\,dt$$

$$= \int_t^{t+T_0} \sin n\omega_0 t \cos m\omega_0 t \quad \text{— (III)}$$

$$= \frac{1}{2}\left[\int_t^{t+T_0} \sin(n+m)\omega_0 t\,dt + \int_t^{t+T_0} \sin(n-m)\omega_0 t\,dt\right]$$

$$= -\frac{1}{2}\left[\frac{\cos(n+m)\omega_0 t}{(n+m)\omega_0}\right]_t^{t+T_0} - \frac{1}{2}\left[\frac{\cos(n-m)\omega_0 t}{(n-m)\omega_0}\right]_t^{t+T_0}$$

{LAST PG}

Q9) P.T. the set of exponentials $1, e^{\pm j\omega_0 t}, e^{\pm 2j\omega_0 t}, e^{\pm j3\omega_0 t}$ ---- is orthogonal over any interval T_0

Ans9) Let $x_m(t) = e^{jm\omega_0 t}$; $x_n(t) = e^{jn\omega_0 t}$

$$\int_t^{t+T_0} x_m(t)\overline{x_n(t)}\,dt$$

$$= \int_t^{t+T_0} e^{j\omega_0 mt} \cdot e^{-jn\omega_0 t}\,dt$$

$$= \int_t^{t+T_0} e^{j(m-n)\omega_0 t}\,dt = \left[\frac{e^{j(m-n)\omega_0 t}}{j(m-n)\omega_0}\right]_t^{t+T_0}$$

$$= \frac{1}{j(m-n)\omega_0}\left[e^{j(m-n)\omega_0(t+T_0)} - e^{j(m-n)\omega_0 t}\right]$$

$$= \frac{1}{j(m-n)\omega_0}\left[e^{j(m-n)\omega_0 t} \cdot e^{j(m-n)\omega_0 T_0} - e^{j(m-n)\omega_0 t}\right]$$

$$= \frac{1}{j(m-n)\omega_0} e^{j(m-n)\omega_0 t}\left[e^{j(m-n)\omega_0 T_0} - 1\right]$$

$\because \omega_0 = 2\pi/T_0 \quad \therefore \omega_0 T_0 = 2\pi$

$$= \frac{e^{j(m-n)\omega_0 t}}{j(m-n)\omega_0}\left[e^{j(m-n)2\pi} - 1\right]$$

For integer values of m & n

$e^{j(m-n)2\pi} = 1$

$$I_1 = \frac{e^{j(m-n)\omega_0 t}}{j(m-n)\omega_0}[1-1]$$

$= 0$

When $m = n$,

$$I_2 = \int_t^{t+T_0} x_m(t) \cdot \overline{x_m(t)}\, dt$$

$$= \int_t^{t+T_0} e^{jm\omega_0 t} \cdot e^{-jm\omega_0 t}\, dt$$

$$= \int_t^{t+T_0} dt = [t]_t^{t+T_0}$$

$= t + T_0 - t = T_0$

$$\int_t^{t+T_0} e^{jm\omega_0 t} e^{-jn\omega_0 t}\, dt = \begin{cases} 0; & m \neq n \\ T_0; & m = n \end{cases}$$

Orthonormal Signals –

Signals $x_1(t), x_2(t), x_3(t), \ldots$ are orthogonal over $[t_1, t_2]$

if $\int_{t_1}^{t_2} x_m(t)\, x_n(t)\, dt = \begin{cases} 0; & m \neq \\ E_n; & m = \end{cases}$

Where E_n is energy of signal $x(t)$. if $E_n = 1$ for all values then signals are orthonormal.

To normalize the orthogonal signal set we divide $x_n(t)$ by

10) Normalise the signal set $e^{jn\omega_0 t}$ for $n = 0, \pm 1, \pm 2, \ldots$ over

s) Let $x_n(t) = e^{j\omega_0 nt}$

$$E_n = \int_t^{t+T_0} x_n(t)\, \overline{x_n(t)}\, dt$$

$$= \int_t^{t+T_0} e^{jn\omega_0 t}\, e^{-jn\omega_0 t}\, dt$$

$$\therefore E_n = \int_t^{t+T_0} dt = [t]_t^{t+T_0} = T_0$$

$$\therefore E_n = T_0$$

$\therefore$ Normalize signal set is

$$\frac{x_n(t)}{\sqrt{E_n}} = \frac{e^{jn\omega_0 t}}{\sqrt{T_0}}, \quad n = 0, \pm 1, \pm 2, \ldots$$

$$\therefore \int_t^{t+T_0} \frac{e^{jn\omega_0 t}}{\sqrt{T_0}} \left[\frac{e^{-jm\omega_0 t}}{\sqrt{T_0}}\right] dt = \begin{cases} 0 & ; m \neq n \\ 1 & ; m = n \end{cases}$$

i) If $x(t)$ & $y(t)$ are orthogonal then S.T. energy of $x(t) + y(t)$ is identical to energy of $x(t)$ plus energy of $y(t)$

Soln) Let energy of $x(t)$ be

$$E_x = \int_{-\infty}^{+\infty} x^2(t)\,dt$$

$$\& \ E_y = \int_{-\infty}^{+\infty} y^2(t)\,dt$$

$$E_{x+y} = \int_{-\infty}^{+\infty} [x(t)+y(t)]^2 dt$$

$$= \int_{-\infty}^{+\infty} [x^2(t)+y^2(t)+2x(t)y(t)] dt$$

$$= \int_{-\infty}^{+\infty} x^2(t) dt + \int_{-\infty}^{+\infty} y^2(t) dt + 2\int_{-\infty}^{+\infty} x(t)y(t) dt$$

$\because$ $x(t)$ & $y(t)$ are orthogonal

$$\therefore \int_{-\infty}^{+\infty} x(t)\, y(t)\, dt = 0$$

$$\therefore E_{x+y} = \int_{-\infty}^{+\infty} x^2(t) dt + \int_{-\infty}^{+\infty} y^2(t) dt$$

$$= E_x + E_y.$$

Q12) S.T. over period 0 to 2π a rectangular funct. is orthogonal to signals $\cos t$, $\cos 2t$, ... $\cos nt$ for integer values of n.

) A rectangular function is given as $f(t) = \begin{cases} 1 & ; 0 \leq t \leq 2\pi \\ 0 & ; \text{otherwise} \end{cases}$

& $x(t) = \cos nt$

Let $I = \int_{t}^{t+T_0} x(t) f(t) \, dt$

$= \int_{t}^{t+T_0} f(t) \cos nt \, dt = \int_{0}^{2\pi} 1 \cdot \cos nt \, dt$

$= \left[\frac{\sin nt}{n} \right]_0^{2\pi} = \frac{\sin 2n\pi - \sin 0}{n}$

for int. values of n

$I = 0$

$\therefore$ $x(t)$ & $f(t)$ are orthogonal for $[0, 2\pi]$

If $n \neq m$; for int. values of m & n for one T.P. $\therefore I_3 = 0$

If $n = m$; $\int_t^{t+T_0} x_n^2(t)\,dt$ {from (III)}

$= \int_t^{t+T_0} \sin^2 n\omega_0 t\,dt = 0$

$\int_t^{t+T_0} \sin n\omega_0 t \cos n\omega_0 t\,dt = 0$, for all values of m & n

GOOD BAD

with YOU at all times

NAVLAKHI's