Total Marks: 80 **Hours: 3 hrs**

Note: 1. Question no. **1** is compulsory.
2. Attempt any **three** questions out of remaining **five** questions.

Q.1. [a] Evaluate $\int_0^{\infty} 5^{-4x^2}\,dx$. [3]

[b] Solve $\frac{dy}{dx} = xy$ with the help of Euler's method, given that $y(0) = 1$, and find y when $x = 0.3$ $(h = 0.1)$. [3]

[c] Evaluate $\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = 0$. [3]

[d] Evaluate $\int_0^1 \sqrt{\sqrt{x} - x}\,dx$. [3]

[e] Solve $(1 + \log xy)dx + \left(1 + \frac{x}{y}\right)dy = 0$. [4]

[f] Evaluate $\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dxdy}{1+x^2+y^2}$. [4]

Q.2.[a] Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. [6]

[b] Find the area inside the circle $r = a\sin\theta$ and outside the cardioide $r = a(1 + \cos\theta)$. [6]

[c] Apply Runge-kutta Method of fourth order to find an approximate value of y when $x = 0.2$ given that $\frac{dy}{dx} = x + y$ when $y = 1$ at $x = 0$ with step size $h = 0.2$. [8]

Q.3.[a] Show that the length of the curve $9ay^2 = x(x-3a)^2$ is $4\sqrt{3}a$. [6]

[b] Change the order of the integration of $\int_0^1 \int_{-\sqrt{2y-y^2}}^{1+\sqrt{1-y^2}} f(x,y)\,dx\,dy$. [6]

[c] Find the volume of the paraboloid $x^2 + y^2 = 4z$ cut off by the plane $z = 4$. [8]

Q.4. [a] Show that $\int_0^1 \frac{x^a - 1}{\log x}dx = \log(a+1)$. [6]

[b] If y satisfies the equation $\frac{dy}{dx} = x^2y - 1$ with $x_0 = 0$, $y_0 = 1$, using Taylor's Series Method find y at $x = 0.1$ (take $h = 0.1$). [6]

[c] Find the value of the integral $\int_0^1 \frac{x^2}{1+x^3}dx$ using (i) Trapezoidal rule (ii) Simpson's $1/3^{rd}$ rule (iii) Simpson's $3/8^{th}$ rule. [8]

Q.5.[a] Solve $(y - xy^2)dx - (x + x^2y)dy = 0.$ [6]

[b] Evaluate $\iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\, dx\, dy\, dz$ over the ellipsoid [6]
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

[c] Evaluate $(2x + 1)^2 \frac{d^2y}{dx^2} - 2(2x + 1)\frac{dy}{dx} - 12y = 6x.$ [8]

Q.6. [a] A resistance of 100 ohms and inductance of 0.5 henries are connected in series with a battery of 20 volts. Find the current at any instant if the relation between L, R, E is $L\frac{di}{dt} + Ri = E.$ [6]

[b] Solve by variation parameter method $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}.$ [6]

[c] Evaluate $\iint xy\,(x - 1)dx\,dy$ over the region bounded by $xy = 4,$ [8]

$y = 0,\quad x = 1$ and $x = 4.$

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