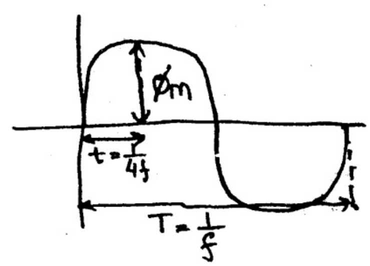
TRANSFORMER

Q1. Derive the expression for emf equation of a transformer.

Ans. Consider a transformer having,

N_1 = No. of turns in primary,
N_2 = No. of turns in secondary,
E_1 = induced emf in primary,
E_2 = induced emf in secondary,
ϕm = maximum flux in core = $Bm \times A$
Bm = maximum flux density.
A = Area of the core,
f = Frequency of ac input.



For maximum flux, time $= \frac{T}{4} = \frac{1}{4f}$.

Average rate of change of flux

$$= \frac{\phi m}{t} = \frac{\phi m}{1/4f} = 4f\phi m$$

For sinusoidal signal,

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}} = 1.11$$

$\therefore$ rms value/turn = $1.11 \times$ average value/turn
$= 1.11 \times$ average rate of change of flux
$= 1.11 \times 4f\phi m = 4.44 f\phi m$

$\therefore$ rms value of induced emf in primary = rms value/turn $\times$ No. of turns in primary

$$\therefore E_1 = 4.44 f \phi m \times N_1 = 4.44 f N_1 Bm A$$

Similarly, $E_2 = 4.44 f N_2 Bm.A$ (where $\phi m = Bm.A$)

NOTE : (1) $\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44 f \phi_m$

$\therefore$ Emf / turn is same in both primary & secondary winding.

(2) For ideal transformer, $V_1 = E_1$ & $V_2 = E_2$ where V_1 & V_2 are terminal voltages.

(3) $\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$; where K = voltage transformation ratio

If $N_2 > N_1$, then $K > 1$ for step up transformer,

If $N_1 > N_2$ then $K < 1$ for step down transformer.

(4) For ideal transformer, efficiency = 100%,

$\therefore$ Input VA = Output VA

$\therefore V_1 I_1 = V_2 I_2$

$\therefore \frac{V_2}{V_1} = \frac{I_1}{I_2}$

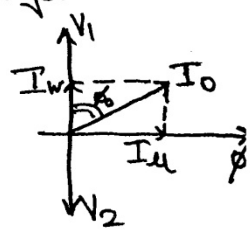
$$\therefore \boxed{\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K}$$ (For ideal transformer)

8) Transformer On No-Load:

At no-load, the primary input current has to supply

(i) Iron losses in core i.e. eddy current loss & hysteresis loss

(ii) Small amount of copper loss in primary.



Thus I_0 lags V_1 by an angle $\phi_0 < 90°$.

Input power $= W_0 = V_1 I_0 \cos\phi_0 = V_1 I_w$

From vector diag. I_0 has two components -

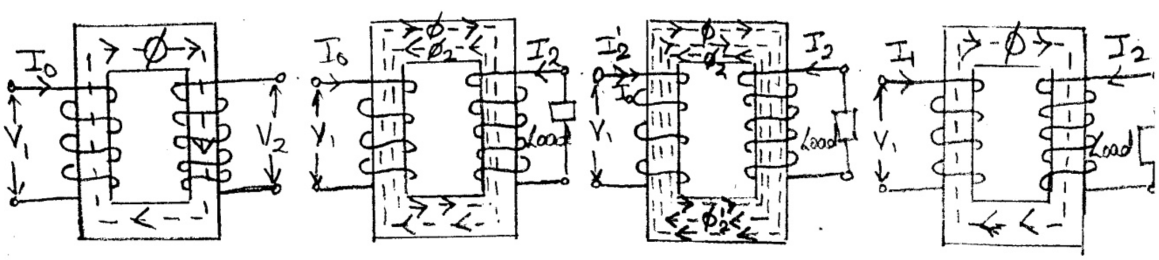
(i) $I_w = I_0 \cos\phi_0$; is in phase with V_1 & supplies iron loss & copper loss & is known as active or working or iron loss component

(ii) $I_\mu = I_0 \sin\phi_0$; is in quadrature with V_1 & sustains the alternating flux in the core & is known as magnetising or wattless component

$\therefore I_0 = \sqrt{I_\mu^2 + I_w^2}$ & ϕ_0 = hysterisis ∠ of advance

Transformer On Load:

Q3) When the secondary is loaded, secondary current I_2 is set up. I_2 is in phase with V_2 if load is non-inductive, it lags if load is inductive & leads if load is capacitive.

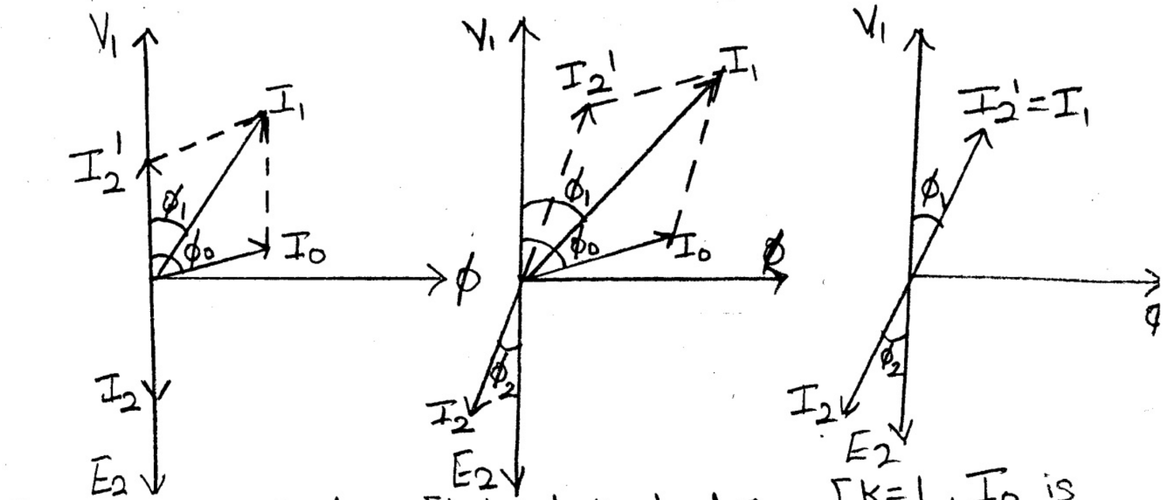


The secondary current sets up its own flux ϕ_2 which is in opposition to the main primary flux ϕ which is due to I_0. The opposing secondary flux ϕ_2 weakens the primary flux ϕ momentarily, hence primary back emf E_1 tends to be reduced. For a moment V_1 gains the upper hand over E_1 & hence causes more current to flow in primary. This current I_2' is known as the load component of primary current and is in antiphase with I_2. It sets it own flux ϕ_2' in opposition to flux ϕ_2. Since ϕ_2 & ϕ_2' are equal in magnitude but opposite in direction the cancel off each other. Thus the magnetic effect of I_2 is neutralised by I_2'. Thus the net flux passing through the core is approximately the same as at no-load.

As $\phi_2 = \phi_2'$ $\therefore N_2 I_2 = N_1 I_2'$ $\therefore I_2' = \frac{N_2}{N_1} \cdot I_2$

$\therefore \boxed{I_2' = k I_2}$ $\therefore \frac{I_2'}{I_2} = k.$

When transformer is on load, the primary current is the vector sum of I_0 & I_2'.



[$K=1$, resistive load, I_2 in phase with E_2] [$K=1$, inductive load, I_2 lags E_2] [$K=1$, I_0 is negligible]

Q4) Transformer With Winding Resistance But No Magnetic Leakage:-

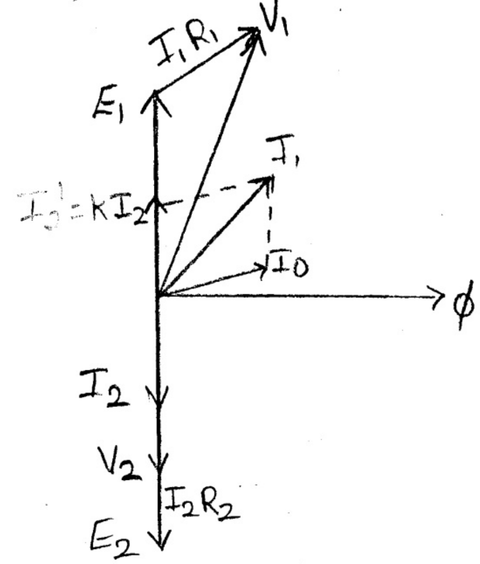
If R_1 = resistance of primary winding,
R_2 = resistance of secondary winding.
E_1 = Primary induced emf,
E_2 = secondary induced emf,
V_1 = Terminal voltage of primary,
V_2 = Terminal voltage of secondary,
I_1 = Primary current,
I_2 = Secondary current.

Then,

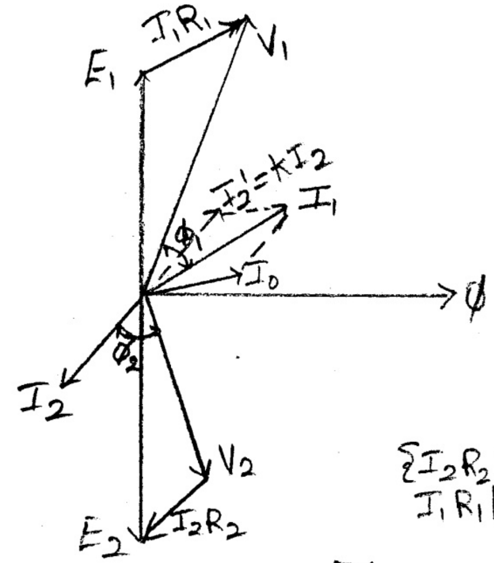
$$\boxed{\begin{aligned} E_2 - V_2 &= I_2 R_2 \\ V_1 - E_1 &= I_1 R_1 \end{aligned}}$$

If R_2' = equivalent secondary resistance as referred to primary,
Then, R_2' in primary would cause the same loss as R_2 in the secondary

Q7)

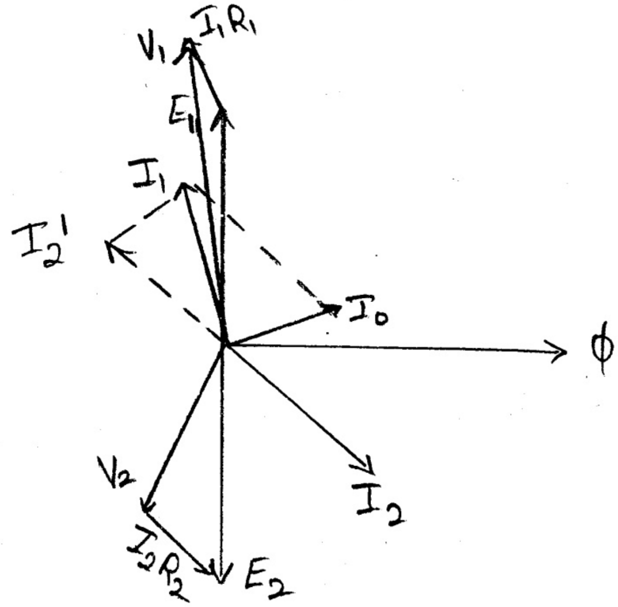


[Resistive Load]



$\{I_2R_2 \,||\, I_2\}$
$I_1R_1 \,||\, I_1$

[Inductive Load]



$\{I_2R_2 \,||\, I_2\}$
$I_1R_1 \,||\, I_1$

[Capacitive Load]

$$I_1^2 R_2' = I_2^2 R_2$$

$$\therefore R_2' = \left(\frac{I_2}{I_1}\right)^2 \cdot R_2$$

$$\boxed{\therefore R_2' = \frac{R_2}{K^2}} \qquad \left\{ \because \frac{I_2}{I_1} = \frac{1}{K} \right\}$$

When component from secondary is shifted to primary then we divide by K^2

$\therefore$ Equivalent or effective resistance of the transformer as referred to primary is,

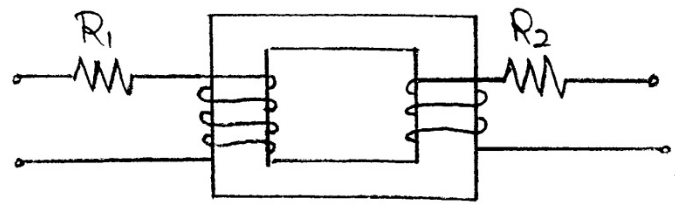
$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$$

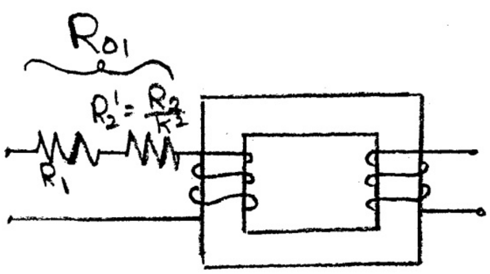
Similarly, if R_1' = equivalent primary resistance as referred to secondary,

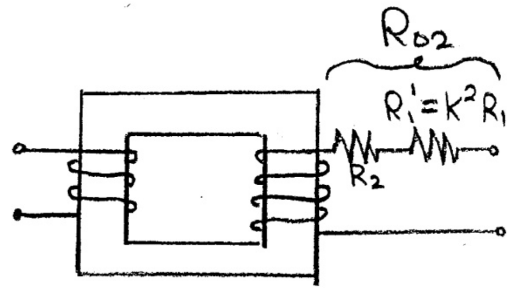
$$\therefore \boxed{R_1' = K^2 R_1}$$

When component from primary is shifted to secondary then we multiply by K^2

$$\therefore R_{02} = R_2 + R_1' = R_2 + K^2 R_1$$



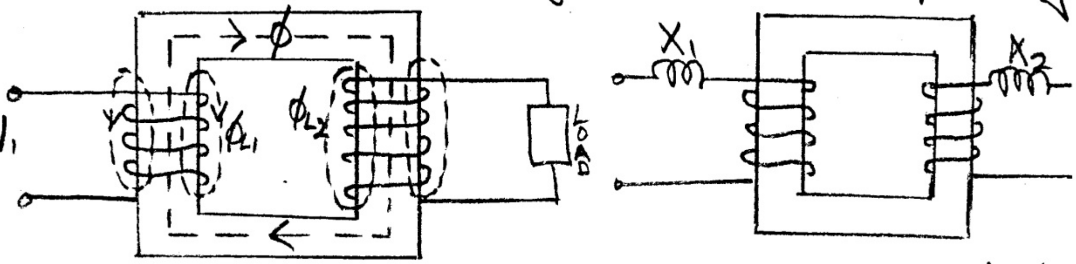




Q5) Magnetic leakage:

All flux linked with primary does not link the secondary but part of it i.e. ϕ_{L1} completes its magnetic circuit by passing through air rather than around the core. This primary leakage flux induces an emf e_{L1}

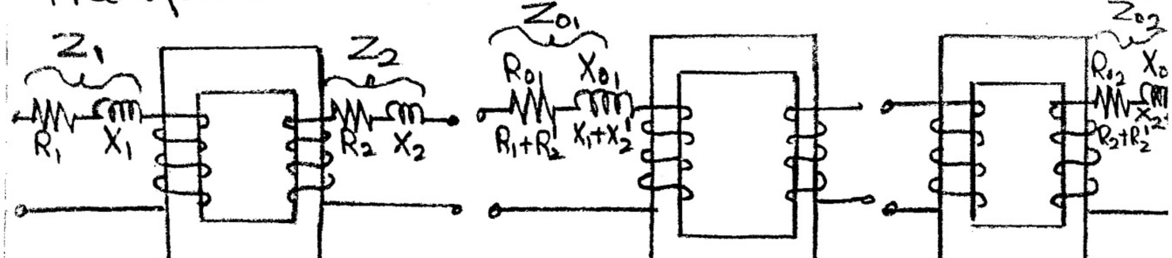
in the primary but not in secondary. Similary ϕ_{L2} induces emf in the secondary e_{L2} but not in primary



A transformer with magnetic leakage is equivalent to an ideal transformer with inductive coils connected in both primary & secondary circuits such that emf in each coil is equal to that due to the corresponding leakage flux in actual transformer.

$$\therefore X_1 = \frac{e_{L1}}{I_1} \quad \& \quad X_2 = \frac{e_{L2}}{I_2}$$

Q6) Transformer With Resistance & Leakage Reactance:



Primary impedance, $Z_1 = \sqrt{R_1^2 + X_1^2}$

Secondary impedance, $Z_2 = \sqrt{R_2^2 + X_2^2}$

$$V_1 = E_1 + I_1(R_1 + jX_1) = E_1 + I_1 Z_1.$$

$$E_2 = V_2 + I_2(R_2 + jX_2) = V_2 + I_2 Z_2.$$

$X_{01} = X_1 + X_2' = X_1 + X_2/K^2$; $R_{01} = R_1 + R_2' = R_1 + R_2/K^2$

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1$; $R_{02} = R_2 + R_1' = R_2 + K^2 R_1$

$$\therefore Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} \quad \& \quad Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}.$$

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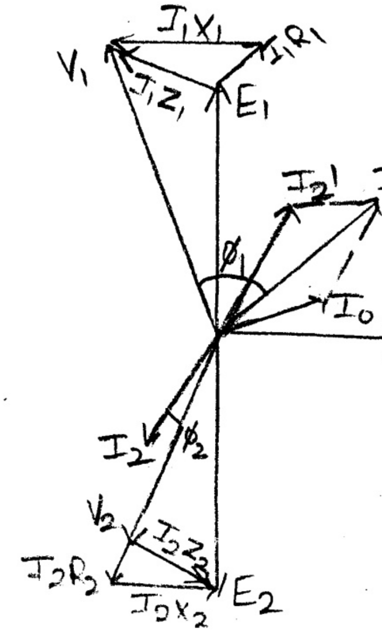
88) Draw phasor diagram of transformer (on load with magnetic and copper loss and inductive load)

8M



[Inductive Load with Leakage]

Step1: Mark x-axis as ϕ and y-axis as E_1 and E_2

Step2: For inductive load draw I_2 lagging E_2 ~~by~~ in the 3rd quadrant

Step3: Between I_2 & E_2 draw V_2 and mark ϕ_2

Step4: Draw $I_2R_2 \parallel I_2$ beginning from V_2 & ending till the horizontal of arrowhead of E_2

Step5: Join by horizontal line the vectors I_2R_2 and E_2

Step6: Draw I_2Z_2 joining V_2 & t

Step7: Draw I_2' opp. to I_2 in 1st quad., I_0 above x-axis & complete ||gm in 1st quad. with I_1 as the resultant.

Step 8: Draw $I_1R_1 \parallel I_1$ beginning from E_1

Step 9: Draw horizontal line I_1X_1

Step 10: Join E_1 & I_1X_1 by I_1Z_1 & complete the tria

Step 11: Complete Δ^{le} of E_1 & I_1Z_1 with V_1 & mark ϕ_1