If t °C is the temperature of the coil, its resistance is 21 Ω.

$$21 = R_o(1 + 0.004\,t) \quad \dots(3)$$

Dividing equation (3) by (1)

$$\frac{21}{18} = \frac{1 + 0.004\,t}{1 + 0.004 \times 20}$$

$$21 + 1.68 = 18 + 0.072\,t$$

$$0.072\,t = 4.68$$

$$t = 65\ °C$$

Rise in temperature = 65 − 15 = 50 °C

Q. Two coils A and B have resistances 120 Ω and 180 Ω respectively at 0 °C are connected in series. Coil A has resistance temperature coefficient of 0.0035/°C while B has 0.008/°C at 0 °C. Find the resistance temperature coefficient of the series combination at 0 °C.

For coil A,

$$R_{A_0} = 120\ \Omega, \quad \alpha_A = 0035/°C$$

For coil B,

$$R_{B_0} = 180\ \Omega, \quad \alpha_B = 0.008/°C$$

We know that,

$$R_{A_1} = R_{A_0}(1 + \alpha_A\,t) \quad \dots(1)$$

$$R_{B_1} = R_{B_0}(1 + \alpha_B\,t) \quad \dots(2)$$

For series combination of R_A and R_B,

$$R_{A_1} + R_{B_1} = (R_{A_0} + R_{B_0})(1 + \alpha\,t) \quad \dots(3)$$

Adding (1) and (2), and comparing with equation (3),

$$R_{A_0}\alpha_A + R_{B_0}\alpha_B = (R_{A_0} + R_{B_0})\,\alpha$$

$$\alpha = \frac{R_{A_0}\alpha_A + R_{B_0}\alpha_B}{R_{A_0} + R_{B_0}}$$

$$= \frac{120 \times 0.0035 + 180 \times 0.008}{120 + 180} = 0.0062/°C$$

Q. Two coils have resistances of 150 Ω and 350 Ω at 30 °C. Their resistance temperature coefficients are 0.003/°C and 0.002/°C at 30 °C. Find (i) resistance of their series combination at 50 °C, (ii) resistance of their parallel combination at 50 °C and (iii) resistance temperature coefficient of individual coils.

At 50 °C,

$$R_1 = 150\,[1 + 0.003\,(50 - 30)]$$

$$= 159\ \Omega$$

$$R_2 = 350\,[1 + 0.002\,(50 - 30)]$$

$$= 364\ \Omega$$

Resistance of the series combination at 50 °C,

$$= 159 + 364$$

$$= 523\ \Omega$$

Resistance of the parallel combination at 50 °C,

$$= \frac{159 \times 364}{159 + 364}$$

$$= 110.66\ \Omega$$

Resistance temperature coefficient of individual coils are given by,

$$\alpha_{50}\ \text{(first coil)} = \frac{\alpha_{30}}{1 + \alpha_{30}\,(50 - 30)}$$

$$= \frac{0.003}{1 + 0.003\,(20)}$$

$$= 0.0028/°C$$

$$\alpha_{50}\ \text{(second coil)} = \frac{0.002}{1 + 0.002\,(20)}$$

$$= 0.00192/°C$$

1.4 SERIES CIRCUIT

Resistances R_1 and R_2 are said to be connected in series when same current flows throu each of the resistance.

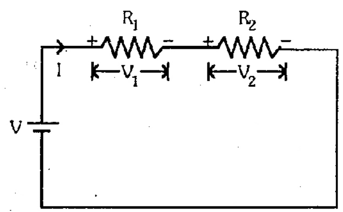


Fig. 1.1

Voltage across $R_1 = V_1 = R_1 \cdot I$

Voltage across $R_2 = V_2 = R_2 \cdot I$

The total voltage applied should be balanced by the sum of voltage drops around the circui

$$V = V_1 + V_2$$

$$= R_1 \cdot I + R_2 \cdot I$$

$$= (R_1 + R_2)\,I$$

$$= R_T \cdot I \quad \text{where } R_T = R_1 + R_2$$

The circuit can be simplified to give total current of 2.5 A as shown in Fig. 1.18 below.

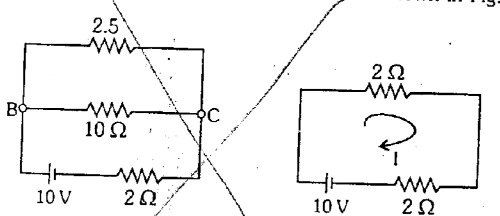


Fig. 1.18

$$I = \frac{10}{4} = 2.5 \text{ A}$$

The current gets divided into two parts at point B.

$$I_{6\,\Omega} = I_{10\,\Omega} = 2.5 \times \frac{2.5}{10 + 2.5} = 0.5 \text{ A}$$

1.6 STAR-DELTA TRANSFORMATION

When a circuit cannot be simplified by normal series-parallel reduction technique, star-delta transformation can be used.

Fig. 1.19 (a) shows three resistances R_A, R_B and R_C connected in delta.

Fig. 1.19 (b) shows three resistances R_1, R_2 and R_3 connected in star.

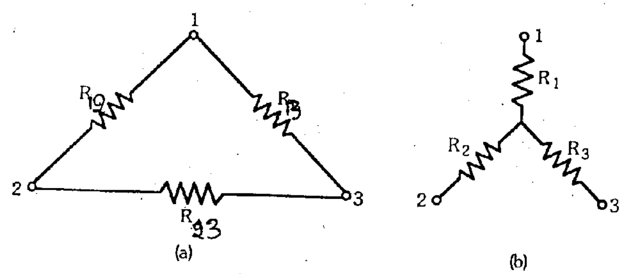


Fig. 1.19

These two networks will be electrically equivalent if the resistance as measured between ny pair of terminals is the same in both the arrangements.

Delta to Star Transformation :

Referring to delta network shown in Fig. 1.19 (a),

Resistance between terminals 1 and 2 = $R_C \,||\, (R_A + R_B)$

$$= \frac{R_C (R_A + R_B)}{R_A + R_B + R_C} \quad \ldots (1)$$

Referring to the star network shown in Fig. 1.19 (b),

Resistance between terminals 1 and 2 = $R_1 + R_2$ ... (2)

Since the two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_C (R_A + R_B)}{R_A + R_B + R_C} \quad \ldots (3)$$

Similarly, $$R_2 + R_3 = \frac{R_A (R_B + R_C)}{R_A + R_B + R_C} \quad \ldots (4)$$

and $$R_3 + R_1 = \frac{R_B (R_A + R_C)}{R_A + R_B + R_C} \quad \ldots (5)$$

Subtracting equation (4) from equation (3),

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \quad \ldots (6)$$

Adding equation (6) and equation (5),

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \quad \ldots (7)$$

Similarly, $$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \quad \ldots (8)$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \quad \ldots (9)$$

Thus, star resistance connected to terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.

Star to Delta Transformation :

Multiplying above equations,

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A + R_B + R_C)^2} \quad \ldots (10)$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \quad \ldots (11)$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A + R_B + R_C)^2} \quad \ldots (12)$$

Adding equations (10), (11) and (12),

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$= \frac{R_A R_B R_C}{R_A + R_B + R_C} = R_A \cdot R_1$$

$$= R_B R_2$$

$$= R_C \cdot R_3$$

Hence $$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$= R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_{13} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3 R_1}{R_2}$$

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

$$= R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Thus, delta resistance between the two terminals is the sum of two star resistances connected to the same terminals plus the product of the two resistances divided by the remaining third star resistance.

Note : When three equal resistances are connected in delta, the equivalent star resistance is given by,

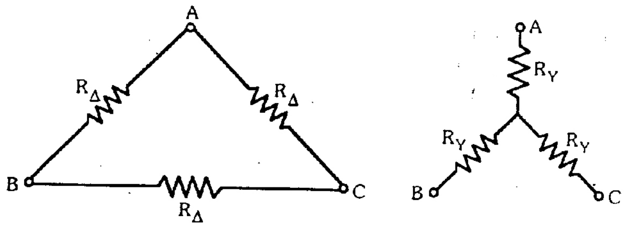


Fig. 1.20

$$R_Y = \frac{R\Delta \cdot R\Delta}{R\Delta + R\Delta + R\Delta}$$

$$= \frac{R\Delta}{3}$$

$$\text{or } R\Delta = 3 R_Y$$

18. Find equivalent resistance between A and B.

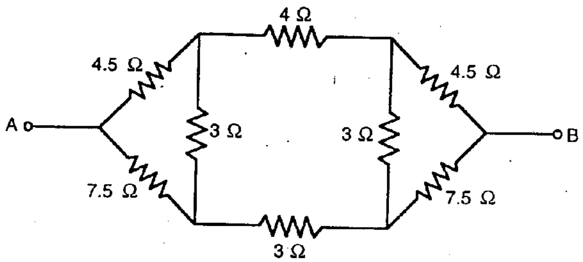


Fig. 1.21

Converting the two delta networks formed by resistors 4.5 Ω, 3 Ω and 7.5 Ω int equivalent star networks,

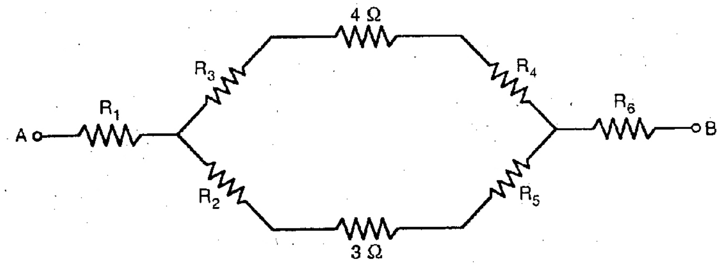


Fig. 1.22

$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\ \Omega$$

$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\ \Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\ \Omega$$

The simplified network is shown in Fig. 1.22.

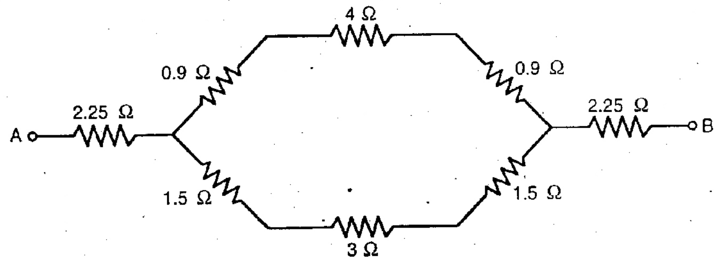
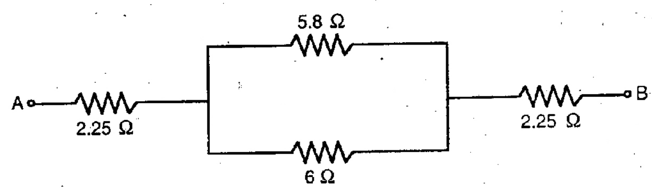


Fig. 1.23

The network can be simplified as follows :



$$I_L = 0.59 \times \frac{2.2}{2.2 + 1}$$
$$= 0.41 \text{ A}$$

MAXIMUM POWER TRANSFER THEOREM

It states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance.

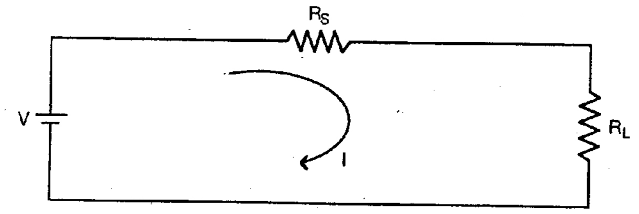


Fig. 1.297

$$I = \frac{V}{R_S + R_L}$$

Power delivered to the load R_L = $P = I^2 R_L = \frac{V^2 R_L}{(R_S + R_L)^2}$

To determine the value of R_L for maximum power to be transferred to the load,

$$\frac{dP}{dR_L} = 0$$

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \frac{V^2}{(R_S + R_L)^2} R_L$$

$$= \frac{V^2 [(R_S + R_L)^2 - (2 R_L)(R_S + R_L)]}{(R_S + R_L)^4}$$

$$(R_S + R_L)^2 - 2 R_L (R_S + R_L)^2 = 0$$

$$R_S^2 + R_L^2 + 2 R_S R_L - 2 R_L^2 - 2 R_S R_L = 0$$

$$R_S = R_L$$

Hence maximum power will be transferred to the load when load resistance is equal to the source resistance.

Steps to be followed in maximum power transfer theorem :

1. Remove the variable load resistor R_L.
2. Find open circuit voltage V_{Th} across points A and B.
3. Find the resistance R_{Th} as seen from point A and B with voltage source and current source replaced by internal resistance.
4. Find the resistance R_L for maximum power transfer.

$$R_L = R_{Th}$$

5. Find maximum power.

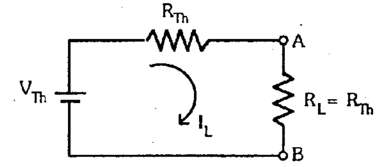


Fig. 1.298

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{max} = I_L^2 R_L$$

$$= \frac{V_{Th}^2}{4R^2_{Th}} \times R_{Th}$$

$$= \frac{V_{Th}^2}{4R_{Th}}$$

93. For the circuit shown, find value of resistance R_L for maximum power and calculate maximum power.

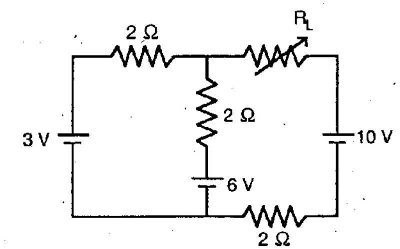


Fig. 1.299

Step I : Calculation of V_{Th}

Removing the variable resistor R_L from the network,

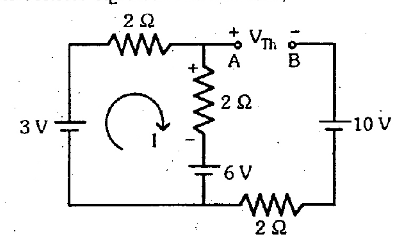


Fig. 1.300

Applying KVL to mesh,

$$3 - 2I - 2I - 6 = 0$$

$$I = -0.75 \text{ A}$$

Many times alternating voltages and currents are represented by a sinusoidal waveform.

A sinusoidal voltage can be represented as,

$$v = V_m \sin\theta$$
$$= V_m \sin \omega t$$
$$= V_m \sin 2\pi f t$$
$$= V_m \sin \frac{2\pi}{T} t$$

2.2 TERMS RELATED WITH ALTERNATING QUANTITY

1. Waveform : A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Fig. 2.1 shows few waveforms.

2. Cycle : One complete set of positive and negative values of an alternating quantity is termed as a cycle.

3. Frequency : The number of cycles per second of an alternating quantity is known as frequency. It is denoted by f and is expressed in hertz (Hz) or cycles per second (c/s).

4. Time Period : The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by T and is expressed in seconds.

$$T = \frac{1}{f}$$

5. Amplitude : The maximum positive or negative value of an alternating quantity is called the amplitude.

6. Phase : The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference.

7. Phase Difference : This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier as compared to the other quantity.

A lagging alternating quantity is one which attains its maximum or zero value later than the other quantity.

A plus (+) sign when used in connection with the phase difference denotes 'lead' whereas a minus (–) sign denotes 'lag'.

$$v_A = V_m \sin \omega t$$
$$v_B = V_m \sin(\omega t + \phi)$$

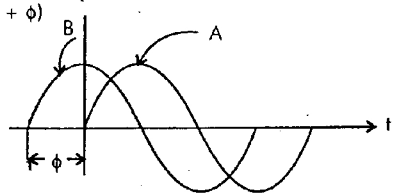


Fig. 2.2

Here quantity B leads A by a phase angle ϕ.

ROOT MEAN SQUARE (RMS) OR EFFECTIVE VALUE

Normally the current is measured by the amount of work it will do or the amount of heat it will produce. Hence rms or effective value of alternating current is defined as that value of steady current (direct current) which will do the same amount of work in the same time or would produce the same heating effect as when the alternating current is applied for the same time.

Fig. 2.3 shows the positive half cycle of a non-sinusoidal alternating current waveform. The waveform is divided in 'm' equal intervals with the instantaneous currents during these intervals being $i_1, i_2, \ldots, i_m$. This waveform is applied to a circuit consisting of resistance R ohms. Then work done in different intervals will be $\left(i_1^2 R \times \frac{t}{m}\right), \left(i_2^2 R \times \frac{t}{m}\right), \ldots \left(i_m^2 R \times \frac{t}{m}\right)$ Joules.

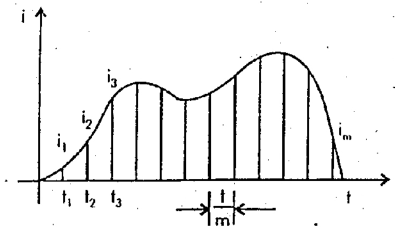


Fig. 2.3

Thus the total work done in t seconds on applying alternating current waveform to a resistance $R = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$ Joules.

Let I be the value of the direct current that while flowing through the same resistance does the same amount of work in the same time t, then

$$I^2 Rt = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m} \times Rt$$

$$I^2 = \frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}$$

Hence rms value of alternating current is given by,

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots + i_m^2}{m}} = \sqrt{\text{Mean value of } (i)^2}$$

RMS value of any current i (t) over the specified interval t_1 and t_2 is expressed mathematically as,

$$I_{rms} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} i^2(t)\,dt}$$

The rms value of an alternating current is of considerable importance in practice because the ammeters and voltmeters record the rms value of alternating current and voltage respectively.

RMS value of sinusoidal waveform :

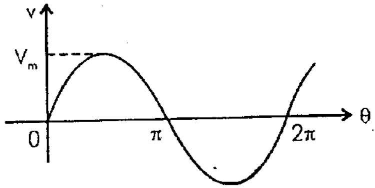


Fig. 2.4

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

Crest or Peak or amplitude factor : It is defined as the ratio of maximum value to rms value the given quantity.

$$\text{Peak factor } (k_p) = \frac{\text{Maximum value}}{\text{RMS value}}$$

AVERAGE VALUE

The average value of an alternating quantity is defined as the arithmetic mean of all the ılues over one complete cycle.

In case of symmetrical alternating waveform (whether sinusoidal or non sinusoidal), the ·erage value over a complete cycle is zero. Hence in such case, the average value is obtained ·er half cycle only.

Referring to Fig. 2.3, the average value of current is given by,

$$I_{avg} = \frac{i_1 + i_2 + \ldots + i_m}{m}.$$

Average value of any current i (t) over the specified interval t_1 and t_2 is expressed mathematically as,

$$I_{avg} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i(t)\, dt$$

Average value of sinusoidal waveform :

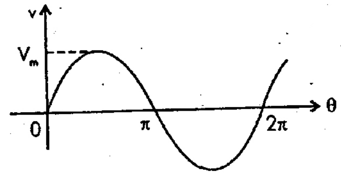


Fig. 2.5

$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

Since this is a symmetrical waveform, the average value is calculated over half the cycle.

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\, d\theta = \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1 + 1] = \frac{2V_m}{\pi}$$

$$= 0.637\, V_m$$

Form factor : It is defined as the ratio of rms value to the average value of the given quantity.

$$\text{Form factor } (k_f) = \frac{\text{RMS value}}{\text{Average value}}$$

1. An alternating current takes 3.375 ms to reach 15 A for the first time after becoming instantaneously zero. The frequency of current is 40 Hz. Find the maximum value of alternating current.

Data :

$i = 15$ A

$t = 3.375$ ms

$f = 40$ Hz

$i = I_m \sin 2\pi ft$

$15 = I_m \sin(2\pi \times 40 \times 3.375 \times 10^{-3})$

$15 = I_m \times 0.75$

$I_m = 20$ A

2. An alternating current of frequency 50 c/s has a maximum value of 100 A. Calculate (a) its value $\frac{1}{600}$ sec. after the instant the current is zero (b) In how many seconds after the zero value, will the current attain the value of 86.6 A?

Data : $f = 50$ c/s

$I_m = 100$ A

(a) $i = I_m \sin 2\pi ft = 100 \sin\left(2\pi \times 50 \times \frac{1}{600}\right)$

$= 100 \sin (30°)$

$= 50$ A

(b) $i = I_m \sin 2\pi ft$

$86.6 = 100 \sin (2\pi \times 50 \times t)$

$\sin (100\pi t) = 0.866$

$100\pi t = 60°$

$t = \frac{60}{100 \times 180} = \frac{1}{300}$ sec.

3. An alternating current varying sinusoidally with a frequency of 50 c/s has an rms value of 20 A. Write down the equation for the instantaneous value and find this value at (a) 0.0025 sec. (b) 0.0125 sec. after passing through zero and increasing positively. (c) At what time, measured from zero, will the value of the instantaneous current be 14.14 A?

Data : $f = 50$ c/s

$I = 20$ A

$I_m = I \times \sqrt{2} = 20 \sqrt{2}$

$= 28.28$ A

Equation of current, $i = I_m \sin 2\pi ft$

$= 28.28 \sin (2\pi \times 50 \times t)$

$= 28.28 \sin 100\pi t$

(a) At $t = 0.0025$ sec.

$i = 28.28 \sin (100\pi \times 0.0025)$

$= 28.28 \sin (45°)$

$= 20$ A

(b) At $t = 0.0125$ sec.

$i = 28.28 \sin (100\pi \times 0.0125)$

$= 28.28 \sin (225°)$

$= -20$ A

(c) $i = 28.28 \sin 100\pi t$

$14.14 = 28.28 \sin 100\pi t$

$\sin 100\pi t = 0.5$

$100\pi t = 30°$

$t = 1.66 \times 10^{-3}$ sec.

4. Find the following parameters of a voltage $v = 200 \sin 314\, t$

(i) frequency (ii) form factor (iii) crest factor.

Data : $v = 200 \sin 314\, t$

$v = V_m \sin 2\pi ft$

$2\pi f = 314$

$f = \frac{314}{2\pi} = 50$ Hz

For a sinusoidal waveform,

$V_{avg} = \frac{2V_m}{\pi}$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

Form factor $= \frac{V_{rms}}{V_{avg}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = 1.11$

Crest factor $= \frac{V_m}{V_{rms}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$

5. A non sinusoidal voltage is having form factor as 1.2 and peak factor as 1.5. If the average value of the voltage is 10 V, calculate (i) rms value (ii) maximum value.

Data : $k_f = 1.2$

$k_p = 1.5$

$V_{avg} = 10$

Form factor $k_f = \frac{V_{rms}}{V_{avg}}$

$1.2 = \frac{V_{rms}}{10}$

$V_{rms} = 12$ V

Peak factor $k_p = \frac{V_m}{V_{rms}}$

$1.5 = \frac{V_m}{12}$

$V_m = 18$ V

6. Find average value and rms value of the waveform shown in Fig. 2.6.

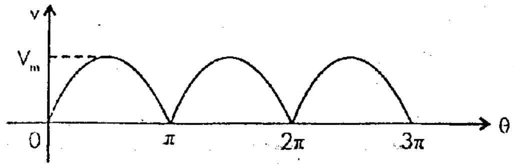


Fig. 2.6

$$v = V_m \sin\theta \qquad 0 < \theta < \pi$$

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\,d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi} = \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi} = 0.637\,V_m$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta} = \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}} = 0.707\,V_m$$

7. Find average and rms value of the waveform shown in Fig. 2.7.

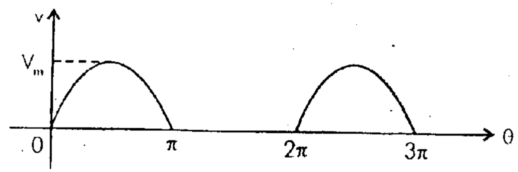


Fig. 2.7

$$v = V_m \sin\theta \qquad 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\,d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\,d\theta$$

$$= \frac{V_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{2\pi}[1+1] = \frac{V_m}{\pi}$$

$$= 0.318\,V_m$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta + \int_{\pi}^{2\pi} 0\,d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{V_m^2}{4}}$$

$$= \frac{V_m}{2} = 0.5\,V_m$$

8. Find average value and rms value of the waveform shown in Fig. 2.8.

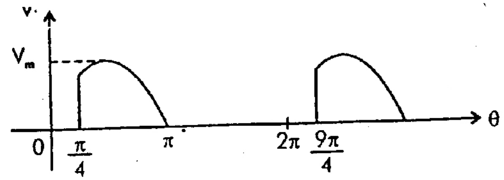


Fig. 2.8

$$v = 0 \qquad 0 < \theta < \pi/4$$
$$= V_m \sin\theta \qquad \pi/4 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta = \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m \sin\theta\, d\theta$$
$$= \frac{V_m}{2\pi}[-\cos\theta]_{\pi/4}^{\pi} = \frac{V_m}{2\pi}[1 + 0.707]$$
$$= 0.272\ V_m$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$
$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta\, d\theta}$$
$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\pi/4}^{\pi}}$$
$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4}\right]}$$
$$= \sqrt{0.227\ V_m^2}$$
$$= 0.476\ V_m$$

9. A full wave rectified wave is clipped at 70.7 % of its maximum value as shown in Fig. 2.9. Find its average and rms value.

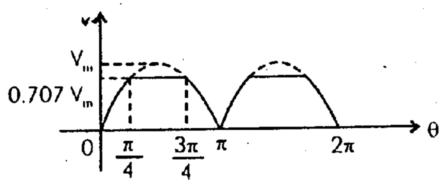


Fig. 2.9

$$v = V_m \sin\theta \qquad 0 < \theta < \pi/4$$
$$= 0.707\ V_m \qquad \pi/4 < \theta < 3\pi/4$$
$$= V_m \sin\theta \qquad 3\pi/4 < \theta < \pi$$

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$
$$= \frac{1}{\pi}\left[\int_0^{\pi/4} V_m \sin\theta\, d\theta + \int_{\pi/4}^{3\pi/4} 0.707\ V_m\, d\theta + \int_{3\pi/4}^{\pi} V_m \sin\theta\, d\theta\right]$$
$$= \frac{V_m}{\pi}\left\{[-\cos\theta]_0^{\pi/4} + 0.707\,[\theta]_{\pi/4}^{3\pi/4} + [-\cos\theta]_{3\pi/4}^{\pi}\right\}$$
$$= \frac{V_m}{\pi}(0.293 + 1.11 + 0.293) = 0.54\ V_m$$

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$
$$= \sqrt{\frac{1}{\pi}\left[\int_0^{\pi/4} V_m^2 \sin^2\theta\, d\theta + \int_{\pi/4}^{3\pi/4} (0.707\ V_m)^2\, d\theta + \int_{3\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta\right]}$$
$$= \sqrt{\frac{V_m^2}{\pi}\left\{\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi/4} + 0.499\,[\theta]_{\pi/4}^{3\pi/4} + \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{3\pi/4}^{\pi}\right\}}$$
$$= \sqrt{0.341\ V_m^2}$$
$$= 0.584\ V_m$$

10. Find rms value of the waveform shown in Fig. 2.10.

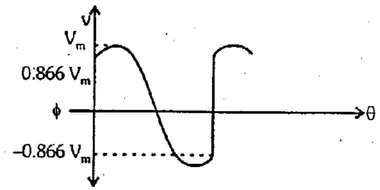


Fig. 2.10

The equation of the waveform is given by $v = V_m \sin(\theta + \phi)$ where ϕ is phase difference.

When $\theta = 0$, $v = 0.866\ V_m$.

$$0.866\ V_m = V_m \sin(0 + \phi)$$
$$\phi = \sin^{-1}(0.866) = \frac{\pi}{3}$$

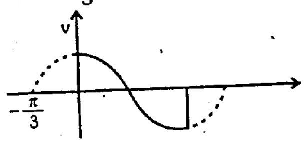


Fig. 2.10 (a)

$$v = V_m \sin\left(\theta + \frac{\pi}{3}\right)$$

Time period of complete sine wave is always 2π. Since some part of the waveform is hopped from both the sides,

Time period $= 2\pi - \frac{\pi}{3} - \frac{\pi}{3} = \frac{4\pi}{3}$

$$V_{rms} = \sqrt{\frac{1}{4\pi/3}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta + \frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3}{4\pi}\int_0^{4\pi/3} V_m^2 \sin^2\left(\theta + \frac{\pi}{3}\right) d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\int_0^{4\pi/3} \left[\frac{1-\cos 2(\theta+\pi/3)}{2}\right] d\theta}$$

$$= \sqrt{\frac{3V_m^2}{4\pi}\left[\frac{\theta}{2} - \frac{\sin 2(\theta+\pi/3)}{4}\right]_0^{4\pi/3}}$$

$$= \sqrt{0.6031\, V_m^2}$$

$$= 0.776\, V_m$$

11. Find rms and average value of the waveform shown in Fig. 2.11.

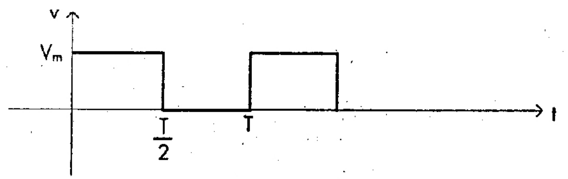


Fig. 2.11

$v = V_m \qquad 0 < t < T/2$

$\;\; = 0 \qquad T/2 < t < T$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\, dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} V_m\, dt + \int_{T/2}^{T} 0\, dt\right]$$

$$= \frac{1}{T}\int_0^{T/2} V_m\, dt$$

$$= \frac{V_m}{T}[t]_0^{T/2}$$

$$= \frac{V_m}{T}\cdot\frac{T}{2} = 0.5\, V_m$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2\, dt}$$

$$= \sqrt{\frac{V_m^2}{T}[t]_0^{T/2}}$$

$$= \sqrt{\frac{V_m^2}{T}\cdot\frac{T}{2}}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= 0.707\, V_m$$

12. Find rms and average value of the waveform shown in Fig. 2.12.

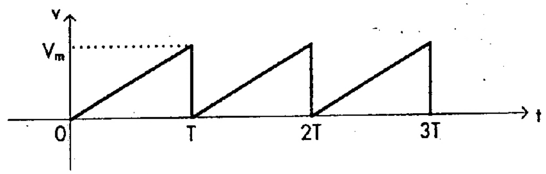


Fig. 2.12

$v = \frac{V_m}{T} t \qquad 0 < t < T$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\, dt = \frac{1}{T}\int_0^T \frac{V_m}{T} t\, dt$$

$$= \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T = \frac{V_m}{T^2}\cdot\frac{T^2}{2}$$

$$= 0.5\, V_m$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2}\cdot t^2\, dt} = \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{T^3}{3}\right]}$$

$$= \sqrt{\frac{V_m^2}{3}}$$

$$= 0.577\ V_m$$

13. Find rms and average value of the waveform shown in Fig. 2.13.

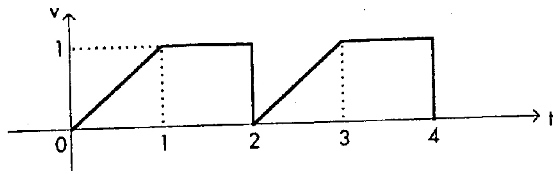


Fig. 2.13

$$v = t \qquad 0 < t < 1$$
$$= 1 \qquad 1 < t < 2$$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{2}\left[\int_0^1 t\,dt + \int_1^2 1\,dt\right]$$

$$= \frac{1}{2}\left\{\left[\frac{t^2}{2}\right]_0^1 + [t]_1^2\right\}$$

$$= \frac{1}{2}\left[\frac{1}{2} - 0 + 2 - 1\right]$$

$$= \frac{3}{4} = 0.75\ V$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 t^2\,dt + \int_1^2 (1)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left\{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2\right\}}$$

$$= \sqrt{\frac{1}{2}\left[\frac{1}{3} - 0 + 2 - 1\right]}$$

$$= \sqrt{\frac{4}{6}}$$

14. Find rms and average value of the waveform shown in Fig. 2.14.

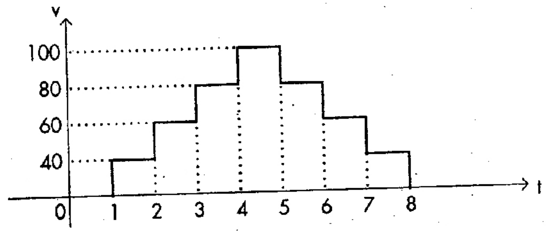


Fig. 2.14

$$V_{avg} = \frac{0 + 40 + 60 + 80 + 100 + 80 + 60 + 40}{8}$$

$$= 57.5\ V$$

$$V_{rms} = \sqrt{\frac{0^2 + (40)^2 + (60)^2 + (80)^2 + (100)^2 + (80)^2 + (60)^2 + (40)^2}{8}}$$

$$= 64.42\ V$$

15. Find rms and average value of the waveform shown in Fig. 2.15.

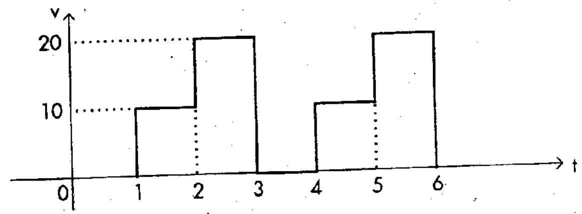
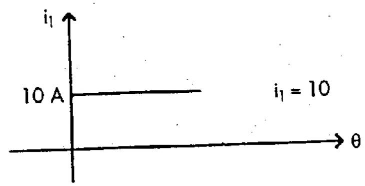


Fig. 2.15

$$V_{avg} = \frac{0 + 10 + 20}{3} = 10\ V$$

$$V_{rms} = \sqrt{\frac{0^2 + (10)^2 + (20)^2}{3}} = 12.9\ V$$

16. Find the effective value of the resultant current which carries simultaneously a dire current of 10 A and a sinusoidally alternating current of peak value 10 A.



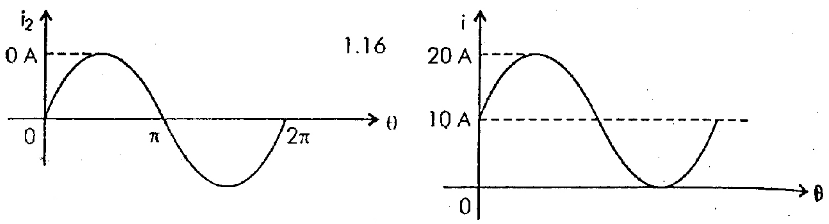


Fig. 2.16

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (10 + 10\sin\theta)^2\, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (100 + 200\sin\theta + 100\sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} (1 + 2\sin\theta + \sin^2\theta)\, d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\int_0^{2\pi} \left[1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right)\right] d\theta}$$

$$= \sqrt{\frac{100}{2\pi}\left[\theta - 2\cos\theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2\cos 2\pi + \frac{2\pi}{2} - \frac{\sin 4\pi}{4} - 0 + 2\cos 0 - 0 + \frac{\sin 0}{4}\right]}$$

$$= \sqrt{\frac{100}{2\pi}\left[2\pi - 2 + \frac{2\pi}{2} + 2\right]} = \sqrt{\frac{100}{2\pi} \times 3\pi}$$

$$= \sqrt{150} = 12.25 \text{ A}$$

17. Find the relative heating effects of two current waves of equal peak value, one sinusoidal and other rectangular in shape.

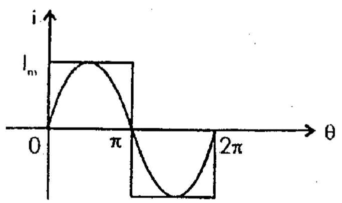


Fig. 2.17

RMS value of the rectangular wave $= I_m$

RMS value of sinusoidal current wave $= \frac{I_m}{\sqrt{2}}$

Heating effect due to rectangular current wave $= (I_m)^2 RT$

Heating effect due to sinusoidal current wave $= \left(\frac{I_m}{\sqrt{2}}\right)^2 RT = \frac{(I_m)^2}{2} RT$

Relative heating effects $= \frac{(I_m)^2}{2} RT : (I_m)^2 RT$

$= \frac{1}{2} : 1 = 1 : 2$

2.5 PHASOR REPRESENTATION OF ALTERNATING QUANTITIES

The alternating quantities are represented by phasors. A phasor is a line of definite length rotating in anticlockwise direction at a constant angular velocity ω. The length of phasor is equal to the maximum value of the alternating quantity and angular velocity is equal to the angular velocity of alternating quantity.

As shown in Fig. 2.18 (a), consider a phasor OP = I_m, where I_m is the maximum value of alternating current. Let this phasor rotate in anticlockwise direction at a uniform angular velocity of ω rad/sec. The projection of phasor OP on Y-axis at any instant gives the instantaneous value of that alternating current.

$$OM = OP \sin \omega t$$
$$= I_m \sin \omega t$$

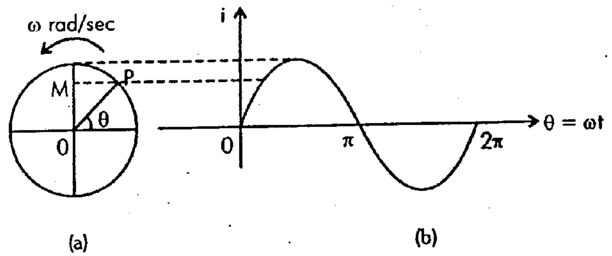


Fig. 2.18

Thus if we plot the projections of the phasor on Y-axis versus its angular position point by point, a sinusoidal alternating current waveform is obtained.

Phasor diagram using rms values : A sinusoidal alternating current and voltages can be represented by phasors. The electrical measuring instruments like ammeter and voltmeter are calibrated to read the rms value of ac quantities. Hence instead of using maximum value, it is more convenient to draw phasor diagrams using rms values of alternating quantities. However such a phasor diagram will not generate sine wave of proper amplitude unless the length of phasor is multiplied by $\sqrt{2}$.

MATHEMATICAL REPRESENTATION OF PHASORS

A phasor can be represented in four forms.

(i) Rectangular form :

$\overline{V} = X \pm jY$

Magnitude of phasor, $V = \sqrt{X^2 + Y^2}$

Phase angle $\phi = \tan^{-1}\left(\frac{Y}{X}\right)$

(ii) Trigonometric form :

$\overline{V} = V(\cos\phi \pm j\sin\phi)$

(iii) Exponential form :

$\overline{V} = V e^{\pm j\phi}$

(iv) Polar form :

$\overline{V} = V \angle \pm \phi$

Significance of operator 'j' :

The operator 'j' is used in rectangular form. It is used to indicate anticlockwise rotation of a phasor through 90°. Mathematically,

$j = \sqrt{-1}$

whenever a phasor is multiplied by 'j', the phasor is rotated once in anticlockwise direction through 90°. The power of 'j' represents the number of times the phasor should be rotated through 90° in anticlockwise direction.

Two sinusoidal currents are given as :
$i_1 = 10\sqrt{2}\sin\omega t$, $i_2 = 20\sqrt{2}\sin(\omega t + 60°)$.
Find the expression for the sum of these currents.

Data : $i_1 = 10\sqrt{2}\sin\omega t$

$i_2 = 20\sqrt{2}\sin(\omega t + 60°)$

Writing currents i_1 and i_2 in phasor form,

$$\overline{I_1} = \frac{10\sqrt{2}}{\sqrt{2}}\angle 0° = 10\angle 0°$$

$$\overline{I_2} = \frac{20\sqrt{2}}{\sqrt{2}}\angle 60° = 20\angle 60°$$

$$\overline{I} = \overline{I_1} + \overline{I_2}$$
$$= 10\angle 0° + 20\angle 60°$$
$$= 26.46\angle 40.89°$$
$$i = 26.46\sqrt{2}\sin(\omega t + 40.89°)$$
$$= 37.42\sin(\omega t + 40.89°)$$

19. The following three sinusoidal currents flow into the junction $i_1 = 3\sqrt{2}\sin\omega t$, $i_2 = 5\sqrt{2}\sin(\omega t + 30°)$ and $i_3 = 6\sqrt{2}\sin(\omega t - 120°)$. Find the expression for the resultant current which leaves the junction.

Data : $i_1 = 3\sqrt{2}\sin\omega t$

$i_2 = 5\sqrt{2}\sin(\omega t + 30°)$

$i_3 = 6\sqrt{2}\sin(\omega t - 120°)$

Writing currents i_1, i_2 and i_3 in phasor form,

$$\overline{I_1} = \frac{3\sqrt{2}}{\sqrt{2}}\angle 0° = 3\angle 0°$$

$$\overline{I_2} = \frac{5\sqrt{2}}{\sqrt{2}}\angle 30° = 5\angle 30°$$

$$\overline{I_3} = \frac{6\sqrt{2}}{\sqrt{2}}\angle -120° = 6\angle -120°$$

The resultant current which leaves the junction is given by,

$$I = \overline{I_1} + \overline{I_2} + \overline{I_3} = 3\angle 0° + 5\angle 30° + 6\angle -120°$$
$$= 5.1\angle -31.9°$$
$$i = 5.1\sqrt{2}\sin(\omega t - 31.9°) = 7.21\sin(\omega t - 31.9°)$$

20. In a circuit four currents are meeting at a point, find the resultant current.

$i_1 = 5\sin\omega t$, $i_2 = 10\sin(\omega t - 30°)$

$i_3 = 5\cos(\omega t - 30°)$ $i_4 = -10\sin(\omega t + 45°)$

Data : $i_1 = 5\sin\omega t$

$i_2 = 10\sin(\omega t - 30°)$

$i_3 = 5\cos(\omega t - 30°) = 5\sin(\omega t + 60°)$

$i_4 = -10\sin(\omega t + 45°) = 10\sin(\omega t + 225°)$

Writing currents i_1, i_2, i_3 and i_4 in phasor form,

$$\overline{I_1} = \frac{5}{\sqrt{2}}\angle 0° = 3.54\angle 0°$$

$$\overline{I_2} = \frac{10}{\sqrt{2}}\angle -30° = 7.07\angle -30°$$

$$\overline{I_3} = \frac{5}{\sqrt{2}}\angle 60° = 3.54\angle 60°$$

$$\overline{I_4} = \frac{10}{\sqrt{2}}\angle 225° = 7.07\angle 225°$$

Resultant current $\overline{I} = \overline{I_1} + \overline{I_2} + \overline{I_3} + \overline{I_4}$

$= 3.54\angle 0° + 7.07\angle -30° + 3.54\angle 60° + 7.07\angle 225°$

$= 8.44\angle -40.36°$

$i = 8.44\sqrt{2}\sin(\omega t - 40.36°)$

$= 11.94 \sin(\omega t - 40.36°)$

21. Find the resultant voltage and its equation for the given voltages.

$e_1 = 20\sin\omega t,\quad e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right),\quad e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right)$

Data : $e_1 = 20\sin\omega t$

$e_2 = 30\sin\left(\omega t - \frac{\pi}{4}\right) = 30\sin(\omega t - 45°)$

$e_3 = 40\cos\left(\omega t + \frac{\pi}{6}\right) = 40\sin(\omega t + 120°)$

Writing voltages e_1, e_2 and e_3 in phasor form,

$\overline{E_1} = \frac{20}{\sqrt{2}}\angle 0° = 14.14\angle 0°$

$\overline{E_2} = \frac{30}{\sqrt{2}}\angle -45° = 21.21\angle -45°$

$\overline{E_3} = \frac{40}{\sqrt{2}}\angle 120° = 28.28\angle 120°$

Resultant voltage $\overline{E} = \overline{E_1} + \overline{E_2} + \overline{E_3}$

$= 14.14\angle 0° + 21.21\angle -45° + 28.28\angle 120°$

$= 17.75\angle 32.33°$

$e = 17.75\sqrt{2}\sin(\omega t + 32.33°)$

$= 25.1\sin(\omega t + 32.33°)$

22. Obtain the sum of three voltages.

$v_1 = 147.3\cos(\omega t + 98.1)$

$v_2 = 294.6\cos(\omega t - 45°)$

$v_3 = 88.4\sin(\omega t + 135°)$

Data : $v_1 = 147.3\cos(\omega t + 98.1) = 147.3\sin(\omega t + 188.1)$

$v_2 = 294.6\cos(\omega t - 45°) = 294.6\sin(\omega t + 45°)$

$v_3 = 88.4\sin(\omega t + 135°)$

Writing the voltages v_1, v_2 and v_3 in phasor form,

$\overline{V_1} = \frac{147.3}{\sqrt{2}}\angle 188.1 = 104.16\angle 188.1°$

$\overline{V_2} = \frac{294.6}{\sqrt{2}}\angle 45° = 208.31\angle 45°$

$\overline{V_3} = \frac{88.4}{\sqrt{2}}\angle 135° = 62.51\angle 135°$

Resultant voltage $\overline{V} = \overline{V_1} + \overline{V_2} + \overline{V_3}$

$= 104.16\angle 188.1° + 208.31\angle 45° + 62.51\angle 135°$

$= 176.82\angle 90°$

$v = 176.82\sqrt{2}\sin(\omega t + 90°)$

$= 250.06\sin(\omega t + 90°)$

23. Find vectorially the resultant of the following four voltages.

$e_1 = 25\sin\omega t,\quad e_2 = 30\sin\left(\omega t + \frac{\pi}{6}\right),$

$e_3 = 30\cos\omega t,\quad e_4 = 20\sin\left(\omega t - \frac{\pi}{6}\right).$

Obtain the answer in similar form.

Data : $e_1 = 25\sin\omega t$

$e_2 = 30\sin\left(\omega t + \frac{\pi}{6}\right) = 30\sin(\omega t + 30°)$

$e_3 = 30\cos\omega t = 30\sin(\omega t + 90°)$

$e_4 = 20\sin\left(\omega t - \frac{\pi}{6}\right) = 20\sin(\omega t - 30°)$

Writing voltages e_1, e_2, e_3 and e_4 in phasor form,

$\overline{E_1} = \frac{25}{\sqrt{2}}\angle 0° = 17.68\angle 0°$

$\overline{E_2} = \frac{30}{\sqrt{2}}\angle 30° = 21.21\angle 30°$

$\overline{E_3} = \frac{30}{\sqrt{2}}\angle 90° = 21.21\angle 90°$

$\overline{E_4} = \frac{20}{\sqrt{2}}\angle -30° = 14.14\angle -30°$

Resultant voltage $\overline{E} = \overline{E_1} + \overline{E_2} + \overline{E_3} + \overline{E_4}$

$= 17.68\angle 0° + 21.21\angle 30° + 21.21\angle 90° + 14.14\angle -30°$

$= 54.26\angle 27.13°$

$e = 54.26\sqrt{2}\sin(\omega t + 27.13°)$

$= 76.74\sin(\omega t + 27.13°)$

24. Two currents are represented by $i_1 = 15\sin\left(\omega t + \frac{\pi}{3}\right)$ and $i_2 = 25\sin\left(\omega t + \frac{\pi}{4}\right)$. These currents are fed into a common conductor. Find the total current in the form $i = I_m \sin(\omega t + \phi)$. If the conductor has a resistance of 10 Ω, what will be the energy loss in 24 hours?

Data : $i_1 = 15\sin\left(\omega t + \frac{\pi}{3}\right)$

$i_2 = 25\sin\left(\omega t + \frac{\pi}{4}\right)$

$R = 10\ \Omega$

$t = 24\ \text{hrs} = 86400\ \text{sec.}$

Writing currents i_1 and i_2 in phasor form,

$$\overline{I_1} = \frac{15}{\sqrt{2}} \angle 60^\circ = 10.61\angle 60^\circ$$

$$\overline{I_2} = \frac{25}{\sqrt{2}} \angle 45^\circ = 17.68\angle 45^\circ$$

Total current $\overline{I} = \overline{I_1} + \overline{I_2} = 10.61\angle 60^\circ + 17.68\angle 45^\circ$

$= 28.06\ \angle 50.62^\circ$

$i = 28.06\sqrt{2}\ \sin(\omega t + 50.62^\circ)$

$= 39.68\ \sin(\omega t + 50.62^\circ)$

Energy loss in 24 hours, $E = I^2Rt$ where I is rms value of current

$E = (28.06)^2 \times 10 \times 86400 = 6.8 \times 10^8\ J$

25. The voltage drops across four series connected impedances are :

$v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right)$ $v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right)$,

$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right)$, $v_4 = V_{4m} \sin(\omega t + \phi_4)$

Calculate the values of V_{4m} and ϕ_4 if the voltage applied across the series circuit is $140 \sin\left(\omega t + \frac{3\pi}{5}\right)$

Data : $v_1 = 60 \sin\left(\omega t + \frac{\pi}{6}\right) = 60 \sin(\omega t + 30^\circ)$

$v_2 = 75 \sin\left(\omega t - \frac{5\pi}{6}\right) = 75 \sin(\omega t - 150^\circ)$

$v_3 = 100 \cos\left(\omega t + \frac{\pi}{4}\right) = 100 \sin(\omega t + 135^\circ)$

$v = 140 \sin\left(\omega t + \frac{3\pi}{5}\right) = 140 \sin(\omega t + 108^\circ)$

Writing voltages v_1, v_2, v_3 and v in phasor form,

$$\overline{V_1} = \frac{60}{\sqrt{2}} \angle 30^\circ = 42.43\angle 30^\circ$$

$$\overline{V_2} = \frac{75}{\sqrt{2}} \angle -150^\circ = 53.03\angle -150^\circ$$

$$\overline{V_3} = \frac{100}{\sqrt{2}} \angle 135^\circ = 70.71\angle 135^\circ$$

$$\overline{V} = \frac{140}{\sqrt{2}} \angle 108^\circ = 98.99\angle 108^\circ$$

For series connected impedances,

$\overline{V} = \overline{V_1} + \overline{V_2} + \overline{V_3} + \overline{V_4}$

$\overline{V_4} = \overline{V} - \overline{V_1} - \overline{V_2} - \overline{V_3}$

$= 98.99\angle 108^\circ - 42.43\angle 30^\circ - 53.03\angle -150^\circ - 70.71\angle 135^\circ$

$= 57.13\ \angle 59.96^\circ$

$v_4 = 57.13\sqrt{2}\ \sin(\omega t + 59.96^\circ)$

$= 80.79\ \sin(\omega t + 59.96^\circ)$

$V_{4m} = 80.79\ V$

$\phi_4 = 59.96^\circ$

26. Two voltages having rms values of 50 and 75 V have phase difference of 60°. Find th resultant sum of these two voltages.

Data : $V_1 = 50\ V$

$V_2 = 75\ V$

$\phi = 60^\circ$

Let $\overline{V_1} = 50\ \angle 0^\circ\ V$

$\overline{V_2} = 75\ \angle -60^\circ\ V$

Resultant voltage $\overline{V} = \overline{V_1} + \overline{V_2}$

$= 50\angle 0^\circ + 75\angle -60^\circ$

$= 108.97\ \angle -36.58^\circ\ V$

27. Two single phase alternators supply 300 and 400 A respectively at a phase differenc of 20° to a common load. Find the resultant current and its phase relation to it component.

Data : $I_1 = 300\ A$

$I_2 = 400\ A$

$\phi = 20^\circ$

Let $\overline{I_1} = 300\ \angle 0^\circ\ A$

$\overline{I_2} = 400\ \angle -20^\circ\ A$

Resultant current $\overline{I} = \overline{I_1} + \overline{I_2}$

$= 300\angle 0^\circ + 400\angle -20^\circ$

$= 689.59\ \angle -11.44^\circ\ A$

28. Two voltage sources have equal emfs and a phase difference α. When they are connected in series, the voltage is 200 V. When one source is reversed, the voltage is 15 V. Find their emfs and phase angle α.

Data : $\overline{E_1} = E\angle 0^\circ$

$\overline{E_2} = E\angle \alpha^\circ$

$E_1 = E_2 = E$

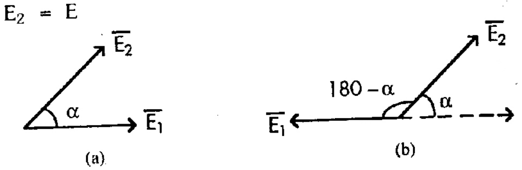


Fig. 2.19

When two sources are connected in series,

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha} = 200$$

$$\sqrt{E^2 + E^2 + 2E^2 \cos\alpha} = 200$$

$$2E^2 + 2E^2 \cos\alpha = 40000 \quad \ldots (1)$$

When one source is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha} = 15$$

$$\sqrt{E^2 + E^2 - 2E^2 \cos\alpha} = 15$$

$$2E^2 - 2E^2 \cos\alpha = 225 \quad \ldots (2)$$

Adding equations (1) and (2),

$$4E^2 = 40225$$

$$E^2 = 10056.25$$

$$E = 100.28\text{ V}$$

$$2E^2 + 2E^2 \cos\alpha = 40000$$

$$20112.5 + 20112.5 \cos\alpha = 40000$$

$$\cos\alpha = 0.988$$

$$\alpha = 8.58^\circ$$

29. Two sinusoidal sources of emf have rms values E_1 and E_2 **and a phase difference α**. When connected in series, the resultant voltage is 41.1 V. When one of **the source** is reversed, the resultant emf is 17.52 V. When phase displacement is made zero, the resultant emf is 42.5 V. Calculate E_1, E_2 and α.

Data : $\overline{E_1} = E_1\angle 0$

$\overline{E_2} = E_2\angle \alpha$

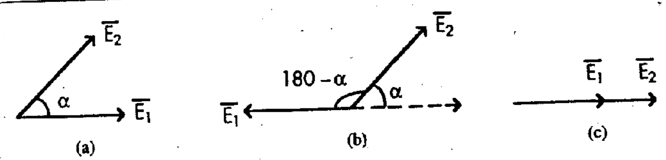


Fig. 2.20

When two sources are connected in series

$$\sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha} = 41.1$$

$$E_1^2 + E_2^2 + 2E_1E_2 \cos\alpha = 1689.21 \quad \ldots (1)$$

When one of the source is reversed,

$$\sqrt{E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha} = 17.52$$

$$E_1^2 + E_2^2 - 2E_1E_2 \cos\alpha = 306.95 \quad \ldots (2)$$

When phase displacement is made zero,

$$\sqrt{E_1^2 + E_1^2 + 2E_1E_2 \cos 0} = 42.5$$

$$E_1 + E_2 = 42.5 \quad \ldots (3)$$

Adding equations (1) and (2),

$$2\left(E_1^2 + E_2^2\right) = 1996.16$$

$$E_1^2 + E_2^2 = 998.08$$

$$(42.5 - E_2)^2 + E_2^2 = 998.08$$

$$1806.25 - 85E_2 + E_2^2 + E_2^2 = 998.08$$

$$E_2^2 - 42.5E_2 + 404.09 = 0 \quad \ldots (4)$$

Solving equation (4),

$E_2 = 28.14\text{ V}$ or $E_2 = 14.36\text{ V}$

$E_1 = 14.36$ or $E_1 = 28.14\text{ V}$

Subtracting equation (2) from equation (1),

$$4E_1E_2 \cos\alpha = 1382.26$$

$$4 \times 14.37 \times 28.14 \cos\alpha = 1382.26$$

$$\cos\alpha = 0.855$$

$$\alpha = 31.24^\circ$$

2.7 BEHAVIOUR OF PURE RESISTOR IN AC CIRCUIT

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig. 2.21. Let the alternating voltage $v = V_m \sin \omega t$.

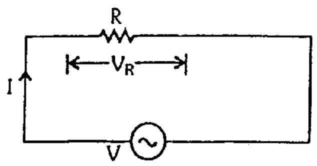


Fig. 2.21

The alternating current i is given by,

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \ldots \left(I_m = \frac{V_m}{R}\right)$$

where I_m is maximum value of alternating current. From voltage and current equation, it is clear that current is in phase with the voltage in a pure resistive circuit.

Waveforms :

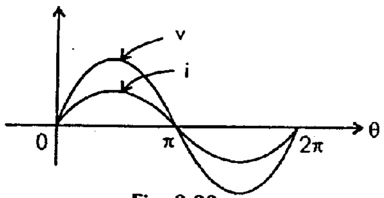


Fig. 2.22

Phasor diagram :

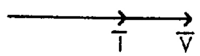


Fig. 2.23

Impedance : It is the resistance offered to the flow of current in ac circuit. In pure resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase difference : Since voltage and current are in phase with each other, phase difference is zero.

$$\phi = 0^\circ$$

Power factor : It is defined as the cosine of the angle between voltage and current phasor.

Power factor $= \cos \phi$

$= \cos (0^\circ) = 1$

Power :

Instantaneous power $p = vi$

$= V_m \sin \omega t \cdot I_m \sin \omega t = V_m I_m \sin^2 \omega t$

$= \frac{V_m I_m}{2} (1 - \cos 2\omega t)$

$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$

The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$. The frequency of fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

Average power $P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = VI$

Thus power in a pure resistive circuit is equal to the product of rms values of voltage and current.

2.8 BEHAVIOUR OF PURE INDUCTOR IN AC CIRCUIT

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. 2.24. Let the alternating voltage $v = V_m \sin \omega t$.

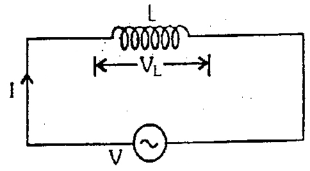


Fig. 2.24

The voltage across inductor is given by,

$$v = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int v\, dt = \frac{1}{L} \int V_m \sin \omega t\, dt$$

$$= \frac{V_m}{\omega L} (-\cos \omega t) = -\frac{V_m}{\omega L} \cos \omega t$$

$$= \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \ldots \left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is maximum value of alternating current. From voltage and current equation, it is clear that current lags behind voltage by 90° in a pure inductive circuit.

Waveform :

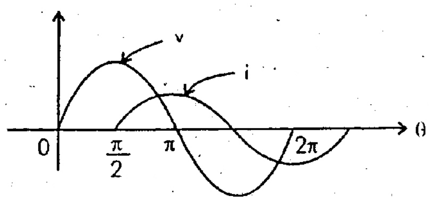


Fig. 2.25

Phasor Diagram :

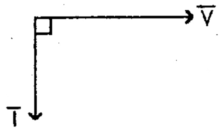


Fig. 2.26

Impedance :

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance and is denoted by X_L and is expressed in ohms.

For a dc supply, $f = 0 \;\therefore\; X_L = 0$

Thus inductor acts as a short circuit for dc supply.

Phase Difference : It is the angle between voltage and current phasor.

$\phi = 90°$

Power Factor : It is defined as the cosine of the angle between voltage and current phasor.

$pf = \cos\phi = \cos(90°) = 0$

Power : Instantaneous power,

$$p = vi$$

$$= V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a pure inductive circuit is zero.

9 BEHAVIOUR OF PURE CAPACITOR IN AC CIRCUIT

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig. 2.27. Let the alternating voltage $v = V_m \sin\omega t$.

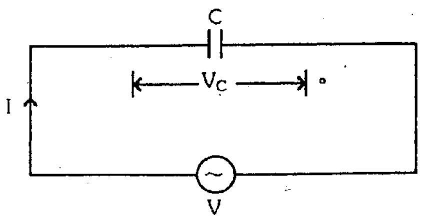


Fig. 2.27

The current through capacitor is given by,

$$i = C\frac{dv}{dt}$$

$$= C\frac{d}{dt}(V_m \sin\omega t)$$

$$= \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°) \qquad \ldots (I_m = \omega C V_m)$$

where I_m is maximum value of alternating current. From voltage and current equation, it is clear that current leads voltage by 90° in pure capacitive circuit.

Waveform :

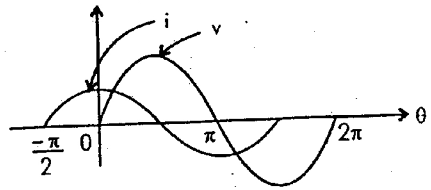


Fig. 2.28

Phasor Diagram :

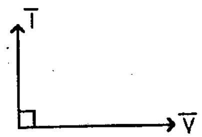


Fig. 2.29

Impedance :

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance and is denoted by X_C and is expressed in ohms.

For a dc supply $f = 0 \;\therefore\; X_C = \infty$

Thus capacitor acts as an open circuit for dc supply.

Phase Difference :

$\phi = 90°$

Power Factor :

$pf = \cos\phi = \cos(90°) = 0$

P = vi
= Vm sin ωt Im sin(ωt+
= Vm Im sin ωt cos ωt
= VmIm/2 sin 2ωt

78. For the circuit shown, calculate (i) total admittance, total conductance and total susceptance (ii) total current and total pf (iii) value of pure capacitance to be connected in parallel with above combination to make total pf unity.

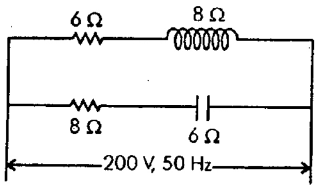


Fig. 2.77

(i)

$$\overline{Z_1} = 6 + j8 = 10 \angle 53.13^\circ\ \Omega$$

$$\overline{Z_2} = 8 - j6 = 10 \angle -36.86^\circ\ \Omega$$

$$\overline{Y_1} = \frac{1}{\overline{Z_1}} = \frac{1}{10 \angle 53.13^\circ} = 0.1 \angle -53.13^\circ\ ℧$$

$$\overline{Y_2} = \frac{1}{\overline{Z_2}} = \frac{1}{10 \angle -36.86^\circ} = 0.1 \angle 36.87^\circ\ ℧$$

$$\overline{Y} = \overline{Y_1} + \overline{Y_2}$$

$$= 0.1 \angle -53.13^\circ + 0.1 \angle 36.87^\circ$$

$$= 0.14 - j0.02\ ℧$$

$$= 0.14 \angle -8.13^\circ\ ℧$$

Total admittance $Y = 0.14\ ℧$

Total conductance $G = 0.14\ ℧$

Total susceptance $B = 0.02\ ℧$

Let $\overline{V} = 200 \angle 0^\circ$ V

(ii)

$$\overline{I} = \overline{V} \cdot \overline{Y}$$

$$= (200 \angle 0^\circ)(0.14 \angle -8.13^\circ)$$

$$= 28 \angle -8.13^\circ \text{ A}$$

Total pf $= \cos(8.13^\circ)$

$= 0.989$ (lagging)

(iii) Since current lags behind voltage, circuit is inductive in nature. In order to make total pf unity, a pure capacitor is connected in parallel.

$$\overline{Y_{req}} = 0.14 - j0.02 + j0.02$$

$$= 0.14$$

$$\frac{1}{X_C} = 0.02$$

$$X_C = 50$$

$$C = \frac{1}{2\pi \times 50 \times 50} = 63.66\ \mu F$$

2.14 RESONANCE

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with the current through it. At resonance, the circuit thus behaves as a pure resistance and the net reactance is zero.

2.15 SERIES RESONANCE

Consider the series R-L-C circuit as shown in Fig. 2.78. The impedance of the circuit is,

$$\overline{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

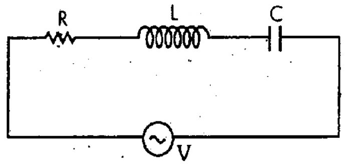


Fig. 2.78

(i) At resonance, Z must be resistive. Therefore the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

$$f = f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called resonant frequency of the circuit.

(ii) Power factor :

$$\text{Power factor} = \cos\phi$$

$$= \frac{R}{Z}$$

$$= \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

But at resonance $\omega L = \frac{1}{\omega C}$

$$\text{Power factor} = \frac{R}{R} = 1$$

(iii) Current : Since impedance is minimum, current is maximum at resonance. Thus, circuit accepts more current and as such R-L-C circuit under resonance is called acceptor circuit.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

(iv) Voltage : At resonance,

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 L\, I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

Thus, potential difference across inductance equal to potential difference across capacitance being equal and opposite cancel each other. Also since I_0 is maximum, V_{L_0} and V_{C_0} will also be maximum. Thus, voltage magnification takes place during resonance. Hence, it is also referred to as voltage magnification circuit.

(v) Phasor diagram :

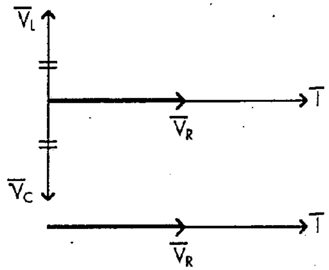


Fig. 2.79

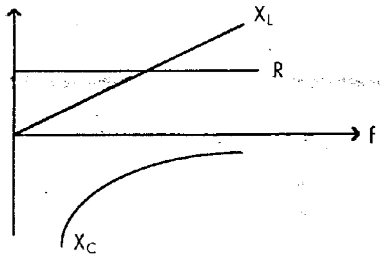


Fig. 2.80

(vi) Behaviour of R, L and C with change in frequency : Resistance remains constant with the change in frequencies. Inductive reactance X_L is directly proportional to frequency f. It can be drawn as a straight line passing through origin. Capacitive reactance X_C is inversely proportional to frequency f. It can be drawn as a rectangular hyperbola in fourth quadrant.

Total impedance

$$\overline{Z} = R + j(X_L - X_C)$$

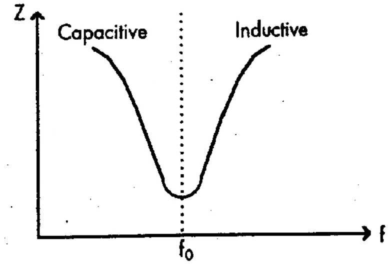


Fig. 2.81

(a) When $f < f_0$, impedance is capacitive and decreases upto f_0. Power factor is leading in nature.

(b) At $f = f_0$, impedance is resistive. Power factor is unity.

(c) When $f > f_0$, impedance is inductive and goes on increasing beyond f_0. Power factor is lagging in nature.

(vii) Bandwidth :

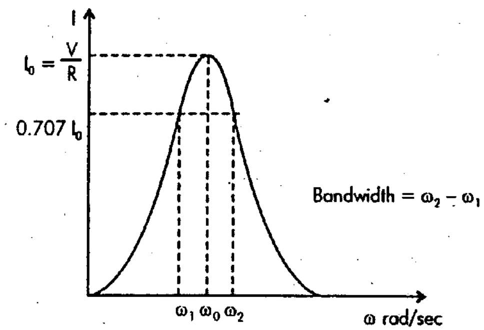


Fig. 2.82

For the series R-L-C circuit, bandwidth is defined as the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance. From the shape of resonance curve, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance. For this reason, these frequencies are referred to as those corresponding to the half power points. The magnitude of the current at each half power point is the same.

Hence $I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$

where subscript 1 denotes the lower half point and subscript 2 the higher half point. It follows then that

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707\, I_0$$

Accordingly, bandwidth may be identified on the resonance curve as the range of frequencies over which the magnitude of the current is equal to or greater than 0.707 of the current at resonance. In above Fig. 2.82, bandwidth is $\omega_2 - \omega_1$.

Expression for the bandwidth : Generally at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 - \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \ldots (1)$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But $$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \quad \ldots (2)$$

From equation (1) and (2), we get

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{1}{\sqrt{2}R}$$

Squaring both sides we get,

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega L - \frac{1}{\omega C} \pm R = 0$$

$$\omega^2 \pm \frac{R}{L}\,\omega - \frac{1}{LC} = 0$$

$$\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by, $\omega = \pm \frac{R}{2L} \pm \sqrt{\frac{1}{LC}} = \pm \frac{R}{2L} \pm \frac{1}{\sqrt{LC}}$

The resonant frequency for this circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega = \pm \frac{R}{2L} + \omega_0 \quad \text{(considering only +ve sign of } \omega_0\text{)}$$

$$\omega_1 = \omega_0 - \frac{R}{2L}$$

and $$\omega_2 = \omega_0 + \frac{R}{2L}$$

$$\text{Bandwidth} = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/sec.}$$

or $$\text{Bandwidth} = f_2 - f_1 = \frac{R}{2\pi L} \text{ c/sec}$$

(viii) Quality factor Q_0 of the R-L-C circuit : It is ratio of resonant frequency to the bandwidth. It is a measure of the selectivity or sharpness of tuning of the series R-L-C circuit.

$$Q_0 = \frac{\omega_0}{\text{Bandwidth}}$$

$$Q_0 = \frac{\omega_0}{R/L} = \frac{\omega_0 L}{R} = \frac{X_{L_0}}{R} = \frac{1}{\omega_0 RC}$$

$$Q_0 = \frac{1/\sqrt{LC}}{R/L} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$Q_0 = \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

where V_{L_0} and V_{C_0} are both measured at resonance. Hence Q_0 is also called as voltage magnification factor.

79. A series R-L-C circuit has the following parameter values, R = 10 Ω, L = 0.01 H, C = 100 μF. Compute the resonant frequency, quality factor of the circuit, bandwidth, lower and upper frequency of the bandwidth.

Data : R = 10 Ω

L = 0.01 H

C = 100 μF

$$\text{Resonant frequency } f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01 \times 100 \times 10^{-6}}}$$

$$= 159.15 \text{ Hz}$$

$$\text{Bandwidth BW} = \frac{R}{2\pi L} = \frac{10}{2\pi \times 0.01} = 159.15 \text{ Hz}$$

$$\text{Lower frequency of bandwidth} \quad f_1 = f_0 - \frac{BW}{2}$$

$$= 159.15 - \frac{159.15}{2}$$

$$= 79.58 \text{ Hz}$$

When capacitance is 600 pF, current reduces to one-half of current at resonance,

$$X_C = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 10^6 \times 600 \times 10^{-12}}$$

$$= 265.26\ \Omega$$

$$I = \frac{1}{2} I_0$$

$$\frac{V}{Z} = \frac{1}{2}\frac{V}{R}$$

$$Z = 2R$$

$$\sqrt{R^2 + (X_L - X_C)^2} = 2R$$

$$R^2 + (314.16 - 265.26)^2 = 4R^2$$

$$3R^2 = 2391.21$$

Resistance of inductor, $R = 28.23\ \Omega$

Quality factor $Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$$= \frac{1}{28.23}\sqrt{\frac{0.05 \times 10^{-3}}{500 \times 10^{-12}}}$$

$$= 11.2$$

2.16 PARALLEL RESONANCE

Consider a parallel circuit consisting of a coil and a capacitor as shown in Fig. 2.84. The impedance of two branches are,

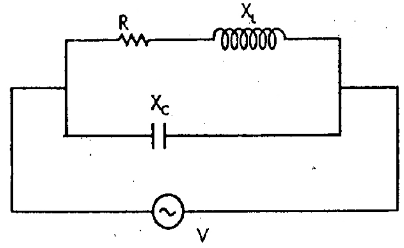


Fig. 2.84

The impedance of two branches are,

$$\overline{Z_1} = R + jX_L$$

$$\overline{Z_2} = -jX_C$$

$$\overline{Y_1} = \frac{1}{\overline{Z_1}} = \frac{1}{R + jX_L}$$

$$= \frac{R - jX_L}{R^2 + X_L^2}$$

$$\overline{Y_2} = \frac{1}{\overline{Z_2}} = \frac{1}{-jX_C}$$

$$= \frac{j}{X_C}$$

Admittance of the circuit $\overline{Y} = \overline{Y_1} + \overline{Y_2}$

$$= \frac{R - jX_L}{R^2 + X_L^2} + j\frac{1}{X_C}$$

$$= \frac{R}{R^2 + X_L^2} - j\left(\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C}\right)$$

At resonance, the circuit is purely resistive. Therefore the condition for resonance is

$$\frac{X_L}{R^2 + X_L^2} - \frac{1}{X_C} = 0$$

$$\frac{X_L}{R^2 + X_L^2} = \frac{1}{X_C}$$

$$X_L \cdot X_C = R^2 + X_L^2$$

$$\omega_0 L \cdot \frac{1}{\omega_0 C} = R^2 + \omega_0^2 L^2$$

$$\omega_0^2 L^2 = \frac{L}{C} - R^2$$

$$\omega_0^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

where f_0 is called the resonant frequency of the circuit.

If R is very small as compared to L, then

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Dynamic Impedance of Parallel Circuit :

At resonance, the circuit is purely resistive. The real part of admittance is $\frac{R}{R^2 \times X_L^2}$. Hence the dynamic impedance at resonance is given by,

$$Z = \frac{R^2 + X_L^2}{R}$$

At resonance,

$$R^2 + X_L^2 = X_L \cdot X_C$$

$$= \frac{L}{C}$$

$$Z = \frac{L}{CR}$$

Comparison of Series and Parallel Resonant Circuits :

Parameter	Series Circuit	Parallel Circuit
Current at resonance	$I = \frac{V}{R}$ and is maximum	$I = \frac{V}{(L/CR)}$ and is minimum
Impedance at resonance	Z = R and is minimum	$Z = \frac{L}{CR}$ and is maximum
Power factor at resonance	Unity	Unity
Resonant frequency	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q - factor	$Q = \frac{2\pi fL}{R}$	$Q = \frac{2\pi fL}{R}$
It magnifies	Voltage across L and C	Current through L and C

88. A coil of 20 Ω resistance has an inductance of 0.2 H is connected in parallel with a condenser of 100 μF capacitance. Calculate the frequency at which this circuit will behave as a non inductive resistance. Find also the value of dynamic resistance.

Data : R = 20 Ω L = 0.2 H

C = 100 μF

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 100 \times 10^{-6}} - \left(\frac{20}{0.2}\right)^2}$$

$$= 31.83 \text{ Hz}$$

$$\text{Dynamic resistance} = \frac{L}{CR}$$

$$= \frac{0.2}{100 \times 10^{-6} \times 20}$$

$$= 100\ \Omega$$

89. A coil having a resistance of 20 Ω and an inductance of 200 μH is connected in parallel with a variable capacitor. This parallel combination is connected in series with a resistance of 8000 Ω. A voltage of 230 V at a frequency of 10^6 Hz is applied across the circuit. Calculate (a) the value of capacitance at resosnance (b) Q factor of the circuit (c) dynamic impedance of the ciruct (d) total circuit current.

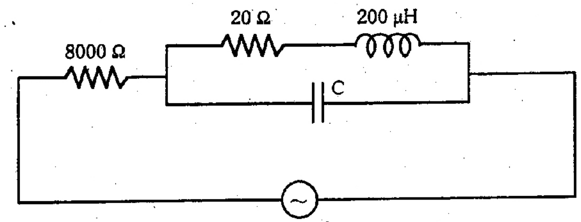


Fig. 2.85

Data : R = 20 Ω L = 200 μH

f = 10^6 Hz V = 230 V

R_S = 8000 Ω

$$X_L = 2\pi fL = 2 \times \pi \times 10^6 \times 200 \times 10^{-6} = 1256.6\ \Omega$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{200 \times 10^{-6} \times C} - \frac{(20)^2}{(200 \times 10^{-6})^2}}$$

$$C = 126.65 \times 10^{-12} \text{ F}$$

$$= 126.65 \text{ pF}$$

$$\text{Quality Factor } Q_0 = \frac{2\pi fL}{R}$$

$$= \frac{2\pi \times 10^6 \times 200 \times 10^{-6}}{20}$$

$$= 62.83$$

In transformer, losses are negligible. Hence input and output can be approximately equated.

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1} = K$$

For step up transformers,

$$N_2 > N_1$$

$$K > 1$$

For step down transformers,

$$N_2 < N_1$$

$$K < 1.$$

4.5 RATING OF TRANSFORMER

Rating of a transformer indicates the output power from it. But for a transformer, load is not fixed and its power factor goes on changing. Hence rating is not expressed in terms of power but in terms of product of voltage and current called as VA rating. This rating is generally expressed in kVA.

kVA rating of transformer $= \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$

We can calculate full load currents of primary and secondary windings from kVA rating of transformer. Full load current is maximum current which can flow through the windings without damaging it.

Full load primary current. $I_1 = \frac{\text{kVA rating} \times 1000}{V_1}$

Full load secondary current $I_2 = \frac{\text{kVA rating} \times 1000}{V_2}$

1. What will be the secondary voltage at no load if the primary of a 5 kVA, 220/110 V, 50 Hz transformer is fed at (i) 110 V, 50 Hz (ii) 220 V dc.

Data : kVA rating = 5 kVA

$E_1 = 220$ V

$E_2 = 110$ V

(i) For a transformer,

$$\frac{V_2}{V_1} = \frac{E_2}{E_1}$$

$$\frac{V_2}{110} = \frac{110}{220}$$

$$V_2 = 55 \text{ V}$$

(ii) When transformer is fed 220 V dc, no emf is induced in primary winding.

$$V_2 = 0$$

2. It is desired to have 4.13 mWb maximum flux in the core of a transformer operating at 110 V and 50 Hz. Determine the required number of turns in the primary.

Data : $\phi_m = 4.13$ mWb

$V_1 = 110$ V

$f = 50$ Hz

For a transformer, $V_1 = E_1 = 110$ V

$$E_1 = 4.44 f \phi_m N_1$$

$$110 = 4.44 \times 50 \times 4.13 \times 10^{-3} \times N_1$$

$$N_1 = 120 \text{ turns}$$

3. A single phase 50 Hz transformer has 80 turns on the primary winding and 280 in the secondary winding. The voltage applied across the primary winding is 240 V at 50 Hz. Calculate (i) maximum flux density in the core (ii) induced emf in the secondary. The net cross-sectional area of the core is 200 cm².

Data : $f = 50$ Hz

$N_1 = 80$

$N_2 = 280$

$V_1 = 240$ V

$A = 200$ cm²

For a transformer, $V_1 \approx E_1 = 240$ V

$$E_1 = 4.44 f \phi_m N_1 = 4.44 f B_m A N_1$$

$$240 = 4.44 \times 50 \times B_m \times 200 \times 10^{-4} \times 80$$

$$B_m = 0.68 \text{ Wb/m}^2$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_2 = \frac{N_2}{N_1} E_1 = \frac{280}{80} \times 240 = 840 \text{ V}$$

4. An 80 kVA, 3200/400 V, 50 Hz single phase transformer has 111 turns on secondary winding. Calculate (i) number of turns on primary winding (ii) secondary current (iii) cross sectional area of the core if the maximum flux density is 1.2 Tesla.

Data : kVA rating = 80 kVA

$E_1 = 3200$ V

$E_2 = 400$ V

$N_2 = 111$ turns

$B_m = 1.2$ Tesla

$f = 50$ Hz

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{400}{3200} = \frac{111}{N_1}$$

$$N_1 = 888$$

$$V_2 \approx E_2 = 400\ V$$

$$I_2 = \frac{kVA\ rating \times 1000}{V_2} = \frac{80 \times 1000}{400} = 200\ A$$

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$

$$400 = 4.44 \times 50 \times 1.2 \times A \times 111$$

$$A = 0.0135\ m^2 = 135\ cm^2$$

5. A 5 kVA, 240/2400 V, 50 Hz single phase transformer has the maximum value of flux density as 1.2 Tesla. If the emf per turn is 8, calculate (i) number of primary turns and secondary turns (ii) cross sectional area of the core (iii) primary and secondary current at full load.

Data : kVA rating = 5 kVA

$E_1 = 240$ V

$E_2 = 2400$ V

$f = 50$ Hz

$B_m = 1.2$ Tesla

$$\frac{E_1}{N_1} = 8$$

$$8 = \frac{240}{N_1}$$

$$N_1 = 30$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{2400}{240} = \frac{N_2}{30}$$

$$N_2 = 300$$

$$E_2 = 4.44\, f\, \phi_m\, N_2 = 4.44\, f\, B_m\, A\, N_2$$

$$2400 = 4.44 \times 50 \times 1.2 \times A \times 300$$

$$A = 0.03\ m^2$$

For a transformer,

$$V_1 \approx E_1 = 240\ V$$

$$V_2 \approx E_2 = 2400\ V$$

$$I_1 = \frac{kVA\ rating \times 1000}{V_1} = \frac{5 \times 1000}{240} = 20.83\ A$$

$$I_2 = \frac{kVA\ rating \times 1000}{V_2} = \frac{5 \times 1000}{2400} = 2.08\ A$$

6. A 250 kVA, 50 Hz single phase transformer has ratio of secondary to primary turns as 0.1. The secondary voltage at no load condition is 240 V. Calculate (i) primary voltage (ii) full load primary and secondary currents.

Data : kVA rating = 250 kVA

$$\frac{N_2}{N_1} = 0.1 \qquad E_2 = 240\ V$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = 0.1$$

$$E_1 = 2400\ V$$

For a transformer,

$$V_1 \approx E_1 = 2400\ V$$

$$V_2 \approx E_2 = 240\ V$$

$$I_1 = \frac{kVA\ rating \times 1000}{V_1} = \frac{250 \times 1000}{2400} = 104.17\ A$$

$$I_2 = \frac{kVA\ rating \times 1000}{V_2} = \frac{250 \times 1000}{240} = 1041.67\ A$$

4.6 IDEAL AND PRACTICAL TRANSFORMER

There are two types of losses in transformer.

(i) Core or iron loss

(ii) Copper loss

(i) **Core or iron loss :** It includes hysteresis and eddy current loss. Hysteresis loss occurs due to setting of alternating flux in the core.

Eddy current loss occurs due to setting of eddy currents in the core which is due to induced emf in the core.

The flux set up in the core is nearly constant. Hence core loss is practically constant at all the loads, from no load to full load.

(ii) **Copper loss :** This loss is due to the resistances of primary and secondary windings.

The copper loss depends on load current and is proportional to square of the load current r kVA rating of the transformer.

For an ideal transformer (i) there will be no core loss and copper loss (ii) winding resistance and leakage flux are zero. But in practical transformer, the windings have some resistance and there is always some leakage flux.

It has been assumed that all the flux linked with primary winding also links the secondary winding. However all the flux linked with primary does not link the secondary. This is known as primary leakage flux. Similarly when flux is set up in the secondary, all the flux linked with secondary does not link the primary, but part of it links with the secondary itself through air. This flux is known as secondary leakage flux. Leakage fluxes produce self induced emf in their respective winding. It is therefore equivalent to an inductive coil in series with the respective windings.

Practical transformer consists of winding resistances R_1 and R_2 and leakage reactances X_1 and X_2 as shown in Fig. 4.2.

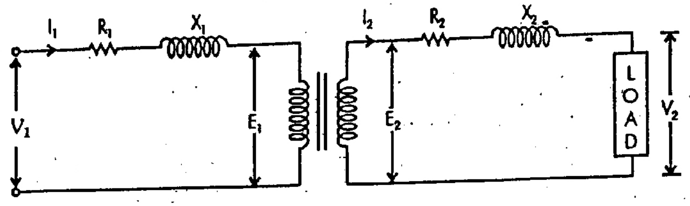


Fig. 4.2

4.7 PHASOR DIAGRAM OF TRAMSFORMER ON NO LOAD

When the transformer is operating at no load, there is iron loss in the core and copper loss in the primary winding. Thus primary input current I_0 has to supply iron loss in the core and a very small amount of copper loss in primary. Hence current I_0 has two components, (i) a magnetizing or reactive component I_μ and (ii) power or active component I_w.

The magnetizing component I_μ is responsible for setting up flux in the core. It is in phase with flux ϕ.

$$I_\mu = I_0 \sin \phi_0$$

The active component I_w is responsible for power loss in the transformer. It is in phase with V_1.

$$I_w = I_0 \cos \phi_0$$

Hence no load current I_0 is phasor sum of I_μ and I_w.

$$\overline{I_0} = \overline{I_\mu} + \overline{I_w}$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2}$$

The no load input power is given by,

$$W_0 = V_1 I_0 \cos \phi_0$$

where $\cos \phi_0$ is power factor at no load.

The no load current I_0 is very small as compared to full load current I_1. Hence copper loss is negligible and no load input power is practically equal to iron loss or core loss in the transformer.

$$\text{Iron loss } W_i = V_1 I_0 \cos \phi_0$$

Phasor Diagram :

Since the flux ϕ is common to both the windings, ϕ is chosen as reference phasor. From emf equation of transformer, it is clear that E_1 and E_2 lag the flux by 90°. Hence emfs E_1 and E_2 are drawn such that these lag behind the flux ϕ by 90°. The magnetizing component I_μ is drawn in phase with the flux ϕ. The applied voltage V_1 is drawn equal and opposite to E_1 as $V_1 = E_1$. The active component I_w is drawn in phase with voltage V_1. The phasor sum of I_μ and I_w gives no load current I_0.

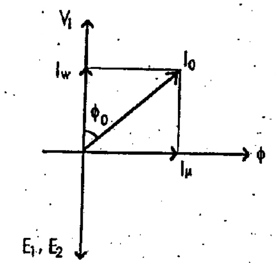


Fig. 4.3

7. A 50 kVA 2300/230 V, 50 Hz transformer takes 200 watts and 0.3 A at no load, when 2300 V are applied to the high voltage side. The primary resistance is 3.5 Ω. Determine (i) no load pf (ii) core loss.

Data : $W_0 = 200$ W $\quad R_1 = 3.5\ \Omega$

$V_1 = 2300$ V $\quad I_0 = 0.3$ A

$$\text{Copper loss in primary} = I_0^2 R_1$$

$$= (0.3)^2 \times 3.5 = 0.315 \text{ W}$$

$$\text{Core loss} = \text{Input power} - \text{Copper loss}$$

$$= 200 - 0.315 = 199.685 \text{ W}$$

$$\text{Input power } W_0 = V_1 I_0 \cos \phi_0$$

$$\cos \phi_0 = \frac{200}{2300 \times 0.3} = 0.29 \text{ (lagging)}$$

8. A single phase transformer has primary voltage of 230 V. No load primary current is 5 A. No load pf is 0.25. Number of primary turns are 200 and frequency is 50 Hz. Calculate (i) maximum value of flux in the core (ii) core loss (iii) magnetizing current.

Data : $I_0 = 5$ A

$\cos\phi_0 = 0.25$

$V_1 = 230$ V

$N_1 = 200$

$f = 50$ Hz

For a transformer, $V_1 \approx E_1 = 230$ V

$E_1 = 4.44 f \phi_m N_1$

$230 = 4.44 \times 50 \times \phi_m \times 200$

$\phi_m = 5.18$ mWb

Neglecting primary copper loss,

Core loss $W_i = V_1 I_0 \cos\phi_0$

$= 230 \times 5 \times 0.25 = 287.5$ W

$\cos\phi_0 = 0.25$

$\sin\phi_0 = 0.97$

Magnetizing current $I_\mu = I_0 \sin\phi_0$

$= 5 \times 0.97 = 4.85$ A

4.8 PHASOR DIAGRAM OF TRANSFORMER ON LOAD

When the transformer is loaded, a current I_2 will flow in the secondary winding. The secondary current I_2 sets up a secondary flux ϕ_2 that tends to reduce the flux ϕ produced by primary current. Hence induced emf E_1 in primary reduces. This causes more current to flow in the primary. Let the additional current in primary be I_2'. This current I_2' is anti phase with I_2 and sets up its own flux ϕ_2' which cancels the flux ϕ_2 produced by I_2.

$\phi_2 = \phi_2'$

$N_2 I_2 = N_1 I_2'$

$I_2' = \frac{N_2}{N_1} I_2 = K I_2$

Hence the primary current I_1 is the phasor sum of no load current I_0 and current I_2'.

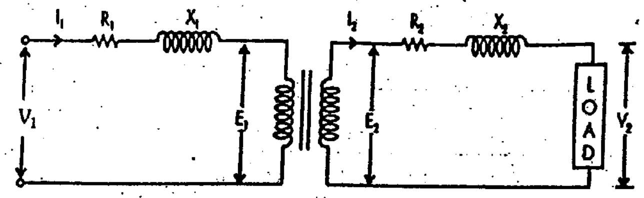


Fig. 4.4

Fig. 4.4 shows a practical transformer on load condition. When a transformer is loaded, current I_2 flows in secondary winding and voltage V_2 appears across the load. Current I_2 is in phase with voltage V_2 if load is resistive, it lags behind it if load is inductive and it leads if load is capacitive.

Writing vector equations for primary and secondary sides,

$\overline{V_1} = \overline{I_1 R_1} + \overline{I_1 X_1} + (-\overline{E_1})$

$\overline{E_2} = \overline{I_2 R_2} + \overline{I_2 X_2} + \overline{V_2}$

where $\overline{I_1} = \overline{I_0} + \overline{I_2'}$

The phasor diagram of transformer on load condition is drawn with the help of above expressions.

Steps for drawing phasor diagrams :

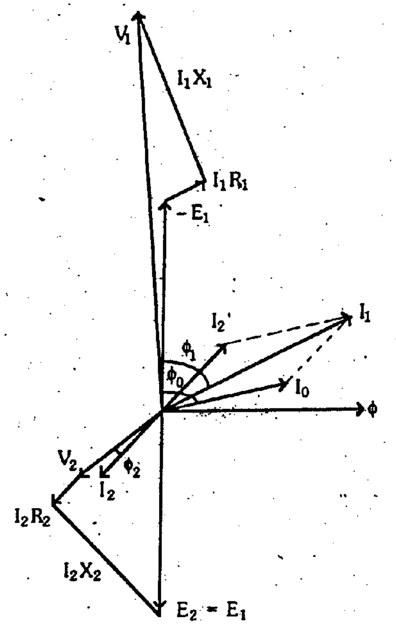
1. First draw $\overline{V_2}$ and then $\overline{I_2}$. Phase angle between $\overline{I_2}$ and $\overline{V_2}$ will depend on the type of load.
2. To $\overline{V_2}$, add resistive drop $\overline{I_2 R_2}$, parallel to $\overline{I_2}$ and inductive drop $\overline{I_2 X_2}$, leading $\overline{I_2}$ by 90° such that

   $\overline{E_2} = \overline{V_2} + \overline{I_2 R_2} + \overline{I_2 X_2}$.
3. Draw $\overline{E_1}$ on the same side such that $E_1 = \frac{E_2}{K}$.
4. Draw $-\overline{E_1}$ equal and opposite to $\overline{E_1}$.
5. For drawing $\overline{I_1}$, first draw $\overline{I_0}$ and $\overline{I_2'}$ such that.

   $I_2' = K I_2$.
6. Add $\overline{I_0}$ and $\overline{I_2'}$ using parallelogram law of vector addition.

   $\overline{I_1} = \overline{I_0} + \overline{I_2'}$.

Case (iii) Capacitive load (leading pf)



K = 1

Fig. 4.7

4.9 EQUIVALENT CIRCUIT

Fig. 4.8 shows a practical transformer. R_1 and R_2 represent the resistances of primary and secondary windings respectively. Similarly X_1 and X_2 represent the leakage reactances of primary and secondary windings respectively.

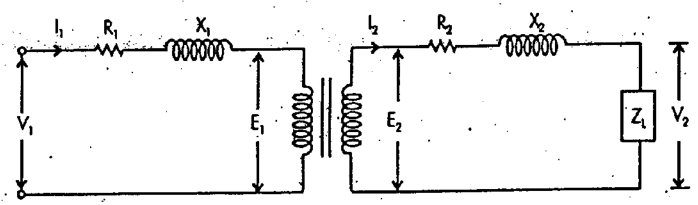


Fig. 4.8

Fig. 4.8 can be further modified to represent no load current I_0 and its component. The current I_0 is the phasor sum of currents I_w and I_μ. Hence current I_0 is simulated by resistance R_0 taking working component I_w and inductance X_0 taking magnetizing component I_μ connected in parallel across the primary circuit.

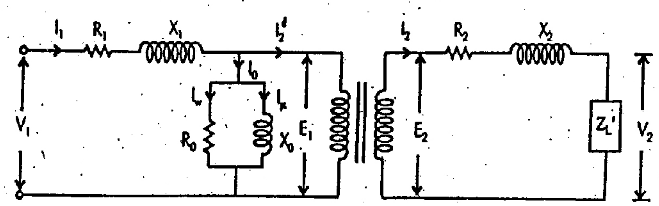


Fig. 4.9

For convenience all the quantities can be shown on only one side by transferring the quantities from one side to other without any power loss. The power loss in secondary is $I_2^2 R_2$. If R_2' is the resistance referred to primary which would have caused the same power loss as R_2 in secondary,

$$I_1^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_1}\right)^2 R_2$$

$$= \frac{R_2}{K^2}$$

Similarly $X_2' = \dfrac{X_2}{K^2}$.

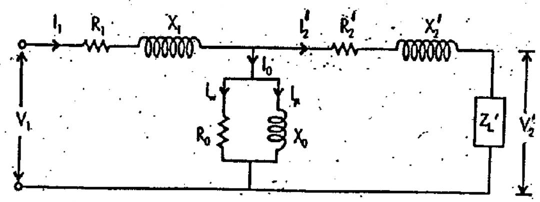


Fig. 4.10

Since all quantities are transferred to primary, transformer need not be shown. The no load current I_0 is very small compared to full load current I_1. Hence drop across R_1 and X_1 due to I_0 can be neglected. Therefore transferring R_0 and X_0 to the extreme left,

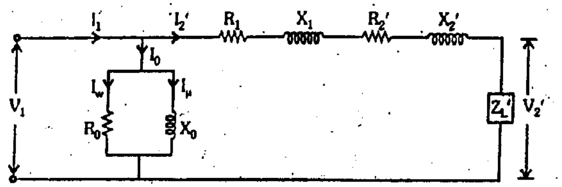


Fig. 4.11

The equivalent resistance referred to primary $R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{K^2}$

The equivalent leakage reactance referred to primary $X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{K^2}$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}$$

The equivalent circuit referred to primary will be as shown in Fig. 4.12.

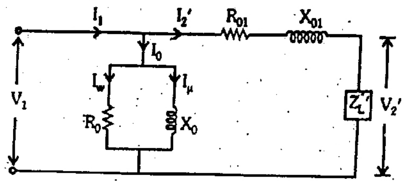


Fig. 4.12

Similarly the equivalent circuit referred to secondary will be as shown in Fig. 4.13.

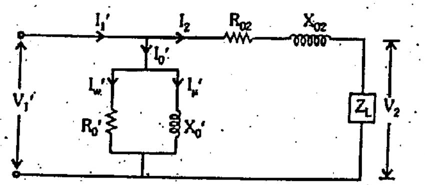


Fig. 4.13

where R_{02} → equivalent resistance referred to secondary

$R_{02} = R_2 + R_1' = R_2 + K^2 R_1 = K^2 R_{01}$

X_{02} → equivalent leakage reactance referred to secondary

$X_{02} = X_2 + X_1' = X_2 + K^2 X_1 = K^2 X_{01}$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = K^2 Z_{01}$$

Note :

(i) While shifting any primary resistance or reactance to the secondary, multiply it by K^2.

(ii) While shifting any secondary resistance or reactance to the primary, divide it by K^2.

9. A transformer has a turn ratio $N_1 : N_2$ of 4. If a 50 Ω resistance is connected across the secondary, what is the resistance referred to primary?

Data : $\frac{N_1}{N_2} = 4$

$R = 50\ \Omega$

$K = \frac{N_2}{N_1} = \frac{1}{4}$

Let resistance R be connected across the secondary.

Equivalent resistance referred to primary $= R' = \frac{R}{K^2} = \frac{50}{(1/4)^2} = 800\ \Omega$

10. A resistance connected across the secondary of an ideal transformer has a value of 800 Ω as referred to the primary. The same resistance when connected across the primary has a value of 3.125 Ω as referred to secondary. Find the ratio of the transformer.

Let R be the resistance connected to secondary.

Then equivalent resistance referred to primary $= \frac{R}{K^2}$

$\frac{R}{K^2} = 800\ \Omega$

If resistance R is connected across primary, the equivalent resistance referred to secondary $= K^2 R$

$K^2 R = 3.125$

$\frac{K^2 R}{R/K^2} = \frac{3.125}{800}$

$K^4 = 3.90624 \times 10^{-3}$

Turn ratio $K = 0.25$

11. A 6600/400 V transformer has primary resistance of 2.5 Ω and reactance of 3.9 Ω. The secondary resistance is 0.01 Ω and reactance is 0.025 Ω. Determine the equivalent circuit parameters referred to primary and secondary.

Data : $E_1 = 6600$ V

$E_2 = 400$ V

$R_1 = 2.5\ \Omega$

$X_1 = 3.9\ \Omega$

$R_2 = 0.01\ \Omega$

$X_2 = 0.025\ \Omega$

$K = \frac{E_2}{E_1} = \frac{400}{6600} = 0.06$

Equivalent resistance referred to primary,

$$R_{01} = R_1 + \frac{R_2}{K^2} = 2.5 + \frac{0.01}{(0.06)^2}$$

$$= 5.28\ \Omega$$

Equivalent reactance referred to primary,

$$X_{01} = X_1 + \frac{X_2}{K^2} = 3.9 + \frac{0.025}{(0.06)^2}$$

$$= 10.84\ \Omega$$

Equivalent resistance referred to secondary,

$$R_{02} = R_2 + K^2 R_1 = K^2 R_{01}$$

$$= (0.06)^2 \times 5.28 = 0.02\ \Omega$$

Equivalent reactance referred to secondary,

$$X_{02} = X_2 + K^2 X_1 = K^2 X_{01}$$

$$= (0.06)^2 \times 10.84 = 0.04\ \Omega$$

12. A 50 kVA, 4400/220 V transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$. The reactances are $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate for the transformer, (i) full load currents on primary and secondary side (ii) equivalent resistances, reactances, impedances referred to primary side and secondary side (iii) total copper loss using individual resistances and equivalent resistances.

Data : $E_1 = 4400$ V $E_2 = 220$ V

$R_1 = 3.45\ \Omega$ $R_2 = 0.009\ \Omega$

$X_1 = 5.2\ \Omega$ $X_2 = 0.015\ \Omega$

$$K = \frac{E_2}{E_1} = \frac{220}{4400} = 0.05$$

Full load primary current $I_1 = \frac{50 \times 1000}{4400} = 11.36$ A

Full load secondary current $I_2 = \frac{50 \times 1000}{220} = 227.27$ A

Equivalent resistance referred to primary,

$$R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(0.05)^2} = 7.05\ \Omega$$

Equivalent reactance referred to primary,

$$X_{01} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{(0.05)^2} = 11.2\ \Omega$$

Equivalent impedance referred to primary,

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(7.05)^2 + (11.2)^2}$$

$$= 13.23\ \Omega$$

Equivalent resistance referred to secondary,

$$R_{02} = K^2 R_{01} = (0.05)^2 \times 7.05 = 0.02\ \Omega$$

Equivalent reactance referred to secondary,

$$X_{02} = K^2 X_{01} = (0.05)^2 \times 11.2 = 0.028\ \Omega$$

Equivalent impedance referred to secondary,

$$Z_{02} = K^2 Z_{01} = (0.05)^2 \times 13.23 = 0.03\ \Omega$$

Copper loss with individual resistances,

$$I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227.27)^2 \times 0.009$$

$$= 445.22 + 464.86 = 910.08\ \text{W}$$

Copper loss with equivalent resistances,

$$I_1^2 R_{01} = I_2^2 R_{02} = (11.36)^2 \times 7.05 = 909.8\ \text{W}$$

4.10 VOLTAGE REGULATION

When a transformer is loaded, the secondary terminal voltage decreases due to drop across secondary winding resistance and leakage reactance. This change in secondary terminal voltage from no load to full load conditions, expressed as a fraction of the no load secondary voltage is called as regulation of the transformer.

$$\text{Regulation} = \frac{\left(\begin{array}{c}\text{Secondary terminal}\\ \text{voltage on no load}\end{array}\right) - \left(\begin{array}{c}\text{Secondary terminal voltage}\\ \text{on full load condition}\end{array}\right)}{\text{Secondary terminal voltage on no load}}$$

$$= \frac{E_2 - V_2}{E_2}$$

$$\text{Percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100.$$

Expression for Voltage Regulation :

Consider phasor diagram of transformer referred to secondary side on load condition (load is assumed to be inductive). With O as centre and radius OC, draw an arc cutting OA produced at M. From point B, draw BD perpendicular on OA produced. Draw CN perpendicular to OM and draw BL parallel to OM.

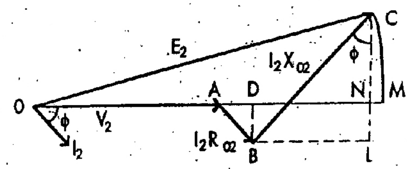


Fig. 4.14

Total voltage drop $= E_2 - V_2 = OC - OA = OM - OA$

$= AM = AN + NM$

Approximate voltage drop $\approx AN$ (∵ NM is very small)

$= AD + DN = AD + BL$

$= I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$

$$\% \text{ Regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} \times 100$$

For leading pf,

Approximate voltage drop $= I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$

$$\% \text{ Regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{E_2} \times 100$$

Hence in general,

$$\% \text{ Regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

'+' sign is used for lagging pf and '–' sign is used for leading pf.

On primary side, we can express regulation as,

$$\% \text{ Regulation} = \frac{I_1 R_{01} \cos\phi \pm I_1 X_{01} \sin\phi}{V_1} \times 100$$

We can also express percentage regulation as,

$$\% \text{ Regulation} = \frac{100\, I_2 R_{02}}{E_2} \cos\phi \pm \frac{100\, I_2 X_{02}}{E_2} \sin\phi$$

$$= v_r \cos\phi \pm v_x \sin\phi$$

where $v_r = \frac{100\, I_2 R_{02}}{E_2}$ = percentage resistive drop

$v_x = \frac{100\, I_2 X_{02}}{E_2}$ = percentage reactive drop.

13. A 200 kVA, 2200/440 V, 50 Hz, single phase transformer is operating at full load, 0.8 lagging pf. The voltage on secondary of the transformer at full load, 0.8 lagging pf is 400 V. Calculate voltage regulation of the transformer.

Data : $E_2 = 440$ V

$V_2 = 400$ V

$$\% \text{ Regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

$$= \frac{440 - 400}{440} \times 100$$

$= 9.09$ %.

14. A single phase, 440/220 V, 10 kVA, 50 Hz transformer has a resistance of 0.2 Ω and reactance of 0.6 Ω on h.v. side. The corresponding values of l.v. side are 0.04 Ω and 0.14 Ω. Calculate the percentage regulation on full load for (i) 0.8 lagging pf (ii) 0.8 leading pf (iii) unity pf.

Data : $E_2 = 220$ V

$E_1 = 440$ V

$R_1 = 0.2\ \Omega$

$X_1 = 0.6\ \Omega$

$R_2 = 0.04\ \Omega$

$X_2 = 0.14\ \Omega$

$I_2 = \frac{10 \times 1000}{220} = 45.45$ A

$K = \frac{E_2}{E_1} = \frac{220}{440} = 0.5$

$R_{02} = R_2 + K^2 R_1$

$= 0.04 + (0.5)^2 \times 0.2 = 0.09\ \Omega$

$X_{02} = X_2 + K^2 X_1$

$= 0.14 + (0.5)^2 \times 0.6 = 0.29\ \Omega$

(i) For 0.8 lagging pf,

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$$\% \text{ Regulation} = \frac{I_2 (R_{02} \cos\phi + X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{45.45\,(0.09 \times 0.8 + 0.29 \times 0.6)}{220} \times 100$$

$= 5.08$ %

(ii) For 0.8 leading pf,

$$\% \text{ Regulation} = \frac{I_2 (R_{02} \cos\phi - X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{45.45\,(0.09 \times 0.8 - 0.29 \times 0.6)}{220} \times 100$$

$= -2.11$ %

(iii) For unity pf,

$\cos\phi = 1$

$\sin\phi = 0$

$$\% \text{ Regulation} = \frac{I_2 (R_{02} \cos\phi \pm X_{02} \sin\phi)}{E_2} \times 100$$

$$= \frac{45.45\,(0.09 \times 1 - 0.29 \times 0)}{220} \times 100$$

$$= 1.86\,\%.$$

15. Calculate the regulation of a transformer in which resistive drop is 1% of the output and reactive drop is 5% of the output, when the pf is (a) 0.8 lagging (b) unity (c) 0.8 leading.

Data : $v_r = 1$

$v_x = 5$

$\% \text{ Regulation} = v_r \cos\phi \pm v_x \sin\phi$

(a) For 0.8 lagging pf,

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$\% \text{ Regulation} = 1 \times 0.8 + 5 \times 0.6$

$= 3.8\,\%$

(b) For unity pf,

$\cos\phi = 1$

$\sin\phi = 0$

$\% \text{ Regulation} = 1 \times 1 + 5 \times 0 = 1\,\%$

(c) For 0.8 leading pf,

$\% \text{ Regulation} = 1 \times 0.8 - 5 \times 0.6 = -2.2\,\%.$

16. A transformer has a reactance drop of 5% and a resistance drop of 2.5%. Find the lagging power factor at which the voltage regulation is maximum and the value of this regulation.

$$\% \text{ Regulation} = V_r \cos\phi + V_x \sin\phi \quad \ldots (1)$$

Differentiating equation (1),

$$\frac{dR}{d\phi} = -V_r \sin\phi + V_x \cos\phi$$

For regulation to be maximum,

$$\frac{dR}{d\phi} = 0$$

$$-V_r \sin\phi + V_x \cos\phi = 0$$

$$\tan\phi = \frac{V_x}{V_r} = \frac{5}{2.5} = 2$$

$$\phi = 63.43°$$

$Pf = \cos\phi = 0.45$

$\sin\phi = 0.89$

Maximum percentage regulation $= V_r \cos\phi + V_x \sin\phi$

$= 2.5 \times 0.45 + 5 \times 0.89$

$= 5.58\%$

17. A 230/460 V, transformer has a primary resistance of 0.2 Ω and reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when supply 10 A at 0.8 pf lagging.

Data : $E_1 = 230\text{ V}$ $\quad E_2 = 460\text{ V}$

$R_1 = 0.2\,\Omega$ $\quad X_1 = 0.5\,\Omega$

$R_2 = 0.75\,\Omega$ $\quad X_2 = 1.8\,\Omega$

$I_2 = 10\text{ A}$ $\quad \cos\phi = 0.8$

$$K = \frac{E_2}{E_1} = \frac{460}{230} = 2$$

$R_{02} = R_2 + K^2 R_1 = 0.75 + (2)^2 \times 0.2 = 1.55\,\Omega$

$X_{02} = X_2 + K^2 X_1 = 1.8 + (2)^2 \times 0.5 = 3.8\,\Omega$

$\cos\phi = 0.8$

$\sin\phi = 0.6$

For lagging pf,

$E_2 - V_2 = I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$

Secondary terminal voltage

$V_2 = E_2 - I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$

$= 460 - 10\,(1.55 \times 0.8 + 3.8 \times 0.6)$

$= 424.8\text{ V}$

4.11 EFFICIENCY

Efficiency is defined as the ratio of output power to input power.

$$\text{Efficiency } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$$

$$= \frac{\text{Output}}{\text{Output + Copper loss + Iron loss}}$$

Also $$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{\text{Input} - \text{Copper loss} - \text{Iron loss}}{\text{Input}}$$

Condition for Maximum Efficiency : We know that,

$$\eta = \frac{\text{Output}}{\text{Output + Losses}}$$

Considering secondary side of the transformer,

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}}$$

Differentiating both sides w.r.t. I_2,

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2I_2 R_{02})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02})^2}$$

For maximum efficiency, $\frac{d\eta}{dI_2} = 0$

$$(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02}) V_2 \cos\phi_2 = V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2I_2 R_{02})$$

$$V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{02} = V_2 I_2 \cos\phi_2 + 2I_2^2 R_{02}$$

$$W_i = I_2^2 R_{02}$$

Similarly on primary side,

$$W_i = I_1^2 R_{01}$$

- Thus when Copper loss = Iron loss, the efficiency of the transformer is maximum.

Load corresponding to maximum efficiency

For maximum efficiency,

$$W_i = I_2^2 R_{02}$$

$$I_{2\,(\text{max. efficiency})} = \sqrt{\frac{W_i}{R_{02}}}$$

Multiplying both sides by V_2,

$$V_2 I_{2\,(\text{max. efficiency})} = V_2 \sqrt{\frac{W_i}{R_{02}}}$$

$$VA_{(\text{max. efficiency})} = V_2 I_2 \sqrt{\frac{W_i}{I_2^2 R_{02}}}$$

$$= V_2 I_2 \sqrt{\frac{W_i}{W_{Cu}}}$$

$$\text{Load kVA}_{(\text{max. efficiency})} = \text{Full load kVA} \sqrt{\frac{W_i}{W_{Cu}}}$$

where W_i = iron loss

W_{Cu} = full load copper loss

Note : The efficiency at any load is given by,

$$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

where x = ratio of actual to full load kVA

W_i = iron loss in kW

W_{Cu} = full load copper loss in kW

18. A 100 kVA, single phase transformer has iron loss of 600 W and a copper loss of 1.5 kW at full load current. Calculate the efficiency at (i) full load and 0.8 lagging pf (ii) half load and unity pf.

Data : Full load kVA = 100 kVA

W_i = 600 W = 0.6 kW

W_{Cu} = 1.5 kW

(i) Efficiency at full load and 0.8 lagging pf

$x = 1$

$pf = 0.8$

$$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.6 + (1)^2 \times 1.5} \times 100$$

$$= 97.44\ \%$$

(ii) Efficiency at half load and unity pf

$x = 0.5$

$pf = 0.1$

$$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 100 \times 1}{0.5 \times 100 \times 1 + 0.6 + (0.5)^2 \times 1.5} \times 100$$

$$= 98.09\ \%.$$

19. A 25 kVA, 2200/220 V, 50 Hz, single phase transformer has a primary resistance of 1.8 Ω and secondary resistance of 0.02 Ω. Calculate the efficiency of the transformer at (i) full load and unity pf (ii) half load and 0.8 lagging pf. Iron loss is 1000 watt.

Data : Full load kVA = 25 kVA

E_1 = 2200 V

E_2 = 220 V

R_1 = 1.8 Ω

R_2 = 0.02 Ω

W_i = 1000 W = 1 kW

$$I_2 = \frac{25 \times 1000}{220} = 113.64\ A$$

$$K = \frac{E_2}{E_1} = \frac{220}{2200} = 0.1$$

$$R_{02} = R_2 + K^2 R_1 = 0.02 + (0.1)^2 \times 1.8 = 0.038\ \Omega$$

$$W_{Cu} = I_2^2 R_{02} = (113.64)^2 \times 0.038 = 0.49\ kW$$

(i) Efficiency at full load and unity pf

$x = 1$

$pf = 1$

$$\% \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 25 \times 1}{1 \times 25 \times 1 + 1 + (1)^2 \times 0.49} \times 100$$

$= 94.38\%$

(ii) Efficiency at half load and 0.8 lagging pf

$x = 0.5$

$pf = 0.8$

$$\% \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 25 \times 0.8}{0.5 \times 25 \times 0.8 + 1 + (0.5)^2 \times 0.49} \times 100$$

$= 89.91\%$.

20. A 250 kVA, single phase transformer has 98.135 % efficiency at full load and 0.8 lagging pf. The efficiency at half load and 0.8 lagging pf is 97.751 %. Calculate the iron loss and full load copper loss.

Data : $\eta_1 = 98.135\%$

$\eta_2 = 97.751\%$

Efficiency at full load and 0.8 lagging pf

$x = 1$

$pf = 0.8$

$$\% \eta_1 = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$98.135 = \frac{1 \times 250 \times 0.8}{1 \times 250 + 0.8 + W_i + (1)^2 W_{Cu}} \times 100$$

$$= \frac{200}{200 + W_i + W_{Cu}} \times 100$$

$W_i + W_{Cu} = 3.8 \text{ kW}$

Efficiency at half load and 0.8 lagging pf

$x = 0.5$

$pf = 0.8$

$$\% \eta_2 = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$97.751 = \frac{0.5 \times 250 \times 0.8}{0.5 \times 250 \times 0.8 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$W_i + 0.25\, W_{Cu} = 2.3 \text{ kW}$ ... (2)

Solving equations (1) and (2),

Full load copper loss $W_{Cu} = 2 \text{ kW}$

Iron loss $W_i = 1.8 \text{ kW}$

21. A 600 kVA, single phase transformer has an efficiency of 92 % at full load and also at half load, working at unity pf. Calculate the efficiency of the transformer at 60 % full load and unity pf.

Data : $\eta_1 = \eta_2 = 92\%$

Efficiency at full load and unity pf

$x = 1$

$pf = 1$

$$92 = \frac{1 \times 600 \times 1}{1 \times 600 \times 1 + W_i + (1)^2 W_{Cu}} \times 100$$

$W_i + W_{Cu} = 52.2$ ... (1)

Efficiency at half load and unity pf

$x = 0.5$

$pf = 1$

$$92 = \frac{0.5 \times 600 \times 1}{0.5 \times 600 \times 1 + W_i + (0.5)^2 W_{Cu}} \times 100$$

$W_i + 0.25\, W_{Cu} = 26.1$ ... (2)

Solving equations (1) and (2),

$W_{Cu} = 34.8 \text{ kW}$

$W_i = 17.4 \text{ kW}$

Efficiency at 60 % full load and unity pf

$x = 0.6$

$pf = 1$

$$\% \eta = \frac{0.6 \times 600 \times 1}{0.6 \times 600 \times 1 + 17.4 + (0.6)^2 \times 34.8} \times 100$$

$= 92.32\%$.

22. A 150 kVA, single phase transformer has iron loss of 1.4 kW and full load copper loss of 1.6 kW. Determine (a) the kVA load for maximum efficiency and the maximum efficiency at 0.8 lagging pf (b) the efficiency at half full load and 0.8 lagging pf.

Data : Full load kVA = 150 kVA

$W_i = 1.4 \text{ kW}$

$W_{Cu} = 1.6 \text{ kW}$

(a) Load kVA for maximum efficiency

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 150 \times \sqrt{\frac{1.4}{1.6}}$$

$$= 140.31 \text{ kVA}$$

For maximum efficiency,

$W_i = W_{Cu} = 1.4$ kW

pf = 0.8

$$\% \eta_{max} = \frac{\text{Load kVA} \times pf}{\text{Load kVA} \times pf + W_i + W_{Cu}} \times 100$$

$$= \frac{140.31 \times 0.8}{140.31 \times 0.8 + 1.4 + 1.4} \times 100$$

$$= 97.57 \%$$

(b) Efficiency at half full load and 0.8 pf

x = 0.5

pf = 0.8

$$\% \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 150 \times 0.8}{0.5 \times 150 \times 0.8 + 1.4 + (0.5)^2 \times 1.6} \times 100$$

$$= 97.08 \%.$$

23. A 100 kVA, single phase transformer has an efficiency of 97 % at full load and 0.8 lagging pf. If the maximum efficiency occurs at 80 % of full load at 0.8 lagging pf. Calculate (i) iron loss (ii) full load copper loss (iii) maximum efficiency.

Data : Full load kVA = 100 kVA

η = 97 %

Efficiency at full load and 0.8 lagging pf

x = 1

pf = 0.8

$$\% \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$97 = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + W_i + (1)^2 W_{Cu}} \times 100$$

$W_i + W_{Cu} = 2.474$ kW ... (1)

Maximum efficiency occurs at 80 % of full load

$W_i = (0.8)^2 W_{Cu} = 0.64 W_{Cu}$ ... (2)

Solving equations (1) and (2),

$W_{Cu} = 1.508$ kW

$W_i = 0.965$ kW

Maximum efficiency at 80 % full load and 0.8 lagging pf

$$\% \eta_{max} = \frac{0.8 \times 100 \times 0.8}{0.8 \times 100 \times 0.8 + 0.965 + 0.965} \times 100$$

$$= 97.07 \%.$$

24. The maximum efficiency of a 500 kVA, 3000/500 V, 50 Hz single phase transformer is 98 % and occurs at 3/4 full load, unity pf. If the impedance is 10 %, calculate the regulation at full load, 0.8 lagging pf.

Data : η_{max} = 98 %

Since maximum efficiency occurs at 3/4 of full load and unity pf, iron loss is equal to copper loss at this load.

$$\% \eta_{max} = \frac{\text{Load kVA} \times pf}{\text{Load kVA} \times pf + W_i + W_i} \times 100$$

$$98 = \frac{3/4 \times 500 \times 1}{3/4 \times 500 \times 1 + 2W_i} \times 100$$

$W_i = 3.826$ kW

Copper loss at 3/4 of full load = 3.826 kW

$$\text{Full load copper loss} = \left(\frac{4}{3}\right)^2 \times 3.826$$

$$= 6.803 \text{ kW}$$

Percentage regulation at full load and 0.8 lagging pf

$$\% \text{ Resistance} = v_r = \frac{I_2 R_{02}}{E_2} \times 100 = \frac{I_1 R_{01}}{V_1} \times 100 = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

= % Cu loss at full load

$$= \frac{6.803}{500} \times 100 = 1.36 \%$$

$$\% \text{ Reactance} = v_x = \sqrt{\% Z^2 - \% R^2}$$

$$= \sqrt{(10)^2 - (1.36)^2}$$

$$= 9.91 \%$$

$\cos \phi = 0.8$

$\sin \phi = 0.6$

$$\% \text{ Regulation} = v_r \cos \phi + v_x \sin \phi$$

$$= 1.36 \times 0.8 + 9.91 \times 0.6$$

$$= 7.034 \%.$$

25. The maximum efficiency of a 100 kVA, 6600/250 V, single phase transformer occurs at half load and is 98 % at unity power factor. If the percentage impedance is 8%, calculate the percentage regulation and efficiency on full load at 0.8 lagging pf.

Data : $\eta_{max} = 98\ \%$

Since maximum efficiency occurs at half load and unity pf, iron loss is equal to copper loss at this load.

$$\%\ \eta_{max} = \frac{\text{Load kVA} \times \text{pf}}{\text{Load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$98 = \frac{1/2 \times 100 \times 1}{1/2 \times 100 \times 1 + 2W_i} \times 100$$

$$W_i = 0.51 \text{ kW}$$

Copper loss at half load = 0.51 kW

Full load copper loss = $(2)^2 \times 0.51$ = 2.04 kW

Efficiency at full load and 0.8 lagging pf

$x = 1$

$pf = 0.8$

$$\%\ \eta = \frac{1 \times 100 \times 0.8}{1 \times 100 \times 0.8 + 0.51 + (1)^2 \times 2.04} \times 100$$

$= 96.91\ \%$

Percentage regulation at full load and 0.8 lagging pf

$$\%\ R = v_r = \%\ \text{Cu loss} = \frac{I_1^2 R_{01}}{V_1 I_1} \times 100$$

$$= \frac{2.04}{100} \times 100$$

$= 2.04\%$

$$\%\ X = v_x = \sqrt{\%\ Z^2 - \%\ R^2}$$

$$= \sqrt{(8)^2 - (2.04)^2}$$

$= 7.74\ \%$

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$\%$ Regulation $= v_r \cos\phi + v_x \sin\phi$

$= 2.04 \times 0.8 + 7.74 \times 0.6$

$= 6.28\ \%$

26. A 300 kVA, single phase transformer has percentage resistance of 1.5% and maximum efficiency occurs at a load of 173.2 kVA. Find the efficiency at full load and 0.8 lagging pf.

Data : $\%\ R = 1.5$

$\%$ Resistance = $\%$ Cu loss

$$= \frac{\text{Full load copper loss}}{\text{Full load kVA}} \times 100$$

$$1.5 = \frac{\text{Full load copper loss}}{300} \times 100$$

Full load copper loss W_{Cu} = 4.5 kW

Also for maximum efficiency,

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$173.2 = 300 \times \sqrt{\frac{W_i}{4.5}}$$

$W_i = 1.5$ kW

Efficiency at full load and 0.8 lagging pf

$x = 1$

$pf = 0.8$

$$\%\ \eta = \frac{1 \times 300 \times 0.8}{1 \times 300 \times 0.8 + 1.5 + (1)^2 \times 4.5} \times 100$$

$= 97.6\ \%$.

27. The parameters of the equivalent circuit of 150 kVA, 2400/240 V transformers are as shown in the Fig. 4.15.

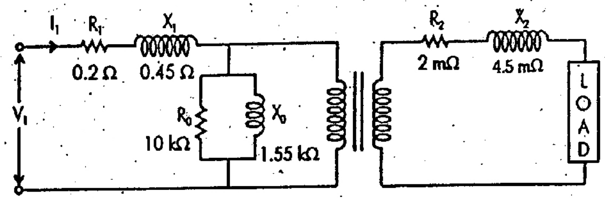


Fig. 4.15

Using the circuit referred to primary, determine voltage regulation and efficiency of the transformer operating at rated load with 0.8 lagging pf.

Percentage regulation at rated load and 0.8 lagging pf

$$K = \frac{240}{2400} = 0.1$$

$$R_{01} = R_1 + \frac{R_2}{K^2} = 0.2 + \frac{2 \times 10^{-3}}{(0.1)^2} = 0.4\ \Omega$$

$$X_{01} = X_1 + \frac{X_2}{K^2} = 0.45 + \frac{4.5 \times 10^{-3}}{(0.1)^2} = 0.9\ \Omega$$

$$I_1 = \frac{150 \times 1000}{2400} = 62.5 \text{ A}$$

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$$\% \text{ Regulation} = \frac{I_1 (R_{01}\cos\phi + X_{01}\sin\phi)}{V_1} \times 100$$

$$= \frac{62.5\,(0.4 \times 0.8 + 0.9 \times 0.6)}{2400} \times 100$$

$$= 2.24\ \%$$

Efficiency at rated load and 0.8 lagging pf

$$I_w = \frac{2400}{10 \times 10^3} = 0.24 \text{ A}$$

$$I_w = I_0 \cos\phi_0 = 0.24 \text{ A}$$

$$W_i = V_1 I_0 \cos\phi_0 = 2400 \times 0.24 = 576 \text{ W} = 0.576 \text{ kW}$$

$$W_{Cu} = I_1^2 R_{01} = (62.5)^2 \times 0.4 = 1562.5 \text{ W} = 1.5625 \text{ kW}$$

$x = 1$

$pf = 0.8$

$$\% \eta = \frac{1 \times 150 \times 0.8}{1 \times 150 \times 0.8 + 0.576 + (1)^2 \times 1.5625} \times 100$$

$$= 98.25\ \%.$$

4.12 ALL DAY EFFICIENCY

The transformers used for distribution are energized for 24 hours of the day. Thus constant losses occur in the transformer for the whole day. These transformers are normally operating on different loads during 24 hours of a day. As such copper losses are different during different periods of the day. Hence efficiency of such transformers should be measured on the energy basis.

All day efficiency,

$$\eta_{\text{all day}} = \frac{\text{Output in kWh}}{\text{Input in kWh}}$$

$$= \frac{\text{Output in kWh}}{\text{Output in kWh} + \text{Iron loss in kWh} + \text{Copper loss in kWh}}$$

28. A 50 kVA, single phase, transformer has load cycle for a day as follows :

(i) 50 kVA at 0.8 pf for 10 hours

(ii) 25 kVA at 0.6 pf for 10 hours

(iii) No load for 4 hours

The iron loss is 1000 W and full load copper loss is 1200 W. Calculate all day efficiency.

Data : $W_i = 1000 \text{ W} = 1 \text{ kW}$

$W_{Cu} = 1200 \text{ W} = 1.2 \text{ kW}$

$$\text{Output in 24 hours} = 50 \times 0.8 \times 10 + 25 \times 0.6 \times 10 + 0 \times 4$$

$$= 550 \text{ kWh}$$

$$\text{Copper loss at any load} = x^2 \times \text{Full load copper loss}$$

where $x = \dfrac{\text{Actual kVA}}{\text{Full load kVA}}$

$$\text{Copper loss in 24 hrs} = (1)^2 \times 1.2 \times 10 + \left(\frac{25}{50}\right)^2 \times 1.2 \times 10 + 0$$

$$= 12 + 3 + 0$$

$$= 15 \text{ kWh}$$

Iron loss remains constant at all load.

$$\text{Iron loss in 24 hrs} = 1 \times 24 = 24 \text{ kWh}$$

$$\eta_{\text{all day}} = \frac{\text{Output in 24 hrs}}{\left(\begin{matrix}\text{Output in}\\ \text{24 hrs}\end{matrix}\right) + \left(\begin{matrix}\text{Iron loss}\\ \text{in 24 hrs}\end{matrix}\right) + \left(\begin{matrix}\text{Copper loss}\\ \text{in 24 hrs}\end{matrix}\right)}$$

$$= \frac{550}{550 + 24 + 15} \times 100$$

$$= 93.37\ \%.$$

29. A 100 kVA lighting transformer has a full load loss of 3 kW the losses being equally divided between iron and copper. During a day, the transformer operates on full load for 3 hours, half load for 4 hours, the output being negligible for the remainder of the day. Calculate all day efficiency.

Data : Full load kVA = 100 kVA

Full load loss = 3 kW

Full load loss = Iron loss + Full load copper loss

But Iron loss = Full load copper loss

$W_i = 1.5 \text{ kW}$

$W_{Cu} = 1.5 \text{ kW}$

Output for 24 hrs $= 100 \times 3 + 50 \times 4 = 500 \text{ kWh}$

Copper loss for 3 hrs on full load $= 1.5 \times 3 = 4.5 \text{ kWh}$

Copper loss for 4 hrs on half load $= (0.5)^2 \times 1.5 \times 4 = 1.5 \text{ kWh}$

Copper loss for 24 hrs $= 4.5 + 1.5 = 6 \text{ kWh}$

Iron loss for 24 hrs $= 1.5 \times 24 = 36 \text{ kWh}$

Input for 24 hrs $= 500 + 6 + 36 = 542 \text{ kWh}$

$$\eta_{all\ day} = \frac{\text{Output for 24 hrs}}{\text{Input for 24 hrs}} \times 100$$

$$= \frac{500}{542} \times 100$$

$$= 92.25\ \%.$$

30. A 5 kVA distribution transformer has a full load efficiency of 95 % at which copper loss is equal to the iron loss. The transformer is loaded in 24 hours as follows :

No load for 10 hours

$\frac{1}{4}$ full load for 7 hours

$\frac{1}{2}$ full load for 5 hours

Full load for 2 hours.

Calculate all day efficiency of the transformer. Assume load pf of unity.

Data : $\eta_{max} = 95\%$

For maximum efficiency,

$$\%\ \eta_{max} = \frac{\text{Load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_{Cu}} \times 100$$

$$95 = \frac{5 \times 1}{5 \times 1 + W_i + W_i} \times 100$$

$W_i = 0.13$ kW

$W_{Cu} = 0.13$ kW

Output in 24 hours $= 0 \times 10 + \frac{1}{4} \times 5 \times 1 \times 7 + \frac{1}{2} \times 5 \times 1 \times 5 + 5 \times 1 \times 2$

$= 0 + 8.75 + 12.5 + 10$

$= 31.25$ kWh

Cu loss for 7 hours on $\frac{1}{4}$ full load $= \left(\frac{1}{4}\right)^2 \times 0.13 \times 7 = 0.06$ kWh

Cu loss for 5 hours on $\frac{1}{2}$ full load $= \left(\frac{1}{2}\right)^2 \times 0.13 \times 5 = 0.16$ kWh

Cu loss for 2 hours on full load $= (1)^2 \times 0.13 \times 2 = 0.26$ kWh

Cu loss for 24 hours $= 0.06 + 0.16 + 0.26 = 0.48$ kWh

Iron loss for 24 hrs $= 0.13 \times 24 = 3.12$ kWh

Input for 24 hrs $= 31.25 + 0.48 + 3.12 = 34.85$ kWh

$$\eta_{all\ day} = \frac{\text{Output for 24 hrs}}{\text{Input for 24 hrs}} \times 100$$

$$= \frac{31.25}{34.85} \times 100 = 89.67\ \%.$$

31. A transformer has its maximum efficiency of 0.98 at 15 kVA at unity pf. During the day it is loaded as follows :

12 hours - 2 kW at 0.5 pf

6 hours - 12 kW at 0.8 pf

6 hours - 18 kW at 0.9 pf

Find its all day efficiency.

[DEC 2007]

Data : $\eta_{max} = 0.98$

For maximum efficiency,

$$\eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i}$$

$$0.98 = \frac{15 \times 1}{15 \times 1 + 2W_i}$$

$W_i = 0.15$ kW

$W_{Cu} = 0.15$ kW

Output for 24 hrs $= 2 \times 12 + 12 \times 6 + 18 \times 6$

$= 204$ kWh

Copper loss for 12 hrs $= \left(\frac{2/0.5}{15}\right)^2 \times 0.15 \times 12$

$= 0.128$ kWh

Copper loss for 6 hrs $= \left(\frac{12/0.8}{15}\right)^2 \times 0.15 \times 6$

$= 0.9$ kWh

Copper loss for 6 hrs $= \left(\frac{18/0.9}{15}\right)^2 \times 0.15 \times 6$

$= 1.6$ kWh

Copper loss for 24 hrs $= 0.128 + 0.9 + 1.6$

$= 2.63$ kWh

Iron loss for 24 hrs $= 0.15 \times 24$

$= 3.6$ kWh

Input for 24 hrs $= 204 + 2.63 + 3.6$

$= 210.23$ kWh

$$\eta_{all\ day} = \frac{\text{Output for 24 hrs}}{\text{Input for 24 hrs}} \times 100$$

$$= \frac{204}{210.23} \times 100$$

$$= 97.04\ \%.$$

4.13 OPEN CIRCUIT (O.C.) TEST

The purpose of this test is to determine (i) iron loss or core loss (W_i) (ii) magnetizing resistance R_0 and (iii) magnetizing reactance X_0.

Fig. 4.16 shows the circuit diagram for conducting O.C. test on the transformer. In this test, one winding (usually high voltage winding) is left open and other winding is connected to a supply of normal voltage and frequency. Ammeter, voltmeter and wattmeter are connected on this side. Ammeter indicates no load current drawn by the transformer. As the no load current is usually 3 to 5% of full load current, copper losses will be negligible and wattmeter indicates iron loss.

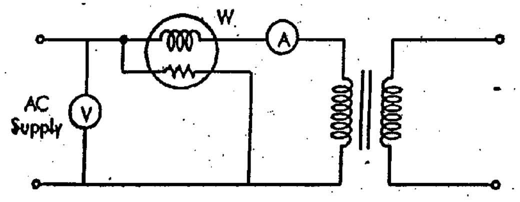


Fig. 4.16

Calculations :

(i) If meters are connected on primary side,

Wattmeter reading = W_i

Voltmeter reading = V_1

Ammeter reading = I_0

$$W_i = V_1 I_0 \cos\phi_0$$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0}$$

$$I_w = I_0 \cos\phi_0$$

$$I_\mu = I_0 \sin\phi_0$$

$$R_0 = \frac{V_1}{I_w}$$

$$X_0 = \frac{V_1}{I_\mu}$$

(ii) If meters are connected on secondary side,

Wattmeter reading = W_i

Voltmeter reading = V_2

Ammeter reading = I_0'

$$W_i = V_2 I_0' \cos\phi_0'$$

$$\cos\phi_0' = \frac{W_i}{V_2 I_0'}$$

$$I_w' = I_0' \cos\phi_0'$$

$$I_\mu' = I_0' \sin\phi_0'$$

$$R_0' = \frac{V_2}{I_w'}$$

$$R_0 = \frac{R_0'}{K^2}$$

$$X_0' = \frac{V_2}{I_\mu'}$$

$$X_0 = \frac{X_0'}{K^2}.$$

4.14 SHORT CIRCUIT (S.C.) TEST

The purpose of this test is to determine

(i) full load copper loss, (ii) equivalent resistance R_{01} or R_{02} and (iii) equivalent reactance X_{01} or X_{02}.

Fig. 4.17 shows the circuit diagram for conducting S.C. test on the transformer. In this test, one winding (usually low voltage winding) is short circuited, while a low voltage is applied to the other winding. The applied voltage is slowly increased until full load current flows in this winding and hence through the other winding. Normally the applied voltage is 5 to 10% of rated voltage of this winding. Hence flux produced in the core will be small and so the iron losses are very small. Thus wattmeter indicates full load copper loss.

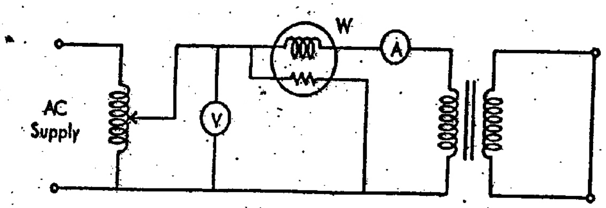


Fig. 4.17

Calculations :

(i) If meters are connected on primary side

Wattmeter reading = W_{sc}

Voltmeter reading = V_{sc}

Ammeter reading = I_{sc}

$$Z_{01} = \frac{V_{sc}}{I_{sc}}$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2}$$

(ii) If meters are connected on secondary side,

$$Z_{02} = \frac{V_{sc}}{I_{sc}}$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2}$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2}$$

Sometimes it is not possible to circulate full load current through the winding. Hence wattmeter reading does not indicate full load copper loss. Full load copper loss is calculated as follows :

$$W_{Cu} = \left(\frac{I_1}{I_{sc}}\right)^2 W_{sc} \quad \text{(If meters are connected on primary)}$$

$$= \left(\frac{I_2}{I_{sc}}\right)^2 W_{sc} \quad \text{(If meters are connected on secondary).}$$

32. A 5 kVA, 1000/200 V, 50 Hz, single phase transformer gave the following test results :

O.C. test (L.V. side)	200 V	1.2 A	90 W
S.C. test (H.V. side)	50 V	5 A	110 W

Determine efficiency at half load at 0.8 pf lagging.

From O.C. test (meters are connected on L.V. side i.e. secondary),

W_i = 90 W = 0.09 kW

From S.C. test (meters are connected on H.V. side i.e. primary),

W_{sc} = 110 W

$$\text{Full load current} = \frac{\text{kVA rating} \times 1000}{V_1}$$

$$I_1 = \frac{5 \times 1000}{1000} = 5\text{ A}$$

$$W_{Cu} = W_{sc} = 110\text{ W} = 0.11\text{ kW}$$

Efficiency at half load and 0.8 pf lagging

x = 0.5

pf = 0.8

$$\%\eta = \frac{x \times \text{Full load kVA} \times pf}{x \times \text{Full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 5 \times 0.8}{0.5 \times 5 \times 0.8 + 0.09 + (0.5)^2 \times 0.11} \times 100$$

$$= 94.45\ \%.$$

33. A 5 kVA, 200/400 V, 50 Hz, single phase transformer gave the following test results :

O.C. test (L.V. side)	200 V	0.7 A	60 W
S.C. test (H.V. side)	22 V	16 A	120 W

(i) Draw the equivalent circuit of the transformer and insert all parameter values.

(ii) Efficiency and regulation at 0.9 pf (lead) if operating at rated load.

(iii) Current at which efficiency is maximum.

(i) From O.C. test (meters are connected on L.V. side i.e. primary),

W_i = 60 W

V_1 = 200 V

I_0 = 0.7 A

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{200 \times 0.7} = 0.43$$

$$\sin\phi_0 = 0.9$$

$$I_w = I_0 \cos\phi_0 = 0.7 \times 0.43 = 0.3\text{ A}$$

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.3} = 666.67\ \Omega$$

$$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.9035 = 0.63\text{ A}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.63} = 317.46\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. secondary),

W_{sc} = 120 W

V_{sc} = 22 V

I_{sc} = 16 A

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{22}{16} = 1.375\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(16)^2} = 0.47\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.375)^2 - (0.47)^2}$$
$$= 1.29\ \Omega$$
$$K = \frac{400}{200} = 2$$
$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.47}{(2)^2} = 0.12\ \Omega$$
$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.29}{(2)^2} = 0.32\ \Omega$$

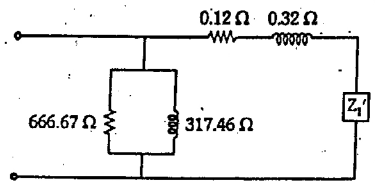


Fig. 4.18

(ii) $W_i = 60\ W = 0.06\ kW$

$W_{sc} = 120\ W$

$I_{sc} = 16\ A$

Since meters are connected on secondary in S.C. test,

$$I_2 = \frac{5 \times 1000}{400} = 12.5\ A$$
$$W_{Cu} = \left(\frac{I_2}{I_{sc}}\right)^2 W_{sc} = \left(\frac{12.5}{16}\right)^2 \times 120 = 73.24\ W$$
$$= 0.073\ kW$$

Efficiency at rated load and 0.9 pf leading

$x = 1$

$pf = 0.9$

$$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$
$$= \frac{1 \times 5 \times 0.9}{1 \times 5 \times 0.9 + 0.06 + (1)^2 \times 0.073} \times 100$$
$$= 97.13\ \%$$

Regulation at rated load and 0.9 pf lead

$\cos\phi = 0.9$

$\sin\phi = 0.44$

$$\%\ \text{Regulation} = \frac{I_2 (R_{02}\cos\phi - X_{02}\sin\phi)}{E_2} \times 100$$
$$= \frac{12.5\,(0.47 \times 0.9 - 1.29 \times 0.44)}{400} \times 100$$
$$= -0.45\ \%$$

(iii) At maximum efficiency,

$$W_i = I_2^2 R_{02}$$
$$I_2 = \sqrt{\frac{W_i}{R_{02}}} = \sqrt{\frac{60}{0.47}} = 11.3\ A$$

34. A 5 kVA, 250/500 V, 50 Hz, single phase transformer gave the following test results :

No load test (L.V. side)	250 V	0.75 A	60 W
Short Circuit (H.V. side)	9 V	6 A	21.6 W

Calculate : (a) the equivalent circuit constants and insert these on the equivalent circuit diagram.

(b) efficiency at 60 % of full load unity pf.

(c) maximum efficiency and the load at which it occurs.

(d) the secondary terminal voltage on full load at pf of 0.8 lagging, unity and 0.8 leading.

(a) From no load test (meters are connected on L.V. side i.e. primary),

$W_i = 60\ W$ $\quad V_1 = 250\ V$ $\quad I_0 = 0.75\ A$

$$\cos\phi_0 = \frac{W_i}{V_1 I_0} = \frac{60}{250 \times 0.75} = 0.32$$
$$\sin\phi_0 = 0.95$$
$$I_w = I_0\cos\phi_0 = 0.75 \times 0.32 = 0.24\ A$$
$$R_0 = \frac{V_1}{I_w} = \frac{250}{0.24} = 1041.66\ \Omega$$
$$I_\mu = I_0\sin\phi_0 = 0.75 \times 0.95 = 0.71\ A$$
$$X_0 = \frac{V_1}{I_\mu} = \frac{250}{0.71} = 351.84\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. secondary),

$W_{sc} = 21.6\ W$

$V_{sc} = 9\ V$

$I_{sc} = 6\ A$

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{9}{6} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{21.6}{(6)^2} = 0.6\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.6)^2} = 1.37\ \Omega$$

$$K = \frac{500}{250} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.6}{(2)^2} = 0.15\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.37}{(2)^2} = 0.34\ \Omega$$

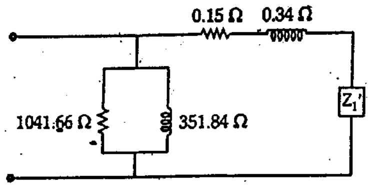


Fig. 4.19

(b) $W_i = 60\ W = 0.06\ kW$

$W_{sc} = 21.6\ W$

$I_{sc} = 6\ A$

Since meters are connected on H.V. side in S.C. test,

$$I_2 = \frac{5 \times 1000}{500} = 10\ A$$

$$W_{Cu} = \left(\frac{I_2}{I_{sc}}\right)^2 W_{sc} = \left(\frac{10}{6}\right)^2 \times 21.6 = 60\ W = 0.06\ kW$$

Efficiency at 60 % of full load and unity factor

$x = 0.6$

$pf = 1$

$$\% \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{0.6 \times 5 \times 1}{0.6 \times 5 \times 1 + 0.06 + (0.6)^2 \times 0.06} \times 100$$

$$= 97.35\ \%$$

(c) **The load corresponding to maximum efficiency**

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 5 \times \sqrt{\frac{60}{60}}$$

$$= 5\ kVA$$

For maximum efficiency,

$W_i = W_{Cu} = 60\ W = 0.06\ kW$

$pf = 1$

$$\% \eta_{max} = \frac{5 \times 1}{5 \times 1 + 0.06 + 0.06} \times 100$$

$$= 97.65\ \%$$

(d) We know that,

$$E_2 - V_2 = I_2 (R_{02} \cos\phi \pm X_{02} \sin\phi)$$

Secondary terminal voltage,

$$V_2 = E_2 - I_2 (R_{02} \cos\phi \pm X_{02} \sin\phi)$$

For $pf = 0.8$ lagging,

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$E_2 = 500\ V$

$V_2 = 500 - 10\,(0.6 \times 0.8 + 1.37 \times 0.6)$

$= 486.98\ V$

For $pf = 0.8$ leading,

$V_2 = 500 - 10\,(0.6 \times 0.8 - 1.37 \times 0.6)$

$= 503.42\ V$

For unity pf,

$V_2 = 500 - 10\,(0.6 \times 0.8 + 0)$

$= 494\ V$

35. A single phase, 50 kVA, 2400/120 V, 50 Hz transformer gave the following results :

O.C. test with instruments on L.V. side	120 V	9.65 A	396 W
S.C. test with instruments on H.V. side	92 V	20.8 A	810 W

Calculate : (i) equivalent circuit constant

(ii) Draw equivalent circuit

(iii) The efficiency when rated kVA is delivered to a load having a pf of 0.8 lagging

(iv) The voltage regulation

(v) kVA at maximum efficiency

(i) From O.C. test (meters are connected on L.V. side i.e. secondary),

$W_i = 396\ W$

$V_2 = 120\ V$

$I_0' = 9.65\ A$

$$\cos\phi_0' = \frac{396}{120 \times 9.65} = 0.34$$

$\sin\phi_0' = 0.94$

$I_w' = I_0' \cos\phi_0' = 9.65 \times 0.34 = 3.28 \text{ A}$

$R_0' = \frac{V_2}{I_w'} = \frac{120}{3.3} = 36.36\ \Omega$

$I_\mu' = I_0' \sin\phi_0' = 9.65 \times 0.94 = 9.07 \text{ A}$

$X_0' = \frac{V_2}{I_\mu'} = \frac{120}{9.07} = 13.23\ \Omega$

$K = \frac{120}{2400} = 0.05$

$R_0 = \frac{R_0'}{K^2} = \frac{36.36}{(0.05)^2} = 14.54\ \text{k}\Omega$

$X_0 = \frac{X_0'}{K^2} = \frac{13.23}{(0.05)^2} = 5.29\ \text{k}\Omega$

From S.C. test (meters are connected on primary),

$W_{sc} = 810 \text{ W}$

$V_{sc} = 92 \text{ V}$

$I_{sc} = 20.8 \text{ A}$

$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{92}{20.8} = 4.42\ \Omega$

$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{810}{(20.8)^2} = 1.87\ \Omega$

$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(4.42)^2 - (1.87)^2} = 4\ \Omega$

$\phi = \tan^{-1}\left(\frac{X_{01}}{R_{01}}\right) = \tan^{-1}\left(\frac{4}{1.87}\right) = 64.94°$

(ii)

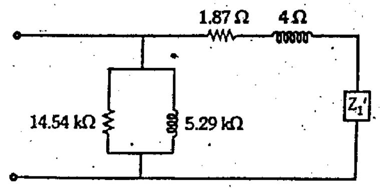


Fig. 4.20

(iii) $W_i = 396 \text{ W} = 0.396 \text{ kW}$

$W_{sc} = 810 \text{ W}$

$I_{sc} = 20.8 \text{ A}$

Since meters are connected on primary side in S.C. test

$I_1 = \frac{50 \times 1000}{2400} = 20.8 \text{ A}$

$W_{Cu} = W_{sc} = 810 \text{ W} = 0.81 \text{ kW}$

Efficiency at full load and 0.8 pf lagging

$x = 1$

$pf = 0.8$

$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} + W_i + x^2 W_{Cu}} \times 100$

$= \frac{1 \times 50 \times 0.8}{1 \times 50 \times 0.8 + 0.396 + (1)^2 \times 0.81} \times 100 = 97.07\ \%$

(iv) **Voltage regulation**

$\cos\phi = 0.8$

$\sin\phi = 0.6$

$\%\ \text{Regulation} = \frac{I_1 (R_{01} \cos\phi + x_{01} \sin\phi)}{V_1} \times 100$

$= \frac{20.8\,(1.87 \times 0.8 + 4 \times 0.6)}{2400} \times 100$

$= 3.38\ \%$

(v) At maximum efficiency,

$\text{Load kVA} = \text{full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$

$= 50 \times \sqrt{\frac{396}{810}}$

$= 34.96 \text{ kVA}$

36. The instrument readings obtained from open circuit test and short circuit test on a 10 kVA, 450/120 V, 50 Hz single phase ac transformer are as follows :

O.C. test (L.V. side) $V_0' = 120$ V, $I_0' = 4.2$ A, $W_0 = 80$ W

S.C. test (H.V. side) $V_{sc} = 9.65$ V, $I_{sc} = 22.2$ A, $W_{sc} = 120$ W

Compute the following : (i) Equivalent circuit constants.

(ii) Draw the equivalent circuit.

(iii) Efficiency and voltage regulation for an 80 % lagging pf load.

(iv) Efficiency at half full load for an 80 % lagging pf load.

(v) The maximum efficiency at 0.8 pf lagging.

From O.C. test (meters are connected on L.V. side i.e. secondary),

$W_0 = 80 \text{ W}$

$V_0 = 120 \text{ V}$

$I_0 = 4.2 \text{ A}$

$$\cos\phi_0' = \frac{W_0}{V_0 I_0} = \frac{80}{120 \times 4.2} = 0.16$$

$$\sin\phi_0' = 0.99$$

$$I_w' = I_0 \cos\phi_0' = 4.2 \times 0.16 = 0.67 \text{ A}$$

$$R_0' = \frac{V_0}{I_w'} = \frac{120}{0.67} = 180.03\ \Omega$$

$$I_\mu' = I_0 \sin\phi_0' = 4.2 \times 0.99 = 4.16 \text{ A}$$

$$X_0' = \frac{V_0}{I_\mu'} = \frac{120}{4.16} = 28.85\ \Omega$$

$$K = \frac{120}{450} = 0.27$$

$$R_0 = \frac{R_0'}{K^2} = \frac{180.03}{(0.27)^2} = 2469.55\ \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{28.85}{(0.27)^2} = 395.75\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. primary),

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{9.65}{22.2} = 0.43\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{120}{(22.2)^2} = 0.24\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(0.43)^2 - (0.24)^2} = 0.36\ \Omega$$

(ii) **Equivalent circuit :**

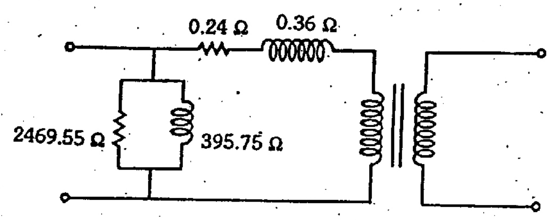


Fig. 4.21

(iii) $W_0 = 80 \text{ W} = 0.08 \text{ kW}$

$W_{sc} = 120 \text{ W}$

$I_{sc} = 22.2 \text{ A}$

Since meters are connected on primary side in S.C. test,

$$I_1 = \frac{10 \times 1000}{450} = 22.22 \text{ A}$$

$$W_{Cu} = W_{sc} = 120 \text{ W} = 0.12 \text{ kW}$$

Efficiency at 80 % lagging pf load :

$x = 1 \qquad pf = 0.8$

$$\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} \times pf + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 10 \times 0.8}{1 \times 10 \times 0.8 + 0.08 + (1)^2 \times 0.12} \times 100$$

$$= 97.56\ \%$$

Voltage regulation

$\cos\phi = 0.8 \qquad \sin\phi = 0.6$

$$\%\ \text{Regulation} = \frac{I_1 (R_{01} \cos\phi + x_{01} \sin\phi)}{V_1} \times 100$$

$$= \frac{22.22\,(0.24 \times 0.8 + 0.36 \times 0.6)}{450} \times 100$$

$$= 2.01\ \%$$

(iv) **Efficiency at half full load for 80 % lagging pf load**

$x = 0.5$

$pf = 0.8$

$$\%\ \eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} + W_0 + x^2 W_{Cu}} \times 100$$

$$= \frac{0.5 \times 10 \times 0.8}{0.5 \times 10 \times 0.8 + 0.08 + (0.5)^2 \times 0.12} \times 100$$

$$= 97.34\ \%$$

(v) For maximum efficiency,

$$\text{Load kVA} = \text{full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 10 \times \sqrt{\frac{80}{120}}$$

$$= 8.16 \text{ kVA}$$

$$\% \eta_{max} = \frac{\ldots}{8.16 \times 0.8 + 0.08 + 0.08} \times 100$$

$$= 97.61\ \%.$$

37. Obtain the equivalent circuit of a 200/400 V, 50 Hz, single phase transformer from the following test :

O.C. test	200 V	0.7 A	70 W	on L.V. side
S.C. test	15 V	10 A	85 W	on H.V. side

Calculate the secondary voltage when delivering 5 kW, 0.8 pf lagging, the primary voltage being 200 V.

From O.C. test (meters are connected on L.V. side i.e. primary),

$W_i = 70$ W

$V_1 = 200$ V

$I_0 = 0.7$ A

$$\cos\phi = \frac{W_i}{V_1 I_0} = \frac{70}{200 \times 0.7} = 0.5$$

$\sin\phi = 0.87$

$I_w = I_0 \cos\phi_0 = 0.7 \times 0.5 = 0.35$ A

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.43\ \Omega$$

$I_\mu = I_0 \sin\phi_0 = 0.7 \times 0.87 = 0.61$ A

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.61} = 327.87\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. secondary),

$W_{sc} = 85$ W

$V_{sc} = 15$ V

$I_{sc} = 10$ A

$$Z_{02} = \frac{V_{sc}}{I_{sc}} = \frac{15}{10} = 1.5\ \Omega$$

$$R_{02} = \frac{W_{sc}}{I_{sc}^2} = \frac{85}{(10)^2} = 0.85\ \Omega$$

$$X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(1.5)^2 - (0.85)^2} = 1.24\ \Omega$$

$$K = \frac{400}{200} = 2$$

$$R_{01} = \frac{R_{02}}{K^2} = \frac{0.85}{(2)^2} = 0.21\ \Omega$$

$$X_{01} = \frac{X_{02}}{K^2} = \frac{1.24}{(2)^2} = 0.31\ \Omega$$

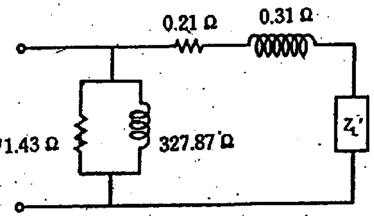


Fig. 4.22

$$I_2 = \frac{5000}{400 \times 0.8} = 15.63\ \text{A}$$

Secondary terminal voltage,

$$V_2 = E_2 - I_2 (R_{02} \cos\phi + X_{02} \sin\phi)$$

$$= 400 - 15.63\,(0.85 \times 0.8 + 1.24 \times 0.6)$$

$$= 377.74\ \text{V}.$$

38. A 40 kVA single phase transformer with voltages 11 kV/440 V has the following test results :

O.C. test (Instruments on L.V. side) 440 V, 1.1 A, 145 W

S.C. test (Instruments on H.V. side) 100 V, 3 A, 90 W

Deduce an approximate circuit referred to L.V. side. For this transformer calculate maximum efficiency.

From O.C. test. (meters are connected on L.V. side i.e. secondary),

$W_0 = 145$ W

$V_0 = 440$ V

$$\cos\phi_0' = \frac{W_0}{V_0 I_0} = \frac{145}{440 \times 1.1} = 0.2996 = 0.3$$

$\sin\phi_0' = 0.95$

$I_w' = I_0 \cos\phi_0' = 1.1 \times 0.3 = 0.33$ A

$$R_0' = \frac{V_0}{I_w'} = \frac{440}{0.33} = 1333.33\ \Omega$$

$I_\mu' = I_0 \sin\phi_0' = 1.1 \times 0.95 = 1.05$ A

$$X_0' = \frac{V_0}{I_\mu'} = \frac{440}{1.05} = 419.05\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. primary)

$W_{sc} = 90$ W

$V_{sc} = 100$ V

$I_{sc} = 3$ A

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{100}{3} = 33.33\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{90}{(3)^2} = 10\ \Omega$$

$$X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(33.33)^2 - (10)^2} = 31.79\ \Omega$$

$$K = \frac{440}{11 \times 10^3} = 0.04$$

$$R_{02} = K^2 R_{01} = (0.04)^2 \times 10 = 0.016\ \Omega$$

$$X_{02} = K^2 X_{01} = (0.04)^2 \times 31.79 = 0.05\ \Omega$$

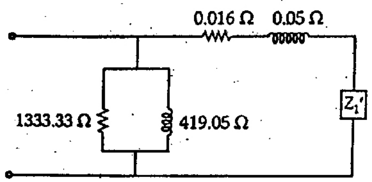


Fig. 4.23

$W_i = 145$ W

$W_{sc} = 90$ W

$I_{sc} = 3$ A

Since meters are connected on primary in S.C. test,

$$I_1 = \frac{40 \times 1000}{11 \times 1000} = 3.64\text{ A}$$

$$W_{Cu} = \left(\frac{I_1}{I_{sc}}\right)^2 W_{sc} = \left(\frac{3.64}{3}\right)^2 \times 90 = 132.5\text{ W}$$

For maximum efficiency,

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 40 \times \sqrt{\frac{145}{132.5}}$$

$$= 41.84\text{ kVA}$$

$$\%\,\eta_{max} = \frac{\text{Load kVA} \times \text{pf}}{\text{Load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{41.84 \times 1}{41.84 \times 1 + 0.145 + 0.145} \times 100$$

$$= 99.31\ \%.$$

39. The results of O.C. and S.C. test on a 25 kVA, 440/220V, 50 Hz transformer are as follows :

O.C. test	220 V	9.6 A	710 W	L.V. side
S.C. test	42 V	57 A	1030 W	H.V. side

Obtain the parameters of exact equivalent circuit referred to high voltage side.

From O.C. test (meters are connected on L.V. side i.e. secondary),

$W_i = 710$ W

$V_2 = 220$ V

$I_0' = 9.6$ A

$$\cos\phi_0' = \frac{W_i}{V_2 I_0'} = \frac{710}{220 \times 9.6} = 0.34$$

$$\sin\phi_0' = 0.94$$

$$I_w' = I_0' \cos\phi_0' = 9.6 \times 0.34 = 3.26\text{ A}$$

$$R_0' = \frac{V_2}{I_w'} = \frac{220}{3.26} = 67.48\ \Omega$$

$$I_\mu' = I_0' \sin\phi_0' = 9.6 \times 0.94 = 9.02\text{ A}$$

$$X_0' = \frac{V_2}{I_\mu'} = \frac{220}{9.02} = 24.39\ \Omega$$

$$K = \frac{220}{440} = 0.5$$

$$R_0 = \frac{R_0'}{K^2} = \frac{67.48}{(0.5)^2} = 269.92\ \Omega$$

$$X_0 = \frac{X_0'}{K^2} = \frac{24.39}{(0.5)^2} = 97.56\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. primary),

$W_{sc} = 1030$ W

$V_{sc} = 42$ V

$I_{sc} = 57$ A

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{42}{57} = 0.74\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{1030}{(57)^2} = 0.32\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(0.74)^2 - (0.32)^2} = 0.67\ \Omega.$$

40. The two windings of a 2400/240 V, 48 kVA, 50 Hz transformer have resistances of 0.6 and 0.025 Ω for the high and low voltage winding respectively. The transformer requires that 238 V be impressed on the high voltage coil in order that rated current be circulated in the short circuit low voltage winding.

(i) Calculate the equivalent leakage reactance referred to high voltage side.

(ii) How much power is needed to circulate valid current on short circuit?

(iii) Compute the efficiency at full load when the pf is 0.8 lagging. Assume that core loss equals the copper loss.

(i) $R_1 = 0.6\ \Omega$

$R_2 = 0.025\ \Omega$

$K = \frac{240}{2400} = 0.1$

Since rated current flows in short circuited low voltage winding i.e. primary winding,

$I_1 = \frac{48000}{2400} = 20\ A$

$I_{sc} = I_1 = 20\ A$

$V_{sc} = 238\ V$

$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{238}{20} = 11.9\ \Omega$

$R_{01} = R_1 + \frac{R_2}{K^2} = 0.6 + \frac{0.025}{(0.1)^2} = 3.1\ \Omega$

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(11.9)^2 - (3.1)^2} = 11.49\ \Omega$

(ii) $W_{sc} = I_{sc}^2 R_{01} = (20)^2 \times 3.1 = 1240\ W$

(iii) $$\%\eta = \frac{x \times \text{full load kVA} \times pf}{x \times \text{full load kVA} + W_i + x^2 W_{Cu}} \times 100$$

$$= \frac{1 \times 48 \times 0.8}{1 \times 48 \times 0.8 + 1.24 + (1)^2 \times 1.24} \times 100$$

$= 93.93\ \%.$

41. A single phase transformer, 50 kVA, 2400/240 V, 60 Hz is subjected to O.C. and S.C. test and the results obtained are :

O.C. test : (i) Secondary excitation voltage 240 V (ii) Current reading 5 A (iii) Power measured to be 186 W.

S.C. test : (i) Excitation voltage 48 V with secondary short circuited. (ii) HT current reading 20.8 A (iii) Power measured to be 617 W.

(a) Draw circuit diagram for each test with appropriate instruments connected indicating their ranges.

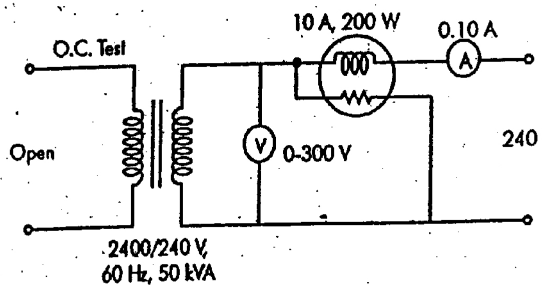
(b) Draw phasor diagram for each test identifying each phasor and identifying the angle involved.

(c) Draw impedance triangle for each test identifying each side and impedance angle and their magnitudes.

(d) State the various assumptions with proper justification while, drawing diagrams for part (b) and (c).

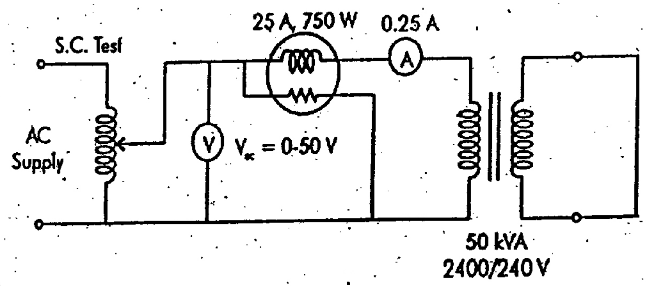
(e) Maximum efficiency.

(a) O.C. test :



(a)

S.C. test :



(b)

Fig. 4.24

(b) From O.C. test (meters are connected on secondary side),

$W_i = 186\ W$

$V_2 = 240\ V$

$I_0' = 5\text{ A}$

$\cos\phi_0' = \frac{W_i}{V_2 I_0'} = \frac{186}{240 \times 5} = 0.155$

$\phi_0' = 81.08°$

$\sin\phi_0' = 0.99$

$I_w' = I_0' \cos\phi_0' = 5 \times 0.155 = 0.775\text{ A}$

$R_0' = \frac{V_2}{I_w'} = \frac{240}{0.775} = 309.68\ \Omega$

$I_\mu' = I_0' \sin\phi_0' = 5 \times 0.99 = 4.95\text{ A}$

$X_0' = \frac{V_2}{I_\mu'} = \frac{240}{4.95} = 48.48\ \Omega$

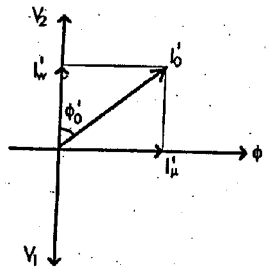


Fig. 4.25

From S.C. test (meters are connected on primary side),

$W_{sc} = 617\text{ W}$

$V_{sc} = 48\text{ V}$

$I_{sc} = 20.8\text{ A}$

$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{48}{20.8} = 2.31\ \Omega$

$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{617}{(20.8)^2} = 1.43\ \Omega$

$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} = \sqrt{(2.31)^2 - (1.43)^2} = 1.81\ \Omega$

$\cos\phi_{sc} = \frac{W_{sc}}{V_{sc} I_{sc}} = \frac{617}{48 \times 20.8} = 0.62$

$\phi_{sc} = 51.68°$

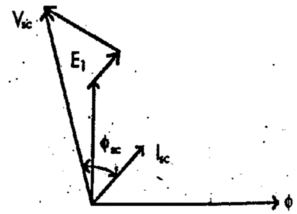


Fig. 4.26

(c) Impedance triangle :

O.C. test : No triangle

S.C. test

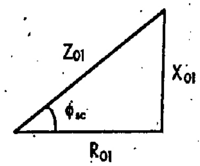


Fig. 4.27.

(d) Assumptions :

1. In O.C. test copper losses are very small and hence neglected.
2. In S.C. test iron losses are very small and hence neglected.

(e) For maximum efficiency,

$W_i = 186\text{ W}$

$= 0.186\text{ kW}$

$W_{sc} = 617\text{ W}$

$I_{sc} = 20.8\text{ A}$

Since meters are connected on primary side in S.C. test,

$\therefore\ I_1 = \frac{50000}{2400} = 20.83\text{ A}$

$W_{Cu} = W_{sc} = 617\text{ W} = 0.617\text{ kW}$

For maximum efficiency,

$\text{Load kVA} = \text{full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$

$= 50 \times \sqrt{\frac{186}{617}}$

$= 27.45\text{ kVA}$

$$\% \eta_{max} = \frac{\text{load kVA} \times \text{pf}}{\text{load kVA} \times \text{pf} + W_i + W_i} \times 100$$

$$= \frac{27.45 \times 1}{27.45 \times 1 + 0.186 + 0.186} \times 100$$

$$= 98.65\ \%.$$

42. A 1000/200 V, 50 Hz, single phase transformer give the following test results :

No load (H.V. side)	1000 V	0.24 A	90 W
S.C. (H.V. side)	50 V	5 A	110 W.

The regulation of the transformer at full load pf 0.8 lagging is 4.46 %. Calculate (i) Rating of the transformer. (ii) Equivalent circuit diagram of transformer with circuit connections. (iii) kVA load for maximum efficiency. (iv) Voltage to be applied on H.V. side on full load at unity pf when the secondary terminal voltage is 200 V.

(i) From no load test (meters are connected on H.V. side i.e. primary),

$W_i = 90$ W

$V_1 = 1000$ V

$I_0 = 0.24$ A

$$\cos \phi_0 = \frac{W_i}{V_1 I_0} = \frac{90}{1000 \times 24} = 0.375$$

$\sin \phi_0 = 0.93$

$I_w = I_0 \cos \phi_0 = 0.24 \times 0.375 = 0.09$ A

$$R_0 = \frac{V_1}{I_w} = \frac{1000}{0.09} = 11111.11\ \Omega$$

$I_\mu = I_0 \sin \phi_0 = 0.24 \times 0.93 = 0.22$ A

$$X_0 = \frac{V_1}{I_\mu} = \frac{1000}{0.22} = 4545.45\ \Omega$$

From S.C. test (meters are connected on H.V. side i.e. primary),

$W_{sc} = 110$ W

$V_{sc} = 50$ V

$I_{sc} = 5$ A

$$Z_{01} = \frac{V_{sc}}{I_{sc}} = \frac{50}{5} = 10\ \Omega$$

$$R_{01} = \frac{W_{sc}}{I_{sc}^2} = \frac{110}{(5)^2} = 4.4\ \Omega$$

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2} - \sqrt{(10)^2 - (4.4)^2} = 8.98\ \Omega$$

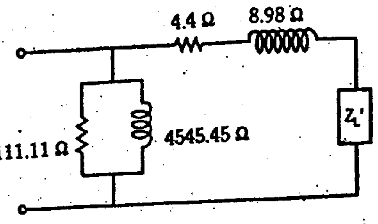


Fig. 4.28

(ii) $K = \frac{200}{1000} = 0.2$

$R_{02} = K^2 R_{01} = (0.2)^2 \times 4.4 = 0.176\ \Omega$

$X_{02} = K^2 X_{01} = (0.2)^2 \times 8.98 = 0.36\ \Omega$

$$\%\ \text{Regulation} = \frac{V_{NL} - V_{FL}}{V_{NL}} \times 100$$

$$4.46 = \frac{200 - V_{FL}}{200} \times 100$$

$V_{FL} = 191.08$ V

$V_2 = 191.08$ V

But $E_2 - V_2 = I_2 (R_{02} \cos \phi + X_{02} \sin \phi)$

$200 - 191.08 = I_2 (0.176 \times 0.8 + 0.36 \times 0.6)$

$I_2 = 25$ A

kVA rating of transformer $= 200 \times 25 = 5000 = 5$ kVA

(iii) $W_i = 90$ W

$W_{sc} = 110$ W

$I_{sc} = 5$ A

Since meters are connected on primary side in S.C. test,

$$I_1 = \frac{5000}{1000} = 5\ \text{A}$$

$W_{Cu} = W_{sc} = 110$ W

For maximum efficiency,

$$\text{Load kVA} = \text{Full load kVA} \times \sqrt{\frac{W_i}{W_{Cu}}}$$

$$= 5 \times \sqrt{\frac{90}{110}}$$

$$= 4.52\ \text{kVA}$$

(iv) Voltage to be applied on primary on full load at unity pf :

$\cos\phi = 1$

$\sin\phi = 0$

$V_1 - E_1 = I_1 (R_{01} \cos\phi + X_{01} \sin\phi)$

$V_1 - 1000 = 5 (4.4 \times 1 + 0)$

$V_1 = 1022$ V.

43. The approximate equivalent circuit of a 4 kVA, 200/400 V, single phase, 50 Hz transformer referred to low voltage side is shown in Fig. 4.29.

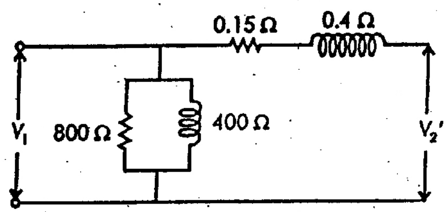


Fig. 4.29

(i) An open circuit is carried out by applying 200 volts to the L.V. side, keeping H.V. side open. Calculate no load current, no load power factor and iron losses.

(ii) A short circuit test is conducted by passing a full load current from H.V. side keeping L.V. side shorted. Calculate voltage required to be applied to the transformer, power factor under short circuit test and copper losses at 50% of full load.

From O.C. test (meters are connected on L.V. side i.e. primary),

$V_1 = 200$ V

$R_0 = 800\ \Omega$

$X_0 = 400\ \Omega$

$I_w = \frac{V_1}{R_0} = \frac{200}{800} = 0.25$ A

$I_\mu = \frac{V_1}{X_0} = \frac{200}{400} = 0.5$ A

No load current $I_0 = \sqrt{I_w^2 + I_\mu^2} = \sqrt{(0.25)^2 + (0.5)^2} = 0.56$ A

No load pf, $\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.25}{0.56} = 0.45$ lagging

Iron losses $W_i = V_1 I_0 \cos\phi_0 = 200 \times 0.56 \times 0.45 = 50.4$ W

From S.C. test (meters are connected on H.V. side i.e. secondary),

$I_2 = \frac{4 \times 1000}{400} = 10$ A

$I_{sc} = I_2 = 10$ A

$R_{01} = 0.15\ \Omega$

$X_{01} = 0.4\ \Omega$

$K = \frac{400}{200} = 2$

$R_{02} = K^2 R_{01} = (2)^2 \times 0.15 = 0.6\ \Omega$

$X_{02} = K^2 X_{01} = (2)^2 \times 0.4 = 1.6\ \Omega$

$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{(0.6)^2 + (1.6)^2} = 1.71\ \Omega$

$V_{sc} = Z_{02} I_{sc} = 1.71 \times 10 = 17.1$ V

$\cos\phi_{sc} = \frac{R_{02}}{Z_{02}} = \frac{0.6}{1.71} = 0.35$ (lagging)

$W_{sc} = I_{sc}^2 R_{02} = (10)^2 \times 0.6 = 60$ W

Copper loss at 50 % of full load $= (0.5)^2 W_{sc}$

$= (0.5)^2 \times 60$

$= 15$ W.

4.15 EFFICIENCY AND VOLTAGE REGULATION BY DIRECT LOADING METHOD

Efficiency and voltage regulation of the transformer at any desired load and power factor can be found by direct loading method. Fig. 4.30 shows the circuit diagram for conducting direct load test on the transformer. In this test the primary of the transformer is connected to an ac supply of rated voltage. Ammeters, voltmeters and wattmeters are connected on both the sides of the transformer. A variable load is connected across the secondary. By varying the load from no load to full load, readings are taken on the meters.

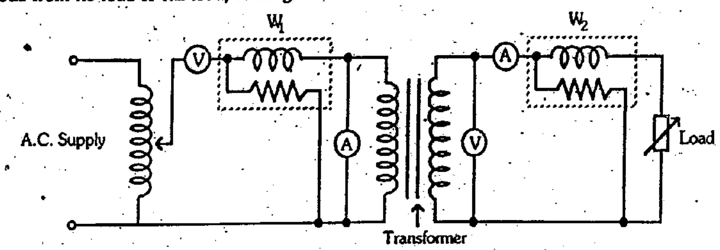


Fig. 4.30 Direct load test on single phase-transformer

Calculations :

The wattmeters on the primary and secondary sides respectively indicate the input power (W_1) and output power (W_2).

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{W_2}{W_1} \times 100$$

If the load is purely resistive (with unity power factor) like lamp-bank, then the wattmeter on the secondary side can be eliminated. This is because with unity power factor loads, output power is also given by the product $V_2 I_2$.

$$\text{Efficiency} = \frac{V_2 I_2}{W_1} \times 100$$

Under no load condition the voltmeter reading on secondary side gives no load secondary terminal voltage E_2. Then for each reading, regulation is given by,

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

where V_2 is secondary terminal voltage on load.

44. A load test is conducted on 1 kVA, 230/115 V, 50 Hz, single-phase transformer using lamp load. Following are the results obtained at a particular load condition.

Primary side			Secondary side	
V_1	I_1	W_1	V_2	I_2
230 V	4.2 A	846 W	111.5 V	7.5 A

If the no load secondary terminal voltage is observed to be 114 V, find the efficiency and regulation of the transformer at the given load condition.

For lamp load having unity power factor,

$$\text{Output power} = V_2 I_2$$

$$= 111.5 \times 7.5$$

$$= 836.25 \text{ W}$$

$$\text{Input power} = 846 \text{ W}$$

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{836.25}{846.0} \times 100$$

$$= 98.85 \%$$

No load secondary terminal voltage $E_2 = 114$ V

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100$$

$$= \frac{114 - 111.5}{114} \times 100$$

$$= 2.19 \%$$

EXERCISE

1. A single phase transformer has 350 primary and 1050 secondary turns. The net cross sectional area of the core is 55 cm². If the primary winding be connected to a 400 V, 50 Hz single phase supply, calculate (i) maximum value of flux density in the core (ii) the voltage induced in the secondary winding.

[0.93 Wb/m², 1200 V]

2. The required no load ratio in a single phase, 50 Hz, core type transformer is 6000/250 V. Find the number of turns in each winding if the flux is to be about 0.06 Wb.

[450, 20]

3. A 25 kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000 V, 50 Hz supply. Find the full load primary and secondary currents, the secondary emf and the maximum flux in the core.

[8.33 A, 83.3 A, 300 V, 27 mWb]

4. A 40 kVA, 3300/240 V, 50 Hz, 1 phase transformer has 660 turns on the primary. Determine :

   (i) The number of turns on the secondary.

   (ii) The maximum value of flux in the core.

   (iii) The approximate value of primary and secondary full load currents.

   Internal drops in the windings are to be ignored.

[48, 0.02 Wb, 12.12 A, 166.67 A]

5. A 2200/250 V, transformer takes 0.5 A at a pf of 0.3 on no load. Find magnetizing and working components of no load primary current.

[0.476 A, 0.15 A]

6. The no load current of a transformer is 4 A at 0.25 pf when supplied at 250 V, 50 Hz. The number of turns on the primary winding is 200. Calculate (i) rms value of flux in the core (ii) core loss (iii) magnetizing current.

[5.63 mWb, 250 W, 3.87 A]

7. The values of the resistances of the primary and secondary windings of a 2200/200 V, 50 Hz, single phase transformer are 2.4 and 0.02 Ω respectively. Find (i) equivalent resistance of primary referred to secondary (ii) equivalent resistance of secondary