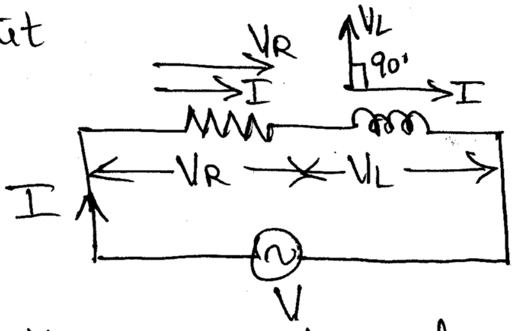
SERIES A.C. CIRCUITS

TYPE I :- A.C. THROUGH RESISTANCE AND INDUCTANCE (R-L CIRCUITS):-

Consider a resistance (R) connected in series with an inductor (L) across a.c. supply as shown in following circuit



If V = r.m.s. value of applied voltage (In volts)

I = r.m.s. value of supply current (In Amperes)

R = resistance of circuit (In Ohms)

L = inductance of circuit (In Henry)

$X_L = 2\pi f L = \omega L$ (In Ohms)

= inductive reactance of circuit

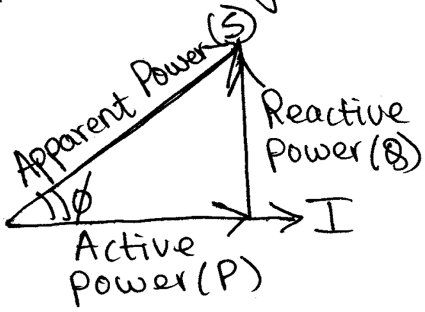
$\omega = 2\pi f$ = Supply frequency (In rad/sec)

f = supply frequency (In Hertz)

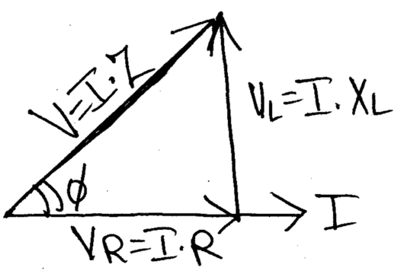
$V_R = I \cdot R$ = Voltage across resistor (In volts)

$V_L = I \cdot X_L$ = Voltage across inductor (In volts)

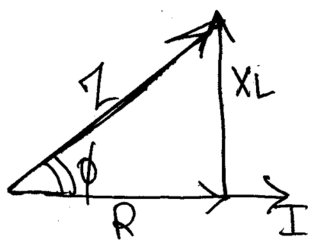
Power Triangle is



Voltage Triangle is



Impedance triangle is



$V = IZ$ = voltage of supply (In volts)

From voltage triangle

$$V = \sqrt{(V_R)^2 + (V_L)^2}$$

$$\therefore IZ = \sqrt{(IR)^2 + (IX_L)^2}$$

$$\therefore IZ = I\sqrt{R^2 + X_L^2}$$

$$\therefore Z = \sqrt{R^2 + X_L^2}$$

where Z = impedance of circuit (In Ohms)

Power factor (p.f.) = $\cos\phi = \frac{R}{Z}$ (Lagging)

Quality factor (Q-factor) = $\frac{1}{p.f.} = \frac{1}{\cos\phi}$

Active power (P or W) = $V.I.\cos\phi$
$= I^2R$

(Units is Watts)

Reactive power (Q) = $V.I.\sin\phi$
$= I^2X_L$

(Units is VAR - volts ampere reactive)

Apparent power (S) = $V.I$
$= I^2Z$

(Units is VA - volts ampere)

Ans 1) $i(t) = 5\sin(314t + 2\pi/3)$

Comparing with

$i(t) = I_m \sin(\omega t + \theta_1)$

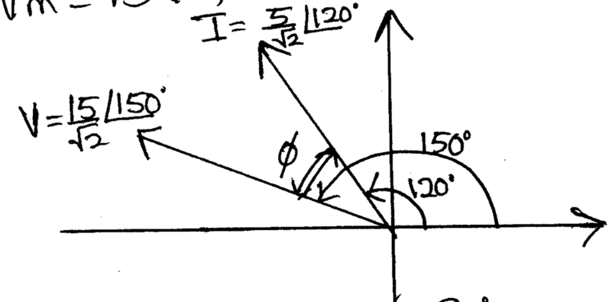
$\therefore I_m = 5A$, $\omega = 314$ rad/sec, $\theta_1 = \frac{2\pi}{3} = 120°$

$V(t) = 15\sin(314t + 5\pi/6)$

Comparing with

$v(t) = V_m \sin(\omega t + \theta_2)$

$\therefore V_m = 15V$, $\omega = 314$ rad/sec, $\theta_2 = \frac{5\pi}{6} = 150°$



$\therefore \phi = \theta_2 - \theta_1 = 150 - 120 = 30°$

$I = \frac{I_m}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54A$

$V = \frac{V_m}{\sqrt{2}} = \frac{15}{\sqrt{2}} = 10.61V$

(a) $Z = \frac{V}{I} = \frac{10.61}{3.54} = 3\Omega$

(b) $\because \cos\phi = \frac{R}{Z}$

$\therefore \cos 30° = \frac{R}{3}$ $\therefore R = 2.6\Omega$

(c) $Z = \sqrt{R^2 + X_L^2}$

$\therefore 3 = \sqrt{(2.6)^2 + X_L^2}$

$\therefore X_L = 1.5\Omega$

$\because X_L = 2\pi f L = \omega L$

$\therefore 1.5 = 314 \times L$

$\therefore L = 4.78 \times 10^{-3} H$

(d) Average power = Active power

$= V \cdot I \cos\phi$

$= 10.61 \times 3.54 \times \cos 30^\circ$

$= 32.53 W$

(e) $p.f. = \cos\phi = \cos 30^\circ$

$= 0.866$ (lag)

Ans 2) $R = 3.5 \Omega$

$L = 0.1 H$

$f = 50 Hz$

$V = 220 \angle 30^\circ$

(i) $X_L = 2\pi f L$

$= 2\pi \times 50 \times 0.1$

$= 31.42 \Omega$

$Z = \sqrt{R^2 + X_L^2}$

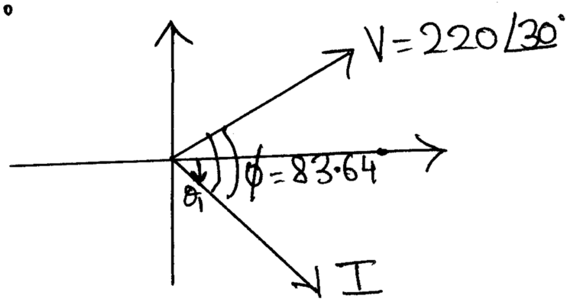
$= \sqrt{(3.5)^2 + (31.42)^2}$

$Z = 31.61 \Omega$

$I = \frac{V}{Z} = \frac{220}{31.61} = 6.96 A$

$\cos\phi = \frac{R}{Z} = \frac{3.5}{31.61} = 0.111$ (lag)

$\therefore \phi = 83.64^\circ$



$\theta_1 = 83.64 - 30$

$= 53.64^\circ$

$\therefore I = 6.96 \angle -53.64^\circ$

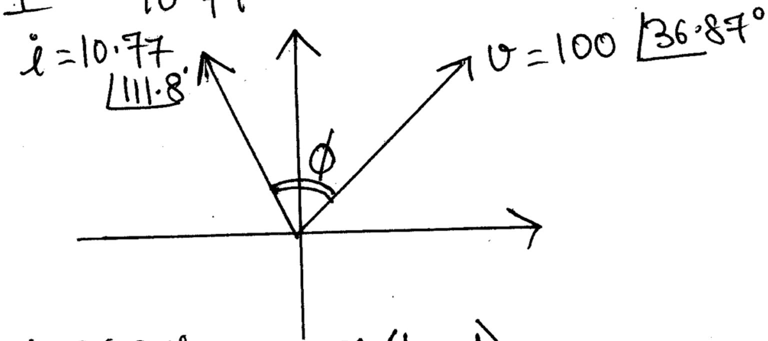
(ii) $p.f. = \cos\phi = \cos(83.64)$

$= 0.111$ (lag)

Q3) $v = 80 + j60 = 100 \angle 36.87^\circ$

$i = -4 + j10 = 10.77 \angle 111.8^\circ$

(i) $Z = \frac{V}{I} = \frac{100}{10.77} = 9.29\,\Omega$



$\phi = 111.8^\circ - 36.87^\circ = 74.93^\circ$ (lead)

$Z = \frac{v}{i} = \frac{80+j60}{-4+j10} = 2.41 - j8.97 = 9.29 \angle -74.93$

(ii) Comparing with $Z = |Z| \angle \phi$

$\therefore \phi = 74.93$ (lead)

$P = VI\cos\phi$

$= 100 \times 10.77 \cos(74.93^\circ)$

$= 280.02\,W$

(iii) Phase angle $= \phi$

$= 74.93$ (lead)

Q4) $V = 100\,V$

$I = 10\,A$

$\phi = 30^\circ$ (lead)

(i) p.f. is leading

(ii) p.f. $= \cos\phi = \cos 30^\circ = 0.866$ (lead)

(iii) Circuit is capacitive

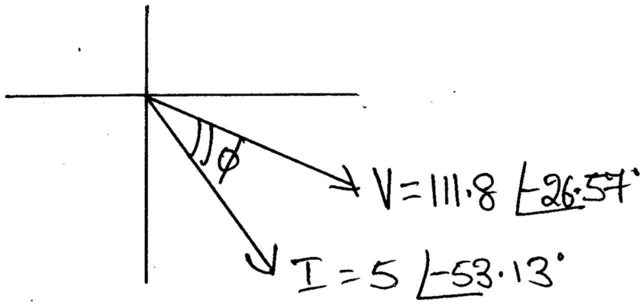
(iv) Active power $= VI\cos\phi$

$= 100 \times 10 \times \cos 30^\circ = 866.03\,W$

Reactive power $= VI\sin\phi$

$= 100 \times 10 \times \sin 30^\circ = 500\,VAR$

Q5) $v = 100 - j50 = 111.8 \angle -26.57^\circ$
$i = 3 - j4 = 5 \angle -53.13^\circ$

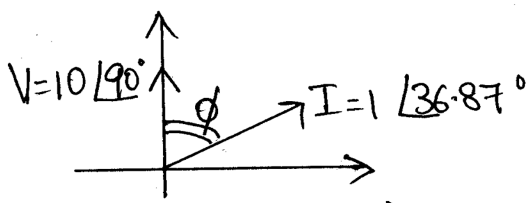


$\phi = 53.13 - 26.57 = 26.56^\circ$ (lag)

Real power = Active power
$= V \cdot I \cos\phi$
$= 111.8 \times 5 \times \cos(26.56^\circ)$
$= 500\,W$

Reactive power $= V \cdot I \sin\phi$
$= 111.8 \times 5 \times \sin(26.56^\circ)$
$= 249.95$ VAR

Q6) $v = 0 + j10 = 10 \angle 90^\circ$
$i = 0.8 + j0.6 = 1 \angle 36.87^\circ$



$\phi = 90 - 36.87 = 53.13^\circ$ (lag)

Since current is lagging the voltage ∴ pf is lagging
∴ Circuit is inductive

$Z = \dfrac{v}{i} = \dfrac{0 + j10}{0.8 + j0.6}$

$= 6 + j8$

Comparing with $Z = R + jX_L$
∴ $R = 6\,\Omega$ and $X_L = 8\,\Omega$

Q7) $e = v = 200\sqrt{2}\sin(\omega t + 20^\circ)$

Comparing with

$v = V_m \sin(\omega t + \theta_1)$

$\therefore V_m = 200\sqrt{2}\ V$

$\theta_1 = 20^\circ$.

Comparing with

$i = 10\sqrt{2}\cos(314t - 25^\circ) = 10\sqrt{2}\sin(314t - 25 + 90)$

$\therefore i = 10\sqrt{2}\sin(314t + 65^\circ)$ $\{\because \sin(90+\theta) = \cos\theta\}$

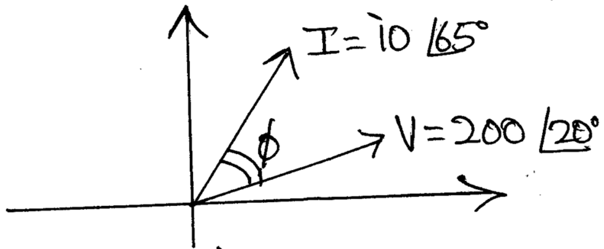
$i = I_m \sin(\omega t + \theta_2)$

$\therefore I_m = 10\sqrt{2}\ A$

$\theta_2 = 65^\circ$

$\omega = 314\ rad/s.$

$V = \frac{V_m}{\sqrt{2}} = \frac{200\sqrt{2}}{\sqrt{2}} = 200V$ and $I = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10A$



$\phi = 65 - 20 = 45^\circ$ (lead)

$Z = \frac{V}{I} = \frac{200}{10} = 20\Omega$

$\cos\phi = \frac{R}{Z}$

$\therefore \cos 45^\circ = \frac{R}{20}$

$\therefore R = 14.14\Omega$

$Z = \sqrt{R^2 + X_C^2}$

$\therefore 20 = \sqrt{(14.14)^2 + X_C^2}$

$\therefore X_C = 14.14\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$ $\therefore 14.14 = \frac{1}{314 \times C}$ $\therefore C = 2.25 \times 10^{-4} F$

Q8) $I = 10A$

Active power $= 1kW = 1000W$

$V = 200V$

$f = 50Hz$

Active power $= VI\cos\phi$

$\therefore 1000 = 200 \times 10 \times \cos\phi$

$\therefore \phi = 60^\circ$ (lag)

$Z = \frac{V}{I} = \frac{200}{10} = 20\Omega$

$\cos\phi = \frac{R}{Z}$

$\therefore \cos 60^\circ = \frac{R}{20}$ $\quad \therefore R = 10\Omega$

$Z = \sqrt{R^2 + X_L^2}$

$\therefore 20 = \sqrt{(10)^2 + X_L^2}$

$\therefore X_L = 17.32\ \Omega$

(i) $Z = R + jX_L$

$= (10 + j17.32)\Omega$

(ii) $Z = 20\angle 60^\circ\ \Omega$

(iii) p.f. $= \cos\phi$

$= \cos 60^\circ$

$= 0.5$ (lag)

(iv) Reactive power $= VI\sin\phi$

$= 200 \times 10 \times \sin 60^\circ$

$= 1{,}732.05$ VAR

(v) Apparent power $= V.I$

$= 200 \times 10$

$= 2000$ VA

89) **Coil A**

$V_1 = 100V$
$f_1 = 50Hz$
$I_1 = 8A$
$P_1 = 120W$

$Z_1 = \frac{V_1}{I_1} = \frac{100}{8} = 12.5\Omega$

$P_1 = V_1 I_1 \cos\phi_1$

$\therefore 120 = 100 \times 8 \times \cos\phi_1$

$\therefore \phi_1 = 81.37$ (lag)

$\cos\phi_1 = \frac{R_1}{Z_1}$

$\therefore \cos 81.37 = \frac{R_1}{12.5}$

$\therefore R_1 = 1.88\Omega$

$Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$

$\therefore 12.5 = \sqrt{(1.88)^2 + X_{L_1}^2}$

$\therefore X_{L_1} = 12.36\ \Omega$

Coil B

$V_2 = 100V$
$f_2 = 50Hz$
$I_2 = 10A$
$P_2 = 500W$

$Z_2 = \frac{V_2}{I_2} = \frac{100}{10} = 10\Omega$

$P_2 = V_2 I_2 \cos\phi_2$

$\therefore 500 = 100 \times 10 \times \cos\phi_2$

$\therefore \phi_2 = 60°$ (lag)

$\cos\phi_2 = \frac{R_2}{Z_2}$

$\therefore \cos 60° = \frac{R_2}{10}$

$\therefore R_2 = 5\Omega$

$Z_2 = \sqrt{R_2^2 + X_{L_2}^2}$

$\therefore 10 = \sqrt{(5)^2 + X_{L_2}^2}$

$\therefore X_{L_2} = 8.66\Omega$

When coil A and coil B are connected in series

Total $R = R_1 + R_2$
$= 1.88 + 5 = 5.88\Omega$

Total $X_L = X_{L_1} + X_{L_2}$
$= 12.36 + 8.66 = 21.02\ \Omega$

Total $Z = \sqrt{R^2 + X_L^2} = \sqrt{(5.88)^2 + (21.02)^2} = 21.83\Omega$

$V = 100V$

$I = \frac{V}{Z} = \frac{100}{21.83} = 4.58A$

$\cos\phi = \frac{R}{Z} = \frac{5.88}{21.83} = 0.27$ (lag)

Power $= VI\cos\phi = 100 \times 4.58 \times 0.27 = 123.36W$

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Q10) $I_{dc} = 6A$

$V_{dc} = 24V$

$V_{dc} = I_{dc} \times R$

$\therefore 24 = 6 \times R$

$\therefore R = 4\Omega$

For a.c. supply,

$I = 6A$

$V = 30V$

$f = 50Hz$.

$Z = \frac{V}{I} = \frac{30}{6} = 5\Omega$

$pf = \cos\phi = \frac{R}{Z} = \frac{4}{5} = 0.8\,(\text{lag})$

$Z = \sqrt{R^2 + X_L^2}$

$\therefore 5 = \sqrt{(4)^2 + X_L^2}$

$\therefore X_L = 3\Omega$

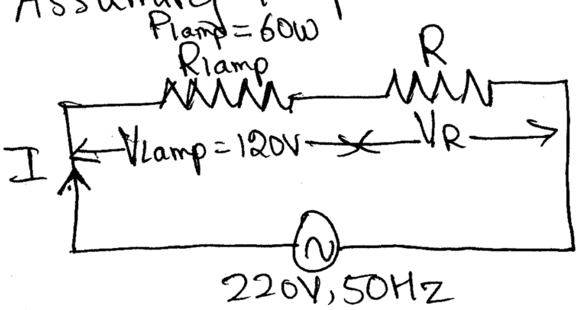
$X_L = 2\pi f L$

$\therefore 3 = 2\pi \times 50 \times L$

$\therefore L = 9.55 \times 10^{-3} H$

Q11) Assuming lamp to be resistive

(i) When resistor is connected in series



$P_{lamp} = V_{lamp} \times I$

$\therefore 60 = 120 \times I$

$\therefore I = 0.5A$

$V_{lamp} = I \times R_{lamp}$

$\therefore 120 = 0.5 \times R_{lamp}$

$\therefore R_{lamp} = 240\Omega$

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$$V = I(R_{lamp} + R)$$

$$\therefore 220 = 0.5(240 + R)$$

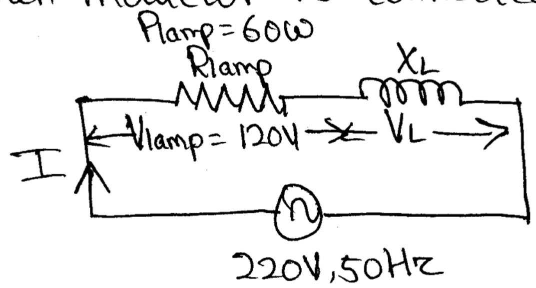
$$\therefore R = 200\Omega$$

$$\text{Total power} = I^2(R_{lamp} + R)$$

$$= 0.5^2(240 + 200)$$

$$= 110W$$

(ii) When inductor is connected in series



$$P_{lamp} = V_{lamp} \times I$$

$$\therefore 60 = 120 \times I$$

$$\therefore I = 0.5A$$

$$V_{lamp} = I \times R_{lamp}$$

$$\therefore 120 = 0.5 \times R_{lamp}$$

$$\therefore R_{lamp} = 240\Omega$$

$$V = IZ$$

$$\therefore 220 = 0.5 \times Z$$

$$\therefore Z = 440\Omega$$

$$Z = \sqrt{R_{lamp}^2 + X_L^2}$$

$$\therefore 440 = \sqrt{(240)^2 + X_L^2}$$

$$\therefore X_L = 368.78$$

$$X_L = 2\pi f L$$

$$\therefore 368.78 = 2\pi \times 50 \times L$$

$$\therefore L = 1.17H$$

$$\cos\phi = \frac{R_{lamp}}{Z} = \frac{240}{440} = \cancel{1.09(lag)}\ 0.546(lag)$$

$$\text{Total power} = VI\cos\phi = 220 \times 0.5 \times 0.546 = 60.06W$$

Method (ii) is preferred since power consumed is less.

Q12) $I = 8A$

$V_R = 100V$

$V_R = I.R$

$\therefore 100 = 8 \times R$

$\therefore R = 12.5\Omega$

$V = 220V$

$I = 8A$

$Z = \frac{V}{I} = \frac{220}{8} = 27.5\Omega$

$Z = \sqrt{R^2 + X_L^2}$

$\therefore 27.5 = \sqrt{(12.5)^2 + X_L^2}$

$\therefore X_L = 24.5\Omega$

$X_L = 2\pi f L$

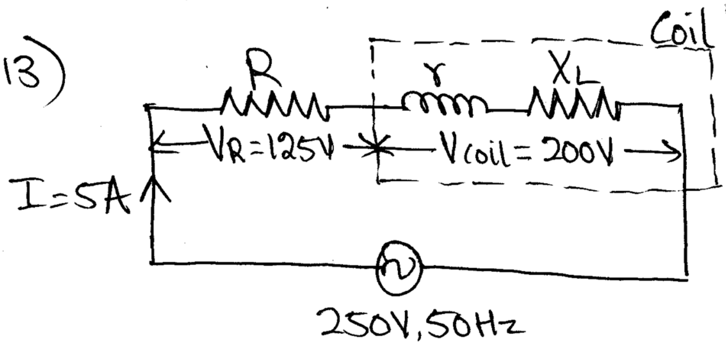
$24.5 = 2\pi \times 50 \times L$

$\therefore L = 0.078H$

$\cos\phi = \frac{R}{Z} = \frac{12.5}{27.5}$

$\therefore \phi = 62.96^\circ$ (lag)

Q13)



$V_R = I.R$

$\therefore 125 = 5 \times R$

$\therefore R = 25\Omega$

$V_{coil} = I.Z_{coil}$

$\therefore 200 = 5 \times Z_{coil}$

$\therefore Z_{coil} = 40\Omega$

$V = I.Z$

$\therefore 250 = 5 \times Z$

$\therefore Z = 50\Omega$

$Z_{coil} = \sqrt{r^2 + X_L^2}$

$\therefore 40 = \sqrt{r^2 + X_L^2}$

$\therefore r^2 + X_L^2 = 1600$ ——— (I)

$Z = \sqrt{(R+r)^2 + X_L^2}$

$\therefore 50 = \sqrt{(25+r)^2 + X_L^2}$

$\therefore 2500 = 625 + 50r + r^2 + X_L^2$

Substituting from eqn (I)

$\therefore 2500 = 625 + 50r + 1600$

$\therefore r = 5.5\Omega$

Substituting in eqn (I)

$(5.5)^2 + X_L^2 = 1600$

$\therefore X_L = 39.62\Omega$

(ii) $\cos\phi_{coil} = \dfrac{r}{Z_{coil}} = \dfrac{5.5}{40} = 0.1375$ (lag)

$P_{coil} = V_{coil} \cdot I \cdot \cos\phi_{coil}$

$= 200 \times 5 \times 0.1375$

$= 137.5W$

(iii) $\cos\phi = \dfrac{R+r}{Z} = \dfrac{25+5.5}{50} = 0.61$ (lag)

$P = VI\cos\phi$

$= 250 \times 5 \times 0.61$

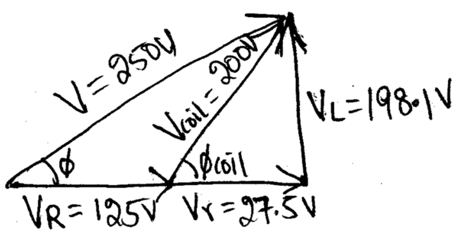
$= 762.5W$

$V_r = I.r = 5 \times 5.5 = 27.5\Omega$

$V_L = I.X_L = 5 \times 39.62 =$

$\phi_{coil} = 82.1°$ (lag)

$\phi = 52.41°$ (lag)



Q14) $V = 240V$

$f = 50Hz$

$R_A = 5\Omega$

$L_B = 0.015H$

Active power $= 3kW = 3000W$

Reactive power $= 2VAR = 2000 VAR$

Active power $= VI\cos\phi$

$\therefore 3000 = 240 \times I \times \cos\phi$ ——— (I)

Reactive power $= VI\sin\phi$

$\therefore 2000 = 240 \times I \times \sin\phi$ ——— (II)

Dividing eqn. (II) by (I)

$\therefore \tan\phi = \frac{2}{3}$ $\quad \therefore \phi = 33.69°$ (lag)

Substituting in eqn (I)

$\therefore 3000 = 240 \times I \times \cos 33.69°$

$\therefore I = 15.02A$

$$Z = \frac{V}{I} = \frac{240}{15.02} = 15.98\Omega$$

$$\cos\phi = \frac{R_A + R_B}{Z}$$

$$\therefore \cos 33.69° = \frac{5 + R_B}{15.98}$$

$\therefore R_B = 8.3\Omega$

$$Z = \sqrt{(R_A + R_B)^2 + (X_{LA} + X_{LB})^2}$$

$\because X_{LB} = 2\pi f \cdot L_B = 4.71\Omega$

$$\therefore 15.98 = \sqrt{(5 + 8.3)^2 + (X_{LA} + 4.71)^2}$$

$X_{LA} = 4.15$

$X_{LA} = 2\pi f \cdot L_A$

$\therefore 4.15 = 2\pi \times 50 \times L_A$

$\therefore L_A = 0.013H$

$Z_{coilA} = \sqrt{R_A^2 + X_{LA}^2}$

$= \sqrt{5^2 + 4.15^2}$

$= 6.5\,\Omega$

$V_{coilA} = I \cdot Z_{coilA}$

$= 15.02 \times 6.5$

$= 97.63\,V$

$Z_{coilB} = \sqrt{R_B^2 + X_{LB}^2}$

$= \sqrt{8.3^2 + 4.71^2}$

$= 9.54\,\Omega$

$V_{coilB} = I \cdot Z_{coilB}$

$= 15.02 \times 9.54$

$= 143.34$

Q15) $e = 141.4 \sin(377t + 30^\circ)$

$R = 4\,\Omega$

$X_{L_1} = 1.25\,\Omega$ at $f_1 = 25\,Hz$

$X_{L_1} = 2\pi f_1 L$

$\therefore 1.25 = 2\pi \times 25 \times L$

$\therefore L = 7.96 \times 10^{-3}\,H$

Comparing with

$e = V_m \sin(\omega t + \theta_1)$

$\therefore V_m = 141.4\,V$

$\omega = 377$ rad/sec

$\theta_1 = 30^\circ$

$V = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 99.99\,V$

$X_L = \omega L = 377 \times 7.96 \times 10^{-3}$

$\therefore X_L = 3\,\Omega$

$$Z = \sqrt{R^2+X_L^2}$$
$$= \sqrt{4^2+3^2}$$
$$\therefore Z = 5\Omega.$$
$$I = \frac{V}{Z} = \frac{99.99}{5} = 20A$$
$$\cos\phi = \frac{R}{Z} = \frac{4}{5} = 0.8$$
$$\therefore \phi = 36.87°$$
$$I_m = I \times \sqrt{2} = 20\times\sqrt{2} = 28.28A$$

$\therefore$ Expression for current is

$$i = I_m \sin(\omega t + \theta_1 - \phi)$$
$$= 28.28 \sin(377t + 30 - 36.87)$$
$$= 28.28 \sin(377t - 6.87°)$$
$$V_R = I.R$$
$$= 20\times 4 = 80V$$
$$V_{Rm} = V_R \times \sqrt{2} = 80\times\sqrt{2} = 113.14V$$

Expression for voltage across resistor is:-

$$v_R = V_{Rm}\sin(\omega t+\theta_1-\phi)$$
$$= 113.14 \sin(377t - 6.87°)$$
$$V_L = I.X_L = 20\times 3 = 60V$$
$$V_{Lm} = V_L\times\sqrt{2} = 60\times\sqrt{2} = 84.85V$$

Expression for voltage across inductor is:-

$$v_L = V_{Lm}\sin(\omega t+\theta_1-\phi+90°)$$
$$= 84.85 \sin(377t - 6.87 + 90°)$$
$$= 84.85 \sin(377t + 83.13°)$$

Q16) Output Power = 7.46 kW = 7,460 W

$V = 400V$

$f = 50Hz$

$\eta = 85\%$

p.f. = $\cos\phi = 0.8$ (lag)

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$\therefore 85 = \frac{7460}{\text{Input power}} \times 100$$

$\therefore$ Input power = 8776.47 W

$\therefore$ Input power = $VI\cos\phi$

$\therefore 8776.47 = 400 \times I \times 0.8$

$\therefore I = 27.43A$

(a) Apparent power = $V.I$

$= 400 \times 27.43$

$= 10972 VA$

$= 10.972 kVA$

(b) Reactive component of input current

$= I\sin\phi$

$= 27.43 \times \cancel{0.4359}\ 0.6$

$= 16.46 A$

(c) Reactive power = $VI\sin\phi$

$= 10,972 \times 0.6$

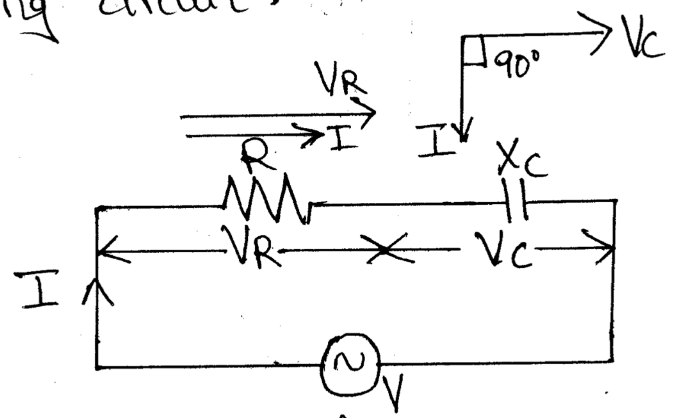
$= 6,583.2 VAR$

$= 6.5832 kVAR$

TYPE II :- R-C CIRCUITS

A.C. THROUGH RESISTANCE AND CAPACITANCE :-

Consider a resistance (R) connected in series with an capacitor (C) across a.c. supply as shown in following circuit :-



If V = r.m.s. value of applied voltage (In volts)

I = r.m.s. value of supply current (In Amperes)

R = resistance of circuit (In Ohms)

C = capacitance of circuit (In Farads)

$X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$ (In Ohms)

= ~~ind~~ capacitive reactance of circuit.

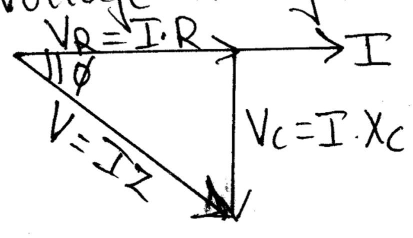
$\omega = 2\pi f$ = supply frequency (In rad/sec)

f = supply frequency (In Hertz)

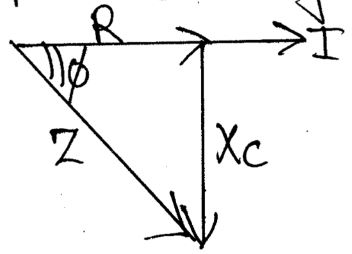
$V_R = I.R$ = Voltage across resistor (In volts)

$V_C = I.X_C$ = voltage across capacitor (In volts)

Voltage Triangle is



Impedance triangle is



$V = IZ$ = voltage of supply (In volts)

From voltage triangle

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$\therefore IZ = \sqrt{(IR)^2 + (IX_C)^2}$$

$$\therefore IZ = I\sqrt{R^2 + X_C^2}$$

$$\therefore Z = \sqrt{R^2 + X_C^2}$$

where Z = impedance of circuit (In Ohms)

Power factor (p.f.) $= \cos\phi = \dfrac{R}{Z}$ (leading)

Quality factor (Q-factor) $= \dfrac{1}{p.f.} = \dfrac{1}{\cos\phi}$

Active power (P or W) $= V.I.\cos\phi$

$= I^2R$

(Units is Watts)

Reactive power (Q) $= V.I.\sin\phi$

$= I^2X_C$

(Units is VAR - volts ampere reactive)

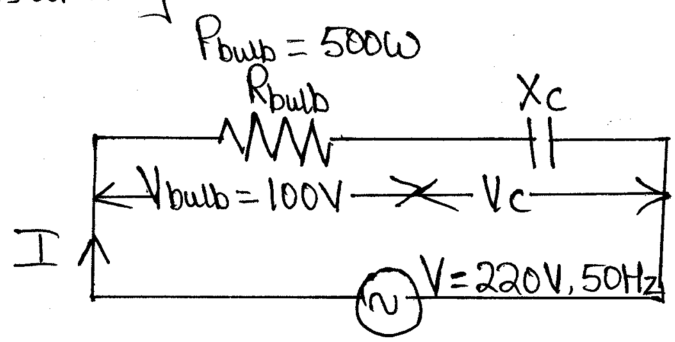
Apparent power (S) $= V.I$

$= I^2Z$

(Units is VA - volts ampere)

Q1). Assuming the bulb to be resistive

(i)



$P_{bulb} = V_{bulb} \times I$

$\therefore 500 = 100 \times I$

$\therefore I = 5A$

$V_{bulb} = I \times R_{bulb}$

$\therefore 100 = 5 \times R_{bulb}$

$\therefore R_{bulb} = 20\Omega$

$V = I.Z$

$\therefore 220 = 5 \times Z$

$\therefore Z = 44\Omega$

$Z = \sqrt{R_{bulb}^2 + X_C^2}$

$\therefore 44 = \sqrt{20^2 + X_C^2}$

$\therefore X_C = 39.19\Omega$

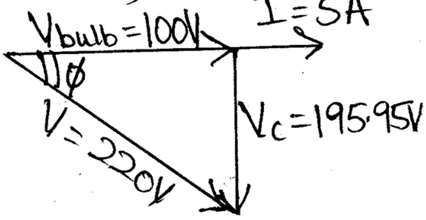
$X_C = \frac{1}{2\pi f C}$

$\therefore 39.19 = \frac{1}{2\pi \times 50 \times C}$

$\therefore C = 81.26 \times 10^{-6} F$

(ii) $P.f. = \cos\phi = \frac{R_{bulb}}{Z} = \frac{20}{44} = 0.4545$ (lead)

(iii) Vector diagram is:-



$V_C = I.X_C = 5 \times 39.19 = 195.95V$

$\phi = 62.96°$ (lead)

Q2) $R = 50\Omega$

$C = 100 \times 10^{-6} F$

$V = 100V$

$f = 50Hz$.

(i) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.85\Omega$

$Z = \sqrt{R^2 + X_C^2} = \sqrt{(50)^2 + (31.85)^2}$

$\therefore Z = 59.28\Omega$

(ii) $I = \frac{V}{Z} = \frac{100}{59.28} = 1.69A$

(iii) $pf = \cos\phi = \frac{R}{Z} = \frac{50}{59.28} = 0.8435$ (lead)

(iv) $\phi = 32.49°$ (lead)

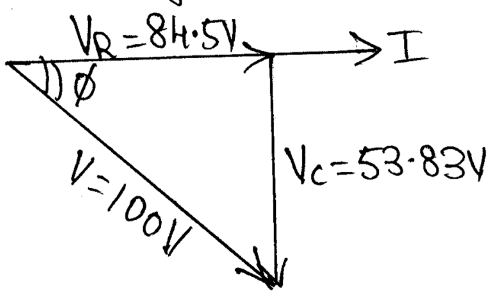
(v) $V_R = I.R$

$= 1.69 \times 50 = 84.5V$

(vi) $V_C = I.X_C$

$= 1.69 \times 31.85 = 53.83V$

Vector or Phasor diagram is :-



$\therefore \phi = 32.49°$ (lead)

Q3) $V = 240V$

$f = 50Hz$

$I = 20A$

Current leads voltage by 1/900 secs.

$T = \frac{1}{f} = \frac{1}{50}$ sec.

$\because \frac{1}{50}$ sec — 360°

$\therefore \frac{1}{900}$ sec — ? $= \frac{1}{900} \times 50 \times 360^\circ = 20^\circ$

$\therefore \phi = 20^\circ$ (lead)

(i) $Pf = \cos\phi = \cos 20^\circ = 0.9397$ (lead)

(ii) Average power = Active power

$= V.I.\cos\phi$

$= 240 \times 20 \times \cos 20^\circ$

$= 4,510.53W$

(iii) $Z = \frac{V}{I} = \frac{240}{20} = 12\Omega$

$\cos\phi = \frac{R}{Z}$

$\therefore \cos 20^\circ = \frac{R}{12}$

$\therefore R = 11.28\Omega$

$Z = \sqrt{R^2 + X_C^2}$

$\therefore 12 = \sqrt{(11.28)^2 + X_C^2}$

$\therefore X_C = 4.09\Omega$

$X_C = \frac{1}{2\pi f C}$

$\therefore 4.09 = \frac{1}{2\pi \times 50 \times C}$ $\therefore C = 778.66 \times 10^{-6} F$

Q4) $v = 100 \sin 314t$

$R = 25\Omega$

$C = 80 \times 10^{-6}$ F

(i) Comparing with

$v = V_m \sin \omega t$

$\therefore V_m = 100V \quad , \quad \omega = 314 \text{ rad/sec}$

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 80 \times 10^{-6}} = 39.81\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(25)^2 + (39.81)^2} = 47.01\Omega$$

$$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71\,V$$

$$I = \frac{V}{Z} = \frac{70.71}{47.01} = 1.5A$$

$$I_m = I \times \sqrt{2} = 1.5 \times \sqrt{2} = 2.12A$$

$$\cos\phi = \frac{R}{Z} = \frac{25}{47.01} = 0.5318 \text{ (lead)}$$

$\therefore \phi = 57.87^\circ$ (lead)

Expression for current is:-

$i = I_m \sin(\omega t + \phi)$

$= 2.12 \sin(314t + 57.87^\circ)$ —— (I)

(ii) Power $= V.I.\cos\phi$

$= 70.71 \times 1.5 \times \cos 57.87$

$= 56.41\,W$

(iii) $i = \frac{1}{2}.I_m = \frac{2.12}{2} = 1.06A$

Subs. in eqn (I)

$\therefore 1.06 = 2.12 \sin(314t + 57.87^\circ)$

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = 314t + 57.87^\circ$

$\therefore 30^\circ = 314t + 57.87^\circ$

$\therefore t = -0.089$ sec.

$V_C = I \cdot X_C$

$= 1.5 \times 39.81 = 59.72V$

$V_{cm} = V_C \times \sqrt{2} = 59.72 \times \sqrt{2}$

$V_{cm} = 84.44V$

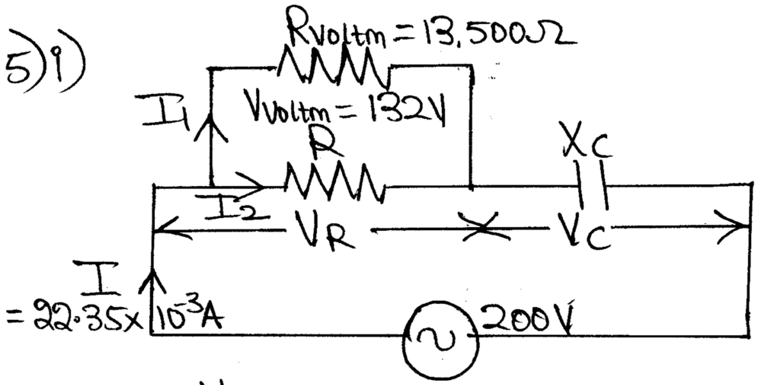
Expression for voltage across capacitor is:-

$v_c = V_{cm} \sin(\omega t + \phi - 90^\circ)$

$= 84.44 \sin(314 \times (-0.089) + 57.87 - 90)$

$v_c = -73.18V$

Q5) i)



$V_R = V_{voltm} = 132V$

$I_1 = \frac{V_{voltm}}{R_{voltm}} = \frac{132}{13500} = 9.78 \times 10^{-3} A$

$I_2 = I - I_1 = 12.57 \times 10^{-3} A$

$R = \frac{V_R}{I_2} = \frac{132}{12.57 \times 10^{-3}} = 10{,}501.19\Omega$

$\frac{1}{R_{Total}} = \frac{1}{R_{voltm}} + \frac{1}{R}$

$= \frac{1}{13{,}500} + \frac{1}{10{,}501.19}$

$\therefore R_{Total} = 5{,}906.63\Omega$

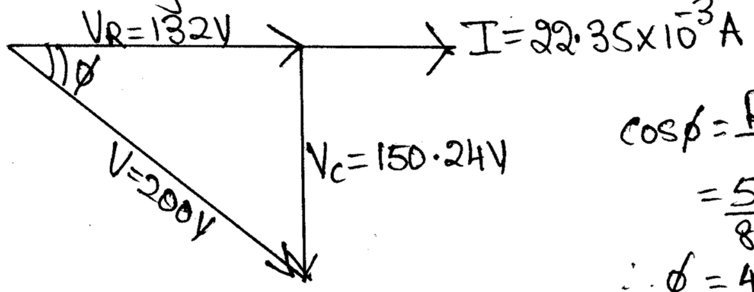
$Z = \frac{V}{I} = \frac{200}{22.35 \times 10^{-3}} = 8{,}948.55\Omega$

$Z = \sqrt{(R_{Total})^2 + X_c^2}$ $\therefore 8948.55 = \sqrt{5906.63^2 + X_c^2}$

$\therefore X_c = 6722.22\Omega$

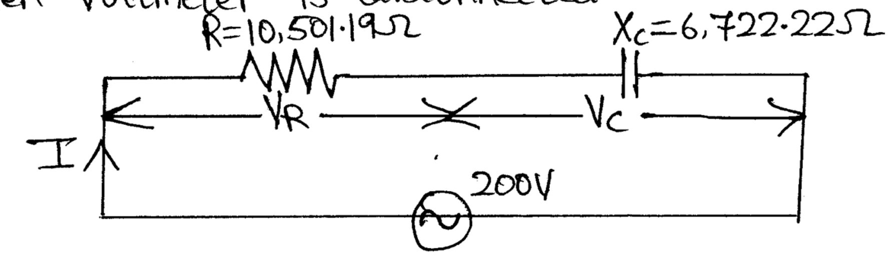
$V_C = I \cdot X_C$
$= 22.35 \times 10^{-3} \times 6722.22$
$= 150.24\,V$

Phasor or vector diagram is :-



$\cos\phi = \frac{R_{Total}}{Z}$
$= \frac{5906.63}{8948.55}$
$\therefore \phi = 48.7°$ (lag)

(ii) When voltmeter is disconnected :-



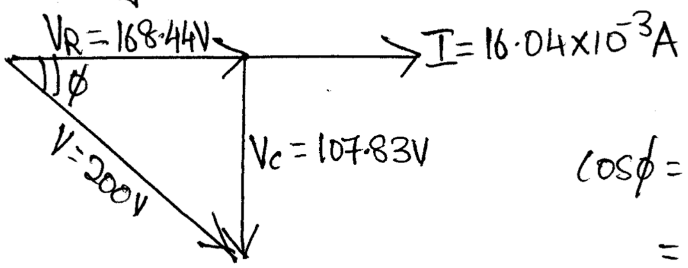
$Z = \sqrt{R^2 + X_C^2} = \sqrt{(10,501.19)^2 + (6722.22)^2}$
$\therefore Z = 12,468.49\,\Omega$

$I = \frac{V}{Z} = \frac{200}{12,468.49} = 16.04 \times 10^{-3}\,A$

$V_R = I \cdot R = 16.04 \times 10^{-3} \times 10,501.19$
$V_R = 168.44\,V$

$V_C = I \cdot X_C = 16.04 \times 10^{-3} \times 6,722.22$
$= 107.83\,V$

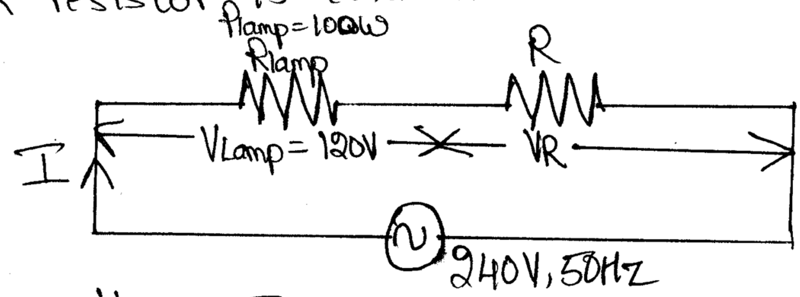
Phasor or vector diagram is :-



$\cos\phi = \frac{R}{Z}$
$= \frac{10,501.19}{12,468.49}$
$\therefore \phi = 32.63°$ (lag)

Q6) Assuming lamp to be resistive

(a) When resistor is connected in series with the lamp



$P_{lamp} = V_{lamp} \cdot I$

$\therefore 100 = 120 \times I$

$\therefore I = 0.83A$

$V_{lamp} = I \cdot R_{lamp}$

$\therefore 120 = 0.83 \times R_{lamp}$

$\therefore R_{lamp} = 144.58\Omega$

$V = I(R_{lamp} + R)$

$\therefore 240 = 0.83(144.58 + R)$

$\therefore R = 144.58\Omega$

$V_R = I \cdot R$

$= 0.83 \times 144.58$

$V_R = 120V$

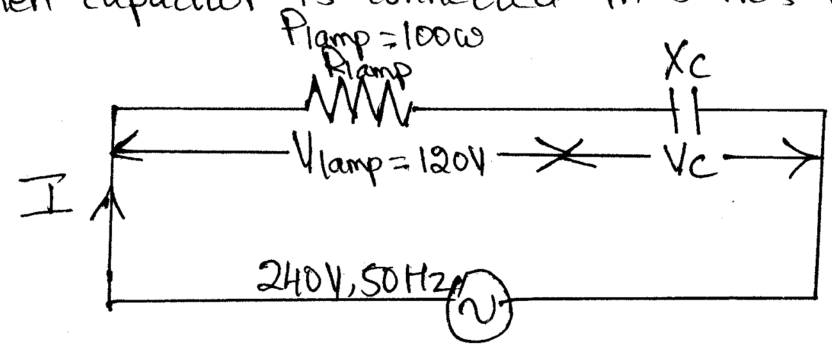
$P.f. = \cos\phi = 1$ ($\because$ Circuit is totally resistive)

$P = I^2(R_{lamp} + R)$

$= (0.83)^2 [144.58 + 144.58]$

$P = 199.2\ W$

(b) When capacitor is connected in series with the lamp



$P_{lamp} = V_{lamp} \cdot I$

$\therefore 100 = \cancel{V_a}\, 120 \times I$

$\therefore I = 0.83A$

$V_{lamp} = I \cdot R_{lamp}$

$\therefore 120 = 0.83 \times R_{lamp}$

$\therefore R_{lamp} = 144.58\Omega$

$$Z = \frac{V}{I} = \frac{240}{0.83} = 289.16\Omega$$

$$Z = \sqrt{(R_{lamp})^2 + (X_C)^2}$$

$$\therefore 289.16 = \sqrt{(144.58)^2 + X_C^2}$$

$\therefore X_C = 250.42\Omega$

$$X_C = \frac{1}{2\pi f \cdot C} \qquad \therefore 250.42 = \frac{1}{2\pi \times 50 \times C} \qquad \therefore C = 1.27 \times 10^{-5} F$$

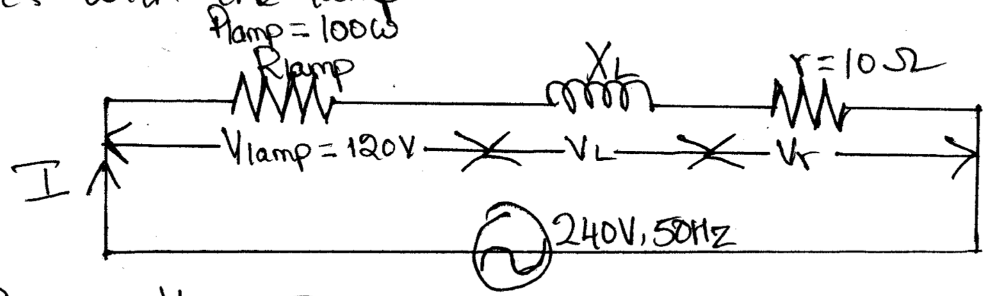
$$\cos\phi = \frac{R_{lamp}}{Z} = \frac{144.58}{289.16} = 0.5 \text{ (lead)}$$

$P = VI\cos\phi$

$= 240 \times 0.83 \times 0.5$

$= 99.6\ W$

(c) When inductor with 10Ω resistor is connected in series with the lamp



$P_{lamp} = V_{lamp} \cdot I$

$\therefore 100 = 120 \times I$

$\therefore I = 0.83A$

$V_{lamp} = I \cdot R_{lamp}$

$\therefore 120 = 0.83 \times R_{lamp}$

$\therefore R_{lamp} = 144.58\Omega$

$Z = \frac{V}{I} = \frac{240}{0.83} = 289.16\Omega$

$Z = \sqrt{(R_{lamp} + r)^2 + X_L^2}$

$\therefore 289.16 = \sqrt{(144.58 + 10)^2 + X_L^2}$

$\therefore X_L = 244.37\Omega$

$X_L = 2\pi f \cdot L$

$\therefore 244.37 = 2\pi \times 50 \times L$

$\therefore L = 0.778H$

$\cos\phi = \frac{R_{lamp} + r}{Z} = \frac{144.58 + 10}{289.16} = 0.535 \text{ (lag)}$

$P = VI\cos\phi$

$= 240 \times 0.83 \times 0.535$

$= 106.49W$

Method (b) is preferred since power consumed is the least.

Q7) $P = 700W$

$pf = 0.707$ (lead)

$v = 141.4 \sin(314t + 30^\circ)$

Comparing with

$v = V_m \sin(\omega t + \theta_1)$

$\therefore V_m = 141.4V, \quad \omega = 314 \text{ rad/sec}, \quad \theta_1 = 30^\circ.$

$V = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 99.99V$

$P = VI\cos\phi$

$\therefore 700 = 99.99 \times I \times 0.707$

$\therefore I = 9.9A$

$Z = \frac{V}{I} = \frac{99.99}{9.9} = 10.1\Omega$

$\cos\phi = \frac{R}{Z}$

$\therefore 0.707 = \frac{R}{10.1} \qquad \therefore R = 7.14\Omega$

$Z = \sqrt{R^2 + X_C^2}$

$\therefore 10.1 = \sqrt{(7.14)^2 + X_C^2}$

$\therefore X_C = 7.14\Omega$

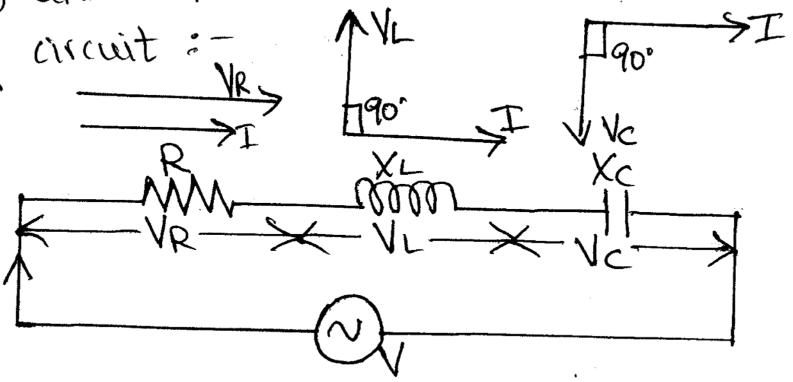
$X_C = \frac{1}{\omega C}$

$\therefore 7.14 = \frac{1}{314 \times C}$

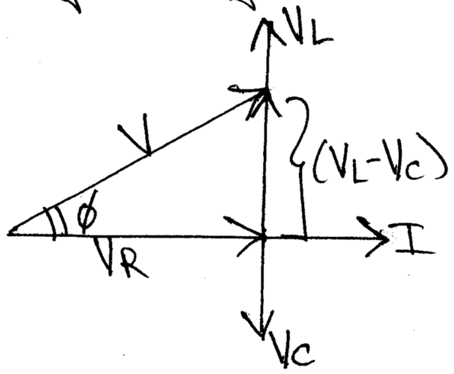
$\therefore C = 4.46 \times 10^{-4} F$

TYPE III :- AC THROUGH RESISTANCE, INDUCTANCE AND CAPACITANCE (R-L-C CIRCUIT):-

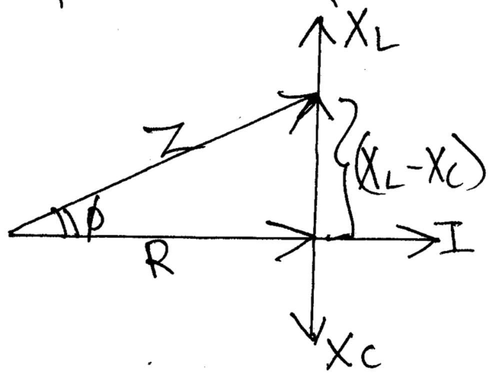
Consider a resistance (R) connected in series with an inductor (L) and capacitor (C) across a.c. supply as shown in following circuit :-



Voltage triangle is



Impedance triangle is



{We assume $V_L > V_C$}

From voltage triangle

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$\therefore IZ = \sqrt{(IR)^2 + (I.X_L - I.X_C)^2}$$

$$\therefore IZ = I.\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$P.f. = \cos\phi = \frac{R}{Z} \quad (\underset{\text{if } X_L > X_C}{\text{lag}} \text{ or } \underset{\text{if } X_C > X_L}{\text{lead}})$$

$$Z = R + j[X_L - X_C] \text{ if } X_L > X_C$$

$$Z = R - j[X_C - X_L] \text{ if } X_C > X_L$$

Type of Impedance	Value of Impedance	Phase Angle of Current	Power Factor
Resistance only	R	0°	1
Inductance only	$X_L = 2\pi f \cdot L$	90° (lag)	0
Capacitance only	$X_C = \frac{1}{2\pi f \cdot C}$	90° (lead)	0
R-L circuit	$Z = \sqrt{R^2 + X_L^2}$	$0^\circ < \phi < 90^\circ$ (lag)	$0 < pf < 1$ (lag)
R-C circuit	$Z = \sqrt{R^2 + X_C^2}$	$0^\circ < \phi < 90^\circ$ (lead)	$0 < pf < 1$ (lead)
R-L-C circuit	$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$0^\circ < \phi < 90^\circ$ (lag or lead)	$0 < pf < 1$ (lag or lead)

Q1) $V = 5V$

$V_R = 3V$

$V_L = 1V$

$$V = \sqrt{V_R^2 + |V_L - V_C|^2}$$

$$\therefore 5 = \sqrt{(3)^2 + |1 - V_C|^2}$$

$$\therefore 16 = |1 - V_C|^2$$

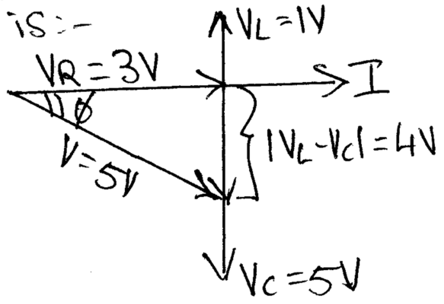
$$\therefore 1 - V_C = \pm 4$$

$$\therefore V_C = -5V \text{ or } V_C = 3V$$

Since voltage across capacitor is vectorly negative

$$\therefore V_C = -5V$$

Phasor Diagram is :-



$$\cos\phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{V_R}{V}$$

$$\therefore \cos\phi = \frac{3}{5} = 0.6 \text{ (lead)}$$

$$\therefore \phi = 53.13^\circ \text{ (lead)}$$

Q2) $R = 10\,\Omega$
$L = 0.1\,H$
$C = 150 \times 10^{-6}\,F$
$V = 200\,V$
$f = 50\,Hz$

(i) $X_L = 2\pi f L$
$= 2\pi \times 50 \times 0.1 = 31.42\,\Omega$

(ii) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.22\,\Omega$

(iii) $Z = \sqrt{R^2 + (X_L - X_C)^2}$
$= \sqrt{(10)^2 + (31.42 - 21.22)^2}$
$Z = 14.28\,\Omega$

(iv) $I = \dfrac{V}{Z} = \dfrac{200}{14.28} = 14\,A$

(v) $Pf = \cos\phi = \dfrac{R}{Z} = \dfrac{10}{14.28} = 0.7$ (lag)

(vi) $Z_{coil} = \sqrt{R^2 + X_L^2} = 32.97\,\Omega$
$V_{coil} = I \cdot Z_{coil} = 14 \times 32.97$
$= 461.58\,V$

$V_C = I \cdot X_C = 14 \times 21.22$
$= 297.08\,V$

Q3) $v = 100 \sin 314t$

$R = 10\Omega$

$L = 0.0318H$

$C = 63.6 \times 10^{-6} F$

Comparing with

$v = V_m \sin \omega t$

$\therefore V_m = 100V, \ \omega = 314 \text{ rad/sec}$

$V = \frac{V_m}{\sqrt{2}} = \frac{100}{\sqrt{2}} = 70.71V$

$X_L = \omega L = 314 \times 0.0318 = 9.99\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 63.6 \times 10^{-6}} = 50.07\Omega$

$Z = \sqrt{R^2 + (-X_C + X_L)^2} = \sqrt{(10)^2 + (-50.07 + 9.99)^2}$

$\therefore Z = 41.31\Omega$

$I = \frac{V}{Z} = \frac{70.71}{41.31} = 1.71A$

$I_m = I \times \sqrt{2} = 2.42A$

(iii) P.f. $= \cos\phi = \frac{R}{Z} = \frac{10}{41.31} = 0.242$ (lead)

(ii) $\therefore \phi = 75.99°$ (lead)

(i) $i(t) = I_m \sin(\omega t + \phi) = 2.42 \sin(314t + 75.99°)$

(iv) Active power $= VI\cos\phi$

$= 70.71 \times 1.71 \cos 75.99°$

$= 29.27W$

(v) Peak value of pulsating energy

$= VI + VI\cos\phi$

$= 70.71 \times 1.71 + 29.27$

$= 150.18W$

Q4) Coil Z_1

$V_1 = 230V$

$f_1 = 50Hz$

$P_1 = 100W$

$\cos\phi_1 = 0.5$ (lag)

$P_1 = V_1 I_1 \cos\phi_1$

$\therefore 100 = 230 \times I_1 \times 0.5$

$\therefore I_1 = 0.87A$

$Z_1 = \frac{V_1}{I_1} = \frac{230}{0.87} = 264.37\Omega$

$\cos\phi_1 = \frac{R_1}{Z_1}$

$\therefore 0.5 = \frac{R_1}{264.37}$

$\therefore R_1 = 132.19\Omega$

$Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$

$\therefore 264.37 = \sqrt{(132.19)^2 + X_{L_1}^2}$

$\therefore X_{L_1} = 228.95\Omega$

Coil Z_2

$V_2 = 230V$

$f_2 = 50Hz$

$P_2 = 60W$

$\cos\phi_2 = 0.6$ (lead)

$P_2 = V_2 I_2 \cos\phi_2$

$60 = 230 \times I_2 \times 0.6$

$I_2 = 0.44A$

$Z_2 = \frac{V_2}{I_2} = \frac{230}{0.44} = 522.73\Omega$

$\cos\phi_2 = \frac{R_2}{Z_2}$

$\therefore 0.6 = \frac{R_2}{522.73}$

$\therefore R_2 = 313.64\Omega$

$Z_2 = \sqrt{R_2^2 + X_{C_2}^2}$

$\therefore 522.73 = \sqrt{(313.64)^2 + X_{C_2}^2}$

$\therefore X_{C_2} = 418.18\Omega$

When coil Z_1 and Z_2 are connected in series

$$Z = \sqrt{(R_1+R_2)^2 + (-X_{C_2} + X_{L_1})^2}$$

$$= \sqrt{(132.19+313.64)^2 + (228.95 - 418.18)^2}$$

$$Z = 484.33\Omega$$

$$I = \frac{V}{Z} = \frac{230}{484.33} = 0.48A$$

$$pf = \cos\phi = \frac{R_1+R_2}{Z} = \frac{132.19+313.64}{484.33} = 0.921 \text{ (lead)}$$

$$P = VI\cos\phi = 230 \times 0.48 \times 0.921$$

$$= 101.68W$$

(ii) In rectangular form:-

$$Z = (R_1 + R_2) + j(X_{L1} - X_{C2})$$
$$= (132.19 + 313.64) + j(228.95 - 418.18)$$
$$= 445.83 - j189.23$$

For overall p.f. to be unity, circuit should be in resonance. ∴ It should be a purely resistive circuit.

Hence we include
$$X_{L3} = +j189.23$$

$$\therefore \text{Total } Z = Z + X_{L3}$$
$$= 445.83 - j189.23 + j189.23$$
$$= 445.83 \text{ (purely resistive)}$$

$$\because X_{L3} = 2\pi f \cdot L_3$$
$$\therefore 189.23 = 2\pi \times 50 \times L_3$$
$$\therefore L_3 = 0.6\,H.$$

Q5) $L = 0.01H$

$v = 400\cos(3000t - 10^\circ)$

$= 400\sin(3000t - 10^\circ + 90^\circ)$

$= 400\sin(3000t + 80^\circ)$

$i = 10\sqrt{2}\cos(3000t - 55^\circ)$

$= 10\sqrt{2}\sin(3000t - 55^\circ + 90^\circ)$

$= 10\sqrt{2}\sin(3000t + 35^\circ)$

Comparing with

$v = V_m \sin(\omega t + \theta_1)$

$\therefore V_m = 400V, \quad \omega = 3000 \text{ rad/sec}, \quad \theta_1 = 80^\circ$

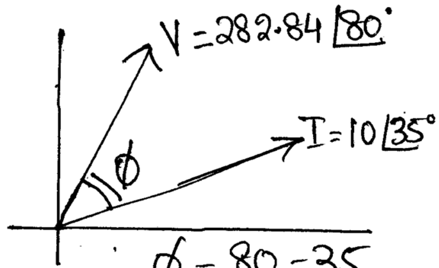
$V = \frac{V_m}{\sqrt{2}} = \frac{400}{\sqrt{2}} = 282.84V$

Comparing with

$i = I_m \sin(\omega t + \theta_2)$

$\therefore I_m = 10\sqrt{2}A, \quad \omega = 3000 \text{ rad/sec}, \quad \theta_2 = 35^\circ$

$I = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10A$



$\therefore \phi = 80 - 35 = 45^\circ (\text{lag})$

$Z = \frac{V}{I} = \frac{282.84}{10} = 28.28\Omega$

$\cos\phi = \frac{R}{Z}$

$\therefore \cos 45^\circ = \frac{R}{28.28}$

$\therefore R = 20\Omega$

$X_L = \omega L = 3000 \times 0.01 = 30\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \therefore 28.28 = \sqrt{20^2 + (30 - X_C)^2}$

$X_C = 10\Omega$

$X_C = \frac{1}{\omega C} \quad \therefore 10 = \frac{1}{3000 \times C}$

$\therefore C = 3.33 \times 10^{-5} F$

Q6) $R = 8\Omega$

$X_L = 16\Omega$

$V = 100V$

$f = 50Hz$

$I = 12.5A$

(i) $Z = \frac{V}{I} = \frac{100}{12.5} = 8\Omega$

$Z = \sqrt{R^2 + (X_L - X_c)^2}$

$\therefore 8 = \sqrt{(8)^2 + (16 - X_c)^2}$

$\therefore 16 - X_c = 0$

$\therefore X_c = 16\Omega$

$X_c = \frac{1}{2\pi f \cdot C}$ $\therefore 16 = \frac{1}{2\pi \times 50 \times C}$ $\therefore C = 1.99 \times 10^{-4} F$

(ii) $V_c = I \cdot X_c$

$= 12.5 \times 16 = 200V$

(iii) $Pf = \cos\phi = \frac{R}{Z} = \frac{8}{8} = 1$

Q8) $i = 4.5 + j12 = 12.82 \angle 69.44°$

$v = 100 + j150 = 180.28 \angle 56.31°$

a) $Z = \frac{v}{i} = \frac{100+j150}{4.5+j12} = 13.7 - j3.2$

Comparing with $Z = R - jX_C$,

$\therefore R = 13.7\,\Omega$, $X_C = 3.2\,\Omega$

Circuit is capacitive $\because Z = 14.06 \angle -13.13$

$\therefore \phi = 13.13$ (lead)

b) $P = V.I.\cos\phi$

$= 180.28 \times 12.82 \times \cos(13.13)$

$P = 2250.77\,W$

c) $\phi = 13.13$ (lead)

Q9) i) $Z_1 = 4 + j3 = 5 \angle 36.87°$

$Z_2 = 6 - j8 = 10 \angle -53.13°$

$Z_3 = 4 = 4 \angle 0°$

$V = 100 \angle 0° = 100 + j0$

Total $Z = Z_1 + Z_2 + Z_3$ (only in rectangular form)

$\therefore Z = 4 + j3 + 6 - j8 + 4$

$\therefore Z = 14 - j5 = 14.87 \angle -19.65°$

$I = \frac{V}{Z} = \frac{100+j0}{14-j5} = 6.33 + j2.26 = 6.73 \angle 19.65°$

ii) $V_1 = I.Z_1 = (6.33+j2.26)(4+j3)$

$= 18.54 + j28.03 = 33.61 \angle 56.52°$

$V_2 = IZ_2 = (6.33+j2.26)(6-j8)$

$= 56.06 - j37.08 = 67.21 \angle -33.48°$

$V_3 = IZ_3 = (6.33+j2.26)(4) = 25.32 + j9.04 = 26.89 \angle 19.65$

10) $R_1 = 10\Omega$

$L_1 = 0.05H$

$R_2 = 20\Omega$

$L_2 = 0.1H$

$C_2 = 50\times10^{-6}F$

$V = 200V, \; f = 50Hz$

$X_{L_1} = 2\pi f L_1 = 2\pi\times50\times0.05 = 15.71\Omega$

$X_{L_2} = 2\pi f L_2 = 2\pi\times50\times0.1 = 31.42\Omega$

$X_{C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi\times50\times50\times10^{-6}} = 63.66\Omega$

$Z_1 = R_1 + jX_{L_1} = 10 + j15.71$

$Z_2 = R_2 + j(X_{L_2} - X_{C_2}) = 20 + j(31.42 - 63.66)$

$= 20 - j32.24$

Total $Z = Z_1 + Z_2$ (Only in rectangular or cartesian form)

$\therefore Z = 30 - j16.53 = 34.25\angle{-28.86^\circ}$

i) $I = \frac{V}{Z} = \frac{200}{30 - j16.53} = 5.11 + j2.82 = 5.84\angle{28.86^\circ}$

ii) $V_1 = IZ_1 = (5.11 + j2.82)(10 + j15.71) = 6.87 + j108.52 = 108.74\angle{86.38^\circ}$

$V_2 = IZ_2 = (5.11 + j2.82)(20 - j32.24) = 193.12 - j108.35 = 221.43\angle{-29.29^\circ}$

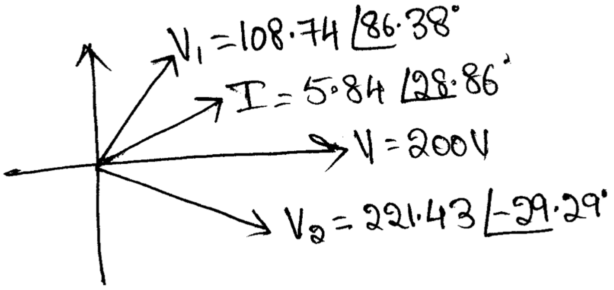
$\cancel{V_3 = IZ_3 = (5.11 + j2.82)(}$

iii) Comparing with $Z = |Z|\angle\phi$

$\therefore \phi = 28.86$ (lead)

$\therefore pf = \cos\phi = \cos 28.86$

$= 0.8758$ (lead)



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11) $\because$ Voltage lags current by 30°

$\therefore$ Current leads voltage by 30°

$\therefore \phi = 30^\circ$ (lead)

a) $\therefore$ p.f. is leading

b) p.f. $= \cos\phi = \cos 30^\circ = 0.866$ (lead)

c) Circuit is capacitive

d) $Z_1 = 3+j4$

$Z_2 = 2+j3.46$

$Z_3 = 1-j7.46$

$V_1 = 10$

i) $I = \frac{V_1}{Z_1} = \frac{10}{3+j4} = 1.2-j1.6 = 2\angle{-53.13^\circ}$

ii) $V_2 = IZ_2 = (1.2-j1.6)(2+j3.46) = 7.94+j0.95 = 7.99\angle{6.84^\circ}$

$V_3 = IZ_3 = (1.2-j1.6)(1-j7.46) = -10.74-j10.55 = 15.05\angle{-135.5}$

iii) $V = V_1+V_2+V_3$

$= 10+7.94+j0.95-10.74-j10.55$

$= 7.2-j9.6 = 12\angle{-53.12^\circ}$

RESONANCE IN SERIES R-L-C CIRCUIT :-

A Series R-L-C circuit is said to resonate if :-

i) $X_L = X_C$

ii) $V_L = V_C$.

iii) $Z = R$ ie. impedance is only resistive.

iv) Current is maximum i.e. $I = \frac{V}{Z} = \frac{V}{R}$

v) P.f. $= \cos\phi = 1$ (Unity)

vi) Resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$

vii) Quality factor (Q-factor) $= \frac{X_L}{R} = \frac{2\pi f_0 L}{R}$

viii) Half power points are at frequencies

$$f_1 = f_0 - \frac{R}{4\pi L} \quad \text{and} \quad f_2 = f_0 + \frac{R}{4\pi L}$$

Q1) $R = 10\,\Omega$

$L = 0.2\,H$

$C = 40 \times 10^{-6}\,F$

$V = 100\,V$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2 \times 40 \times 10^{-6}}} = 56.27\,Hz$$

At resonance

$$I = \frac{V}{R} = \frac{100}{10} = 10\,A$$

$pf = \cos\phi = 1$

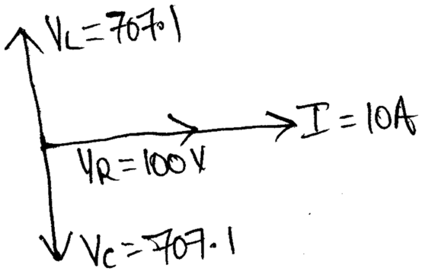
$P = VI\cos\phi = 100 \times 10 \times 1 = 1000\,W$

$V_R = IR = 10 \times 10 = 100\,V$

$V_L = IX_L = 10 \times 2\pi f_0 L = 10 \times 2 \times \pi \times 56.27 \times 0.2 = 707.1\,V$

$V_C = V_L = 707.1\,V$

Phasor Diagram is :-



Q2) $R = 1000\,\Omega$

$L = 100\times10^{-3}\,H$

$C = 10\times10^{-12}\,F$

$V = 100V$

i) $f_0 = \dfrac{1}{2\pi\sqrt{LC}} =$

ii) Q factor $= \dfrac{X_L}{R} = \dfrac{2\pi f_0 L}{R} =$

iii) Frequency at half power points is at

$f_1 = f_0 - \dfrac{R}{4\pi L} =$

$f_2 = f_0 + \dfrac{R}{4\pi L} =$

Q3) Since current becomes maximum

$\therefore$ Resonance frequency is $f_0 = \dfrac{600}{2\pi}$ Hz.

$f' = \dfrac{400}{2\pi}$ Hz at $I' = \dfrac{1}{2} I_{max}$

$R = 3\,\Omega$

$\because f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore \dfrac{600}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$ $\quad\therefore \sqrt{LC} = \dfrac{1}{600}$ $\quad\therefore LC = \dfrac{1}{360000}$ —(I)

$\because I' = \dfrac{1}{2} I_{max}$

$\therefore \dfrac{V}{Z} = \dfrac{1}{2}\dfrac{V}{R}$ $\quad\therefore Z = 2R$

$\therefore \sqrt{R^2 + (X_L - X_C)^2} = 2R$

$\therefore \sqrt{9 + \left(2\pi f' L - \dfrac{1}{2\pi f' C}\right)^2} = 2(3)$

$\therefore 9 + \left(\dfrac{4\pi^2 f'^2 LC - 1}{2\pi f' C}\right)^2 = 36$

$$\therefore \frac{4\pi^2 f'^2 LC - 1}{2\pi f' C} = \pm\sqrt{27}$$

Subs. from eqn (I)

$$\therefore \frac{4\pi^2 \times \left(\frac{400}{2\pi}\right)^2 \times \frac{1}{360000} - 1}{2\pi f' C} = \pm\sqrt{27}$$

$$\therefore \frac{\frac{16\pi^2}{36\pi^2} - 1}{2\pi f' C} = -5.196$$

$$\therefore \frac{0.4444}{2\pi\left(\frac{400}{2\pi}\right)C} = -5.196$$

$$\therefore C = 2.14 \times 10^{-4} F$$

Subs. in eqn (I)

$$\therefore L = 0.0137$$

PARALLEL A.C. CIRCUITS.

For R-L Circuits in Series

$$Z = R + jX_L$$

For Parallel R-L Circuits

$$Y = G - jB$$

where Admittance $(Y) = \frac{1}{\text{Impedance } (Z)}$

Conductance $(G) = \frac{1}{\text{Resistance } (R)}$

Susceptance $(B) = \frac{1}{\text{Reactance } (X_L)}$

For R-C circuits in Series

$$Z = R - jX_C$$

For parallel R-C Circuits

$$Y = G + jB$$

81) $Z_1 = 6+j8 = 10\angle 53.13^\circ$

$Z_2 = 8-j6 = 10\angle -36.87^\circ$

$V = 100V$

i) $I_1 = \frac{V}{Z_1} = \frac{100}{6+j8} = 6-j8 = 10\angle -53.13^\circ$

$I_2 = \frac{V}{Z_2} = \frac{100}{8-j6} = 8+j6 = 10\angle 36.87^\circ$

Comparing with $Z_1 = |Z_1|\angle\phi_1 \quad \therefore \phi_1 = 53.13^\circ$ (lag)

& $Z_2 = |Z_2|\angle\phi_2 \quad \therefore \phi_2 = 36.87^\circ$ (lead)

$\therefore Pf_1 = \cos\phi_1 = \cos 53.13^\circ = 0.6$

$Pf_2 = \cos\phi_2 = \cos 36.87^\circ = 0.8$

ii) Since Z_1 & Z_2 are connected in parallel

$\therefore \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$\therefore \frac{1}{Z} = \frac{1}{6+j8} + \frac{1}{8-j6}$

$\therefore Z = 7+j = 7.07\angle 8.13^\circ$

$I = \frac{V}{Z} = \frac{100}{7+j} = 14-j2 = 14.14\angle -8.13^\circ$

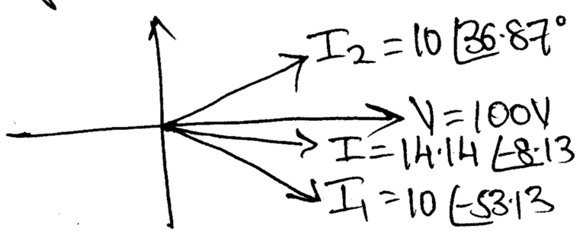
Comparing with $Z = |Z|\angle\phi \quad \therefore \phi = 8.13$ (lag)

$\therefore pf = \cos\phi = \cos 8.13 = 0.99$ (lag)

iii) $P_1 = VI_1\cos\phi_1 = 600W$

$P_2 = VI_2\cos\phi_2 = 800W$

Phasor Diagram is :-



Q2) $i_a = 7.07 \sin(314t - \pi/4)$

$i_b = 21.2 \sin(314t + \pi/3)$

$V = 354 \sin 314t$

Comparing with

$i_a = I_{am} \sin(\omega t + \theta_1)$

$\therefore I_{am} = 7.07A$, $\omega = 314$ rad/sec, $\theta_1 = -\pi/4^c = -45^\circ$

$I_a = \frac{I_{am}}{\sqrt{2}} = 5A$

$\therefore I_a = 5\angle{-45^\circ} = 3.53 - j3.53$

Comparing with

$i_b = I_{bm} \sin(\omega t + \theta_2)$

$\therefore I_{bm} = 21.2A$, $\omega = 314$ rad/sec, $\theta_2 = \frac{\pi}{3}^c = 60^\circ$

$I_b = \frac{I_{bm}}{\sqrt{2}} = 14.99A$

$\therefore I_b = 14.99\angle{60^\circ} = 7.5 + j12.98$

Comparing with

$V = V_m \sin \omega t$

$\therefore V_m = 354V$, $\omega = 314$ rad/sec

$V = \frac{V_m}{\sqrt{2}} = 250.32V$

$\therefore V = 250.32\angle{0^\circ} = 250.32 + j0$

$Z_a = \frac{V}{I_a} = \frac{250.32}{3.53 - j3.53} = 35.46 + j35.46$

$\because Z_a = R_a + jX_{La}$ $\therefore R_a = 35.46\Omega$, $X_{La} = 35.46\Omega$

$Z_b = \frac{V}{I_b} = \frac{250.32}{7.5 + j12.98} =$

$I = I_a + I_b$ (Only in rectangular or cartesian form)

$=$