1. For a particle performing S.H.M. according to the equation $x = 10 \sin \pi t$, where x is in meters, find the time taken by the particle to move from the position of equilibrium to the position of maximum displacement. (0.5 s)

2. The differential equation for linear S.H.M. of a particle of mass 2 g is $\frac{d^2x}{dt^2} + 16x = 0$. Find the force constant. (0.032 N/m)

[Comment : Compare with standard equation of S.H.M. We get $\omega^2 = 16$. Also $k = m\omega^2$.]

3. A particle in S.H.M. has a velocity of 10 cm/s when it crosses the mean position. If the amplitude of its oscillations is 2 cm, find the velocity when it is midway between the mean and extreme positions. ($5\sqrt{3}$ cm/s)

4. The acceleration of a particle in S.H.M. is 8 cm/s^2 when the displacement is 2 cm. If the amplitude of motion is 5 cm, find the velocity when the displacement is 3 cm. (8 cm/s)

5. A particle executing S.H.M. has a period of 6 s and its maximum velocity during oscillations is 6.28 cm/s. Find the time taken by it to describe a distance of 3 cm from its equilibrium position. (0.5 s)

6. A particle performs S.H.M. of amplitude a and period Ts. The particle takes a time t_1 s to move from the mean position to a position $\frac{a}{2}$. To cover the farther distance $\frac{a}{2}$ the time taken is t_2 s. Find t_1/t_2. (1/2)

7. A particle performs S.H.M. of amplitude 10 cm. Its maximum velocity during oscillations is 100 cm/s. What is its displacement when the velocity is 60 cm/s ? (8 cm)

8. A particle starting from the extreme position performs S.H.M. of amplitude 12 cm and frequency 40 oscillations per minute. Find the displacement of the particle two seconds after start. (– 6 cm)

9. A particle executes S.H.M. of period 12 s and of amplitude 8 cm. What time will it take to travel 4 cm from the extreme position and what will be the velocity then? (2 s; $2\pi/\sqrt{3}$ cm/s)

10. A particle executes linear S.H.M. of period 4 s and amplitude 4 cm. Find the time taken by it to describe a distance of 1 cm from the positive extremity. (0.46 s)

11. A particle performs S.H.M. of period 12 s along a path 16 cm long. If it is initially at the positive extremity, how much time will it take to cover a distance of 6 cm from that position? (2.52 s)

12. A particle performing linear S.H.M. has a period of 6.28 seconds and a pathlength of 20 cm. What is the velocity when its displacement is 6 cm from mean position? (8 cm/s)

13. A particle performing linear S.H.M. has maximum velocity of 25 cm/s and maximum acceleration of 100 cm/s^2. Find the amplitude and period of oscillation. ($\pi = 3.142$)

(6.25 cm ; 1.571 s)

14. A particle performing S.H.M. has velocities of 8 cm/s and 6 cm/s at displacements of 3 cm and 4 cm respectively. Find its amplitude and frequency of oscillations. Calculate its maximum velocity. What is the phase of its motion when the displacement is 2.5 cm?

(5 cm; 0.3185 s^{-1} : 10 cm/s; 30°)

15. The maximum velocity of a particle performing linear S.H.M. is 0.16 m/s. If its maximum acceleration is 0.64 m/s^2, calculate its period.

(1.57s)

16. A particle in S.H.M. has a period of 2 seconds and amplitude of 10 cm. Calculate the acceleration when it is at 4cm from its positive extreme position.

($6\pi^2$ cm/s^2)

17. A block is on a piston which is moving vertically up and down with S.H.M. of period one second. At what amplitude of motion will the block and piston separate ? At which point in the path of motion will the separation take place?

(24.85 cm; uppermost)

18. A particle of mass 10 g performs S.H.M. of amplitude 10 cm and period 2 s. Determine its kinetic and potential energies when it is at a distance of 8 cm from its equilibrium position.

(18×10^{-6} J; 32×10^{-6} J)

19. A particle performs a linear S.H.M. of amplitude $10\sqrt{2}$ cm. Find at what distance from the mean position its PE is equal to its KE.

(± 10 cm)

20. For particle performing S.H.M. of amplitude a at which position is K.E. equal to P.E.? At which position is the speed of the particle equal to half the maximum speed?

$\left(\pm \frac{a}{\sqrt{2}} ; \pm \frac{\sqrt{3}}{2} a\right)$

21. A particle executes S.H.M. with a period of 10 seconds. Find the time in which its potential energy will be half of its total energy.

[1.25s]

22. Find the relation between amplitude and displacement at the instant when the K.E. of a particle performing S.H.M. is three times its P.E.

$\left(x = \pm \frac{a}{2}\right)$

23. When a particle executing S.H.M. is 2 cm away from the mean position, its KE is double the PE. At what distance from the mean position will the PE be double the KE?

(2.82 cm)

24. The total energy of a particle of mass 200 g performing S.H.M. is 10^{-3} J. Find the maximum velocity and the period if the amplitude is 4 cm.

(0.1 m/s; 2.512 s)

25. The bob of a simple pendulum weighs 25 g. The tension in the cord is 20 g-wt at a certain position when the pendulum is vibrating. Find the restoring force. assume ext. NOTcertain pos)

(15 g-wt)

26. The bob of a simple pendulum of length 1 m has a mass of 20 g and oscillates with an amplitude of 6 cm. What is its P.E. when at the extreme position? $g = 9.8$ m/s^2. (3.528×10^{-4} J)

27. When the length of a simple pendulum is decreased by 20 cm, the period changes by 10%. Find the original length and period of pendulum. $g = 9.8$ m/s^2.

(1.05 m; 2.05 s)

28. A simple pendulum has a length of one metre and amplitude of 4 cm when vibrating. Find the velocity with which the bob passes through the mean position and acceleration at the extreme position. $g = 9.8$ m/s^2

(0.12 m/s; 0.39 m/s^2)

29. The maximum velocity attained by the bob of a simple pendulum is 15.52 cm/s. If the amplitude of oscillations is 4 cm and $g = 9.8$ m/s^2, find the length of the pendulum and the tension in the string at the lowest position when the pendulum is vibrating. Mass of the bob = 0.1 kg. 0.651m

(99.97 cm ; 0.9816 N)

30. Calculate the maximum velocity which an oscillating pendulum of length one metre will attain if its amplitude is 8 cm. Take $g = 9.8$ m/s^2.

(25.04 cm/s)

31. If the length of a second's pendulum is decreased by 0.5%, how many oscillations will it gain or lose in a day?

(108)

32. A clock regulated by seconds pendulum, keeps correct time. During summer, length of pendulum increases to 1.005 m. How many oscillations will the clock gain or lose in one day?

(215)

33. A clock regulated by a second's pendulum keeps correct time. During summer the length of the pendulum increases to 1.01 m. How much will the clock gain or lose in one day? $g = 9.8\ m/s^2$.

(Lose 691 s per day)

34. The equation of motion of a particle performing S.H.M. is written as $x = 8 \sin\left(5\pi t + \frac{\pi}{3}\right)$ cm. Write down the values of path length, period and phase constant. What is the phase angle $\frac{1}{20}$ s after start? Find the displacement, velocity and acceleration of the particle at $t = 2$ s. Find also the values of frequency, maximum velocity and maximum acceleration.

$\left(16\ cm; 0.4\ s; \pi/3; \frac{7\pi}{12}; 4\sqrt{3}\ cm; 20\pi/s; 106\ cm/s; 2.5\ Hz; 40\pi\ cm/s; 1971.9\ cm/s^2\right)$

35. The displacement of a particle performing S.H.M. is given by $x = 3 \sin 5\pi t + 4 \cos 5\pi t$ metres. Calculate the amplitude and initial phase of motion.

(5 m; 53° 8′)

36. The displacement of a particle executing S.H.M. is given by $x = a \sin\left(\frac{\pi}{6} t\right) + b \cos\left(\frac{\pi}{6} t\right)$ where $a = 3$ cm and $b = 4$ cm. Find the amplitude and initial phase of S.H.M.

(5 cm; $\tan^{-1}(4/3)$)

37. Show that $y = \sqrt{a} \sin \omega t - \sqrt{b} \cos \omega t$ represents an S.H.M. Find the amplitude of its motion.

($\sqrt{a+b}$)

38. For a simple pendulum of length 0.7 m carrying a bob of mass 10 g and vibrating with an amplitude of 6 cm, what is the PE at the extreme end of the string?

(2.52×10^{-4} J)

39. A simple pendulum is suspended from an inaccessible point. It has a period of 4 s. When the length is shortened by 118 cm, the period is decreased by 0.5 s. Find the original length of the pendulum and g at the place.

(621 cm; 980.6 cm/s^2)

40. For a particle performing S.H.M., the ratio of its K.E. to P.E. is 3 in a given position. If the amplitude of motion is 20 cm, find the distance of the point from the mean position. **(10 cm)**

41. The equation of motion of a particle performing S.H.M. is $x = 3 \sin 6\pi t$ in cm. Determine its amplitude of motion, frequency, velocity and acceleration at the position of maximum displacement.

(3 cm; 3 Hz; zero; 108 $\pi^2\ cm/s^2$)

42. A particle performs S.H.M. of amplitude 10 cm and period 12s. Find its phase and displacement after 14s if it starts from the extreme position.

(5 cm; $7\pi/3$)

43. A particle of mass 8×10^{-3} kg oscillates under a PE given by $V(x) = 0.08\ x^2$ J where x is in metres. The total energy of the vibrating particle is 8×10^{-4} J. Show that the motion is S.H. What would be the equation of S.H.M. of the particle?

(0.1 sin (4t + α))

44. An airtight piston fits into a cylindrical vessel closed at the one end. In the equilibrium position, the piston comes to rest at a height h above the closed end. The piston is now pressed down a little and released. Neglecting atmospheric pressure and assuming that the changes are isothermal, show that the motion of the piston is S. H. and find the periodic time of its oscillations.

$\left(2\pi\sqrt{\frac{h}{g}}\right)$

[**Comment :** If P is the pressure exerted by the piston in equilibrium position, a slight increase p of pressure will decrease volume to $V - v$. Since change is isothermal, $PV = (P + p)(V - v)$ giving $p = \frac{Pv}{V} = \frac{PAdh}{V}$ neglecting product of small quantities pv. Restoring force on piston is $pA = -\frac{PA^2dh}{V}$]

45. A loaded block of wood floating vertically in water has 20 cm of its length immersed in water. It is depressed a little and then released. Find the period of its oscillations. $g = 980\ cm/s^2$. **$\left(\frac{2\pi}{7} s\right)$**

[**Comment :** In the equilibrium position, the weight of cylindrical tube is balanced by water upthrust. An additional depression x of tube provides an additional upthrust $ax\rho g$. This acts as the restoring force on the tube when the depressing agency is withdrawn.]

46. Assuming the existence of straight frictionless tunnel passing through the centre of the earth, prove that a body dropped into the tunnel will perform S.H.M. and find an expression for its period. Radius of earth 6000 km. $g = 980$ cm/s^2. **(4917 s)**

[Comment : If at any instant when a body of mass m is at a distance x from the earth's centre, a sphere is drawn with x as radius of the sphere and with centre the same as earth's centre. The force of attraction on this sphere is $F = G\frac{(\frac{4}{3}\pi x^3\rho)\,m}{x^2} = \left(\frac{4}{3}\pi G\rho m\right)x$ directed towards the earth's centre. The acceleration is given by $\frac{F}{m} = \frac{4}{3}\pi G\rho x$ in magnitude. This proves motion is SH]

47. A vertical U-tube contains mercury column 60 cm long. The mercury level in one limb is depressed a little and released. Show that the liquid will perform S.H.M. and find the period of its oscillations. $g = 980$ cm/s^2. Density of mercury = 13.6 gm/cc. **(1.1 s)**

[Comment : If the mercury column in the tube of area of cross-section A is depressed on one side through a distance x, the difference of level on the two sides of the tube is $2x$. The excess weight of liquid in a limb of tube is $2xAg\rho$. This forms the restoring force when the external agency is withdrawn.]

48. A small bar magnet is vibrating in a field of induction of which the magnitude only can be changed. Its frequency is found to reduce to half the original frequency when the magnetic induction changes to 1.25×10^{-5} Wb/m^2. Calculate the original magnetic induction. **(5×10^{-5} Wb/m^2)**

49. A bar magnet is vibrating in a field of induction B about transverse axis through its centre. When B is reduced by 3×10^{-5} Wb/m^2, the period of the magnet is doubled. Find B. **(4×10^{-5} Wb/m^2)**

50. A load of 200 g increases the length of a light spring by 10 cm. Find the period of its vertical oscillations when a mass of one kg is attached to the free end of the spring. Take $g = 10$ m/s^2. **(1.4 s)**

51. A vertical light spring is stretched by 2 cm when a weight of 10 g is attached to its free end. The weight is further pulled down by 1cm and released. Compute its frequency and amplitude. **(3.52 Hz; 0.01 m)**

52. A mass of 250 g is suspended from a spring of constant 9N/m. The mass is pulled 10 cm from its equilibrium position and released. Find its speed when it crosses the equilibrium position. **(0.6 m/s)**

53. A mass M attached to a light spring oscillates with a period of 2 s. If the mass is increased by 2 kg, the period increases by 1 s. Find M. **(1.6 kg)**

54. A body of mass 1 kg is made to oscillate on a spring of force constant 16 N/m. Calculate :
   a) Angular frequency
   b) Frequency of vibration **(4 rad/s ; 2/ Hz)**

55. A particle of mass 50 g performs S.H.M. along a line 10 cm long. If its force constant is 12N/m, find its total energy. What are the kinetic and potential energies at $x = 2$cm? What is the maximum velocity of its oscillation? **(1.5×10^{-2} J; 1.26×10^{-2} J; 0.24×10^{-2} J; 0.774 m/s)**

56. For a particle performing S.H.M. of period 4s, find the time in which half its energy will be potential after starting from the centre from rest. **(1/3 s)**

57. At what displacement is the PE of a S.H. oscilator half its total energy? **($a/\sqrt{2}$)**

58. A simple pendulum of length one meter has a bob of mass 15 g. If the amplitude of vibrations of the pendulum is 4 cm, find its PE in the extreme position. **(1.176×10^{-4} J)**

59. When the length of a simple pendulum is increased by 25cm, the period changes by 20%. What was its original length? **(56.82 cm)**

60. A block is resting on a horizontal surface which makes S.H.M. horizontally. The amplitude of oscillations is a. If the coefficience of friction between the block and the surface is μ, show that the block starts to slip when the frequency of oscillation is $\frac{1}{2\pi}\sqrt{\frac{\mu g}{a}}$.

61. The PE of a particle of mass 1 kg oscillating along X-axis is written as $U = 4(1 - \cos 2x)$ in S.I. units. Assuming that the oscillations are small, find the period. ($\pi/2$ s)