

(1) Data :

$$A = 5 \text{ cm}$$

$$T = 2 \text{ s}$$

$$v/a = \frac{a_{\max}}{2} = ?$$

Solution :

$$a_{\max} = \omega^2 A$$

$$\therefore a = \frac{a_{\max}}{2} = \frac{\omega^2 A}{2}$$

$$\therefore \omega^2 x = \frac{\omega^2 A}{2}$$

$$\therefore x = \frac{A}{2}$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$\therefore v = \omega \sqrt{A^2 - \frac{A^2}{4}}$$

$$\therefore v = \frac{2\pi}{2} \sqrt{5^2 - \frac{5^2}{4}}$$

$$v = \pi \sqrt{25 - \frac{25}{4}}$$

$$v = \pi \sqrt{\frac{75}{4}}$$

$$v = \frac{\pi}{2} \times 5\sqrt{3}$$

$$\therefore v = 13.6 \text{ cm/s}$$

(2) Data :

$$T = 12 \text{ s}$$

$$A = 8 \text{ cm}$$

$$t = 0, x = +A$$

$$t = ? \text{ when } x = 2 \text{ cm.}$$

Solution :

$$x = A \cos \omega t$$

$$\therefore 2 = 8 \cos\left(\frac{2\pi}{12} \times t\right)$$

$$\frac{1}{4} = \cos\left(\frac{\pi}{6} \times t\right)$$

$$75^\circ 31' = \frac{180}{6} \times t$$

$$75^\circ 31' = 30^\circ \times t$$

$$\frac{75 + \frac{31}{60}}{30} = t$$

$$2.517 \text{ s} = t$$

(3) Data :

$$x = A/3$$

$$KE/TE = ?$$

$$PE/TE = ?$$

Solution :

$$TE = \frac{1}{2} k A^2$$

$$PE = \frac{1}{2} k x^2$$

$$\therefore \frac{PE}{TE} = \frac{x^2}{A^2} = \frac{(A/3)^2}{A^2} = \frac{1}{9}$$

$$\therefore \frac{KE}{TE} = 1 - \frac{PE}{TE} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$(4) x = 6 \sin\left(\frac{3\pi t + 5\pi}{6}\right)$$

$$A = 6 \text{ m.}$$

$$2\pi f = 3\pi$$

$$\therefore f = \frac{3}{2} = 1.5 \text{ Hz}$$

$$\alpha = \frac{5\pi}{6}$$

Data:

$$(5) T_2 = 1.2 T_1$$

$$l_2 = l_1 + 44 \text{ cm}$$

Solution:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$\therefore T \propto \sqrt{l}$$

$$\therefore \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$$

$$\frac{1.2 T_1}{T_1} = \sqrt{\frac{l_1 + 44}{l_1}}$$

$$1.44 = \frac{l_1 + 44}{l_1}$$

$$1.44 l_1 = l_1 + 44$$

$$1.44 l_1 - l_1 = 44$$

$$0.44 l_1 = 44$$

$$l_1 = \frac{44}{0.44}$$

$$\therefore l_1 = 100 \text{ cm} = 1 \text{ m.}$$

$$\therefore l_2 = l_1 + 44 = 100 + 44 = 144 \text{ cm.}$$

$$T_1 = 2\pi \sqrt{\frac{l_1}{g}}$$

$$= 2 \times 3.14 \sqrt{\frac{1}{9.8}}$$

$$= 2.006 \text{ s}$$

(6) Data:

$$T_1 = 2 \text{ s}$$

$$l_2 = 1.01 \text{ m}$$

Solution

In a day we have
24 hours

$$\therefore 24 \times 3600 \text{ s}$$

$$\text{ie } \frac{24 \times 3600 \text{ osc}}{2}$$

[\because 1 osc is of 2's]

$$\therefore 43200 \text{ osc. in a day}$$

$$\begin{aligned} \therefore T_2 &= 2\pi \sqrt{\frac{L_2}{g}} \\ &= 2 \times 3.14 \sqrt{\frac{1.01}{9.8}} \\ &= 2.016 \end{aligned}$$

$$\begin{aligned} \text{Time lost per osc} &= 2.016 - 2 \\ &= 0.016 \text{ s} \end{aligned}$$

$$\begin{aligned} \therefore \text{Time lost in a day} &= 0.016 \times 43200 \\ &= 691.2 \text{ s} \end{aligned}$$

Note: Taking π as 3.142 gives a more accurate answer

(7) Data :

$$\begin{aligned} m &= 0.5 \text{ kg} \\ k &= 10 \text{ N/m} \\ A &= 3 \times 10^{-2} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{(a) TE} &= \frac{1}{2} k A^2 \\ &= \frac{1}{2} \times 10 \times 9 \times 10^{-4} \\ &= 45 \times 10^{-4} \text{ J} \end{aligned}$$

$$\begin{aligned} \text{(b) } v_{\max} &= \omega A \\ &= \sqrt{\frac{k}{m}} \times A \\ &= \sqrt{\frac{10}{0.5}} \times 3 \times 10^{-2} \\ &= \sqrt{20} \times 3 \times 10^{-2} \\ &= 0.1342 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{(c) } v \Big|_{x=2\text{cm}} &= \omega \sqrt{A^2 - x^2} \\ &= \sqrt{\frac{k}{m}} \sqrt{A^2 - x^2} \\ &= \sqrt{\frac{10}{0.5}} \sqrt{(3 \times 10^{-2})^2 - (2 \times 10^{-2})^2} \\ &= \sqrt{20 \times 10^{-2}} \times \sqrt{9 - 4} \\ &= \sqrt{20 \times 10^{-2}} \times \sqrt{5} \\ &= \sqrt{100} \times 10^{-2} \\ &= 10 \times 10^{-2} \\ &= 0.1 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \text{(d) PE} &= \frac{1}{2} k x^2 \\ &= \frac{1}{2} \times 10 \times (2 \times 10^{-2})^2 \\ &= 0.002 \text{ J} \\ \text{KE} &= \frac{1}{2} k (A^2 - x^2) \\ &= \frac{1}{2} \times 10 \left((3 \times 10^{-2})^2 - (2 \times 10^{-2})^2 \right) \\ &= 0.0025 \text{ J} \end{aligned}$$

(8) Data :

$$L = 0.51 \text{ m}$$

$$\theta = 10^\circ$$

$$T = 1.44 \text{ s}$$

Solution : $\therefore \theta$ is small

$$\therefore T = 2\pi \sqrt{\frac{L}{g}}$$

$$1.44 = 2\pi \sqrt{\frac{0.51}{g}}$$

$$\frac{1.44}{2\pi} = \sqrt{\frac{0.51}{g}}$$

$$g = \frac{0.51 \times 4 \times \pi^2}{1.44^2}$$

$$= 9.7 \text{ m/s}^2$$

(9) Data :

$$A = 10 \text{ cm}$$

$$T = 10 \text{ s}$$

(i) $\pi = 5 \text{ cm}$

$$v = \pm \omega \sqrt{A^2 - \pi^2}$$

$$= \pm \frac{2\pi}{T} \sqrt{A^2 - \pi^2}$$

$$= \pm \frac{2\pi}{10} \sqrt{10^2 - 5^2}$$

$$v = \pm \frac{2 \times 3.14}{10} \sqrt{100 - 25}$$

$$= \pm 0.628 \sqrt{75}$$

$$= \pm 5.44 \text{ cm/s}$$

(ii) $\pi = 5 \text{ cm}$

$$a = -\omega^2 \pi$$

$$= -\left(\frac{2\pi}{T}\right)^2 \pi$$

$$= -\left(\frac{2 \times 3.14}{10}\right)^2 \times 5$$

$$= - (0.628)^2 \times 5$$

$$= -1.972 \text{ cm/s}^2$$

(10) Data :

$$A = \frac{0.12}{2} = 0.06 \text{ m}$$

$$v_{\text{max}} = 0.12 \text{ m/s}$$

$$T = ?$$

$$v/\pi = \sqrt{3} \times 10^{-2} \text{ m} = ?$$

Solution :

$$v_{\text{max}} = \omega A$$

$$0.12 = \frac{2\pi}{T} \times 0.06$$

$$z = \frac{2\pi}{T}$$

$$\therefore T = \pi$$

$$= 3.1425$$

$$v = \omega \sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{T} \sqrt{A^2 - x^2}$$

$$= \frac{2\pi}{\pi} \sqrt{(0.06)^2 - (\sqrt{3} \times 10^{-2})^2}$$

$$= 0.1149 \text{ m/s}$$

(11) Data:

$$T = 8 \text{ s}$$

$$t = ?$$

$$PE = \frac{TE}{2}$$

Solution:

$$PE = \frac{TE}{2}$$

$$\frac{1}{2} k x^2 = \frac{1}{2} \left(\frac{1}{2} k A^2 \right)$$

$$\therefore x = 0.7071 A$$

$$x = A \sin(\omega t)$$

$$0.7071 A = A \sin\left(\frac{2\pi}{8} t\right)$$

$$\frac{1}{\sqrt{2}} = \sin\left(\frac{1}{4} \times 2\pi \times t\right)$$

$$45 = 45 \times t$$

$$\therefore t = \underline{\underline{1 \text{ s}}}$$