ILLUSTRATIVE EXAMPLES

(1) Calculate the radius of the first Bohr orbit from given data and hence find radius of 3rd Bohr orbit.

Data :

$m = 9 \times 10^{-31} kg$,

$e = 1.6 \times 10^{-19} C$,

$h = 6.63 \times 10^{-34} Js$, $\varepsilon_0 = 8.85 \times 10^{-12} C^2 / Nm^2$

Solution :

For first orbit

$n = 1$,

$$r_1 = \left(\frac{h^2 \varepsilon_0}{\pi m e^2}\right)$$

$$r_1 = \frac{8.85 \times 10^{-12} \times (6.63 \times 10^{-34})^2}{3.142 \times 9 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$r_1 = 5.374 \times 10^{-11} m$$

$$r_1 = 0.5374 \overset{o}{A}$$

In general $r_n = r_1 n^2$

$$r_3 = r_1 \times (3)^2$$

$$r_3 = 0.537 \times 9$$

$$r_3 = 4.837 \overset{o}{A}$$

(2) The energy of an excited hydrogen atom is −3.4 eV. Find the angular momentum of the electron.

Data : $E_n = -3.4$ eV

Solution :

$$E_n = \frac{-13.6}{n^2} eV$$

$$-3.4 = \frac{-13.6}{n^2}$$

$$n^2 = 4$$

$$n = 2$$

$$\text{Angular momentum} = \frac{nh}{2\pi}$$

$$= \frac{2 \times 6.63 \times 10^{-34}}{2 \times 3.142} \text{ joule-second}$$

$$= 2.11 \times 10^{-34} \text{ joule-second}$$

(3) Energy of electron in 2nd bohr orbit is −3.4 eV. Find the energy of electron in 3rd bohr orbit and 1st Bohr orbit.

Data : $E_2 = -3.4$ eV, $E_1 = ?$ and $E_3 = ?$

Solution: We have, $E_n \propto \frac{1}{n^2}$

Therefore, $E_2 \propto \frac{1}{2^2}$ and $E_3 \propto \frac{1}{3^2}$

or $$\frac{E_3}{E_2} = \frac{2^2}{3^2} = \frac{4}{9}$$

or $$E_3 = \left(\frac{4}{9}\right) \times E_2$$

$$E_3 = \left(\frac{4}{9}\right) \times (-3.4 \text{ eV})$$

$$E_3 = -1.51 \text{ eV}$$

Similarly, $$E_1 = \left(\frac{4}{1}\right) \times E_2$$

$$E_1 = 4 \times (-3.4 \text{ eV})$$

$$E_1 = -13.6 \text{ eV}$$

(4) Energy of electron in 2nd Bohr orbit is −3.4 eV. Calculate its K.E. and P.E. in the 3rd orbit.

Data : $E_2 = -3.4$ eV

Solution :

We have $$\frac{E_3}{E_2} = \frac{2^2}{3^2} = \frac{4}{9}$$

or $$E_3 = \left(\frac{4}{9}\right) \times E_2$$

$$E_3 = \left(\frac{4}{9}\right) \times (-3.4 \text{ eV})$$

$$\mathbf{E_3 = -1.51 \text{ eV}} \quad \textbf{(total energy)}$$

Since T.E. = P.E. + K.E.

If total energy is (−x), then K.E. = x; and

P.E. = −2x;

i.e. K.E. = − (T.E.)

$= -(-1.51 \text{ eV}) = \mathbf{+1.51 \text{ eV}}$

P.E. = 2 × (T.E.)

P.E. = $2 \times (-1.51 eV) = \mathbf{-3.02 \text{ eV}}$

(5) An electron is orbiting in 4th Bohr orbit. Find the ionization energy for this atom if the ground state energy is −13.6eV.

Data : Given $E_1 = -13.6\text{eV}, E4 = ?$

Solution: We have $\frac{E_4}{E_1} = \frac{1^2}{4^2} = \frac{1}{16}$

The ionisation energy $= E_\infty - E_4$

$= 0 - (-0.85)$

$= \mathbf{+0.85\ eV}$

(6) On the basis of classical electromagnetic theory find the frequency of light emitted by an electron revolving around proton in hydrogen atom.

Data : $r_1 = 0.537\ \text{Å}$

Solution : The speed of electron in 1st orbit is given by –

$$v_1 = \frac{e^2}{2\varepsilon_0 hn}$$

$$= \frac{(1.6\times10^{-19})^2}{2\times8.85\times10^{-12}\times6.63\times10^{-34}\times1}$$

$$v_1 = 2.18\times10^6\ \text{m/s} \quad \text{and}$$

$$f_1 = \frac{v_1}{2\pi r_1}$$

$$f_1 = \frac{2.18\times10^6}{2\times3.142\times0.537\times10^{-10}}$$

$$f_1 = 6.460\times10^{15}\ \text{Hz}$$

(7) The series limit for Lyman series is 912 Å. Find series limit for Paschen and Brackett series.

Solution: Given: Series limit $\lambda_L = 912\ \text{Å}$.

Series limit for Lyman series is given by-

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1^2}{\infty^2}\right)$$

$$\text{or} \quad \lambda_L = \frac{1}{R}$$

Series limit for Paschan series is given by -

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2} - \frac{1^2}{\infty^2}\right)$$

$$\text{or} \quad \lambda_p = \frac{9}{R}$$

Paschen series limit $\lambda_p = \frac{9}{R} = 9\times\lambda_L$

$$\left[\text{Since } \lambda_L = \frac{1}{R}\right]$$

$$\lambda_p = 9\times912\ \text{Å} = 8208\ \text{Å}$$

Similarly, Brackett series limit is -

$$\frac{1}{\lambda_B} = R\left(\frac{1}{4^2} - \frac{1}{\infty^2}\right)$$

Hence, Brackett series limit is -

$$\lambda_B = \frac{16}{R} = 16\times\lambda_L$$

$$\lambda_B = 16\times912\ \text{Å} = \mathbf{14592}\ \text{Å}$$

(8) An electron beam has wavelength of 0.5 Å. Find its velocity and accelerating voltage required to impart that velocity.

Data : $\lambda_e = 0.5\times10^{-10}\ \text{m}$,

$m = 9\times10^{-31}\ \text{kg}$,

$e = 1.6\times10^{-19}\ \text{C}$,

$h = 6.63\times10^{-34}\ \text{Js}$.

Solution:

We have,

$$\lambda = \frac{h}{mv}$$

$$\text{or} \quad v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}}{9\times10^{-31}\times0.5\times10^{-10}}$$

$$v = 1.4733\times10^7\ \text{m/s}.$$

Also, $\frac{1}{2}mv^2 = eV$.

$$\text{or} \quad V = \frac{mv^2}{2e} = \frac{9\times10^{-31}\times(1.473\times10^7)^2}{2\times1.6\times10^{-19}}\ \text{V}$$

$$V = 6.102\times10^2\ \text{volt}$$

$$V = 610.2\ \text{volt}$$

(9) What is de Broglie wavelength of an electron accelerated through 25000 volt ? (neglect relativistic effects).

Data : $V = 25000$ volt

Solution :

We have, $\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å}$

$$\lambda = \frac{12.27}{\sqrt{25000}}$$

$$\lambda = \mathbf{0.07766}\ \mathring{A}$$

(10) Calculate the deBroglie wavelength of electron moving with one fourth of the speed of light. (neglect relativistic effects.)

Data : $v = \left(\frac{1}{4}\right) \times c$

$$= \frac{(3 \times 10^8)}{4} = 0.75 \times 10^8 \text{ m/s}$$

Solution :

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9 \times 10^{-31} \times 0.75 \times 10^8}$$

$$\lambda = 0.9822 \times 10^{-11} \text{ m}$$

$$\lambda = 0.09822\ \mathring{A}$$ **(Ans.)**

PROBLEMS FOR PRACTICE

(1) Find the ratio of diameter of electron in 1st Bohr orbit to that in 4th bohr orbit

(Ans : 1:16)

(2) Calculate the change in angular momentum of electron when it jumps from 3rd orbit to 1st orbit in hydrogen atom.

(Ans : 2.111×10^{-34}Js or kgm^2s^{-1})

(3) Find the longest wavelength in Paschen series. (Given R= $1.097 \times 10^7 m^{-1}$) (Ans : 18752 Å)

(4) Find the frequency of revolution of electron in 2nd Bohr orbit, if the radius and the speed of electron in that orbit is 2.14×10^{-10} m and 1.09×10^6 m/s respectively.

(Ans : 8.11×10^{14}Hz)

(5) Find the value of Rydberg's constant if the energy of electron in second orbit in hydrogen atom is −3.4 eV.

(Ans: $1.095 \times 10^7 m^{-1}$)

(6) Find the shortest wavelength in Paschen series if, the longest wavelength in Balmer series is 6563 Å.

(8204 Å) ~~(Ans : 8.23×10^6 Å)~~

(7) Find the ratio of longest to shortest wavelength in Paschen series.

(Ans : 2.286:1)

(8) The moving electron and a photon has same de Broglie wavelength. Show that the electron possesses more energy than carried by the photon.

(9) A monochromatic wavelength of wavelength λ is incident on an hydrogen atom that lifts it to 3rd orbit from ground level. Find the wavelength and frequency of incident photon.

(Given : $E_3 = -1.51$ eV, $E_1 = -13.6$ eV)

(Ans. : 1028 Å ; 2.9176×10^{15} Hz)

(10) An electron in hydrogen atom stays in 2nd orbit for 10^{-8} s. How many, revolutions will it make till it jumps to the ground state ?

(Ans. : 8.23×10^6)

(11) Find momentum of the electron having de Broglie wavelength of 0.5 Å

(Ans: 1.326×10^{-23} $kgms^{-1}$)

(12) Find the wavelength of a proton accelerated by a potential difference of 50V.

[Given $m_p = 1.673 \times 10^{-27}$ kg]

(Ans: 0.04 Å)

(13) A cracker of mass M at rest explodes in two parts of masses m_1 and m_2 with non-zero velocities. Find the ratio of the de Broglie wavelength of two particles

(Ans: 1)