## PROBLEMS for PRACTICE

### SECTION – I

1. If the earth had its radius suddenly decreased by half when spinning about its axis, what would the length of the day be ? (6 hours)
2. A uniform cylinder of diameter 6 cm and length 6 cm weighs 400 g. Calculate its M.I. about an axis passing through its centre perpendicular to its length and about the axis of the cylinder. $(2.1 \times 10^{-4}\ kg\ m^2;\ 1.8 \times 10^{-4}\ kg\ m^2)$
3. A uniform solid sphere has a radius 0.1 m and density $6 \times 10^3$ kg/m$^3$. Find its moment of inertia about a tangent to its surface. $(0.352\ kg\ m^2)$
4. Assuming the earth to be a sphere, calculate its M.I. about the axis of rotation. Calculate the angular momentum and rotational K.E. of the earth about its axis.
   Mass of earth = $6 \times 10^{24}$ kg; R = 6400 km.
   $(9.83 \times 10^{37}\ kg\ m^2;\ 71.76 \times 10^{32}\ kg\ m^2/s;\ 2.619 \times 10^{29}\ J)$
5. Four point masses $m$, $2m$, $3m$, and $4m$ are kept at the corners A, B, C and D respectively of a square ABCD of side $l$ m. Find the M.I. and radius of gyration of the system about AB as the axis of rotation. $(7m\ l^2;\ \sqrt{7/10}\ l)$
6. A wheel of M.I. 3 kg m$^2$ rotating at 60 rad/s is brought to rest in 5 minutes due to friction. Find the work done and the average frictional torque acting. Also find the angular momentum of the disc 2.5 minutes before the wheel stops rotating. (0.6 Nm; 5400 J; 90 kg m$^2$/s)
7. The M.I. of the earth about its axis of rotation is $9.83 \times 10^{37}$ kgm$^2$ and its angular velocity is $7.3 \times 10^{-5}$ rad/s. Calculate its KE of rotation and radius of gyration. Mass of the earth = $6 \times 10^{24}$ kg. $(2.62 \times 10^{29}\ J;\ 4.05 \times 10^6\ m)$
8. M. I. of a ring about an axis passing through its centre and at right angles to its plane is given by MR$^2$. Find its M. I. about a diameter and also about a tangent (1) perpendicular (2) parallel to the plane of ring. $\left(\frac{1}{2}\ MR^2;\ 2MR^2;\ \frac{3}{2}\ MR^2\right)$
9. M. I. of a disc about an axis passing through its centre and at right angles to its plane is $\frac{1}{2}$ MR$^2$. Find its M. I. about a diameter. What would be its M. I. about an axis tangential to and lying in the plane of disc and perpendicular to the plane of disc?
   $\left(\frac{1}{4}\ MR^2;\ \frac{5}{4}\ MR^2;\ \frac{3}{2}\ MR^2\right)$
10. Calculate the M.I. of a solid sphere of mass 10 kg and radius 0.5 m rotating about an axis parallel to a diameter and passing through a point distant 0.2 m from the centre of the sphere. $(1.4\ kg\ m^2)$
11. Radius of gyration of a body about an axis at a distance of 12 cm from its centre of mass is 13 cm. Find its radius of gyration about a parallel axis through the centre of mass. (5 cm)
12. The M.I. of a uniform circular disc about an axis passing through its centre and perpendicular to its plane is $\frac{1}{2}$ MR$^2$. Find the distance of a parallel axis from the centre of mass about which the M.I. of the disc is MR$^2$. Radius of disc = $\sqrt{8}$ cm. (2 cm)
13. Radius of gyration of a disc about an axis passing through its centre and perpendicular to its plane is 2 cm. Find its radius of gyration about a diameter. $(\sqrt{2}$ cm)
14. A light rod 1.2 m long carries two identical spherical masses 0.2 kg each at its ends. Find the M. I. and the K.E. when the rod rotates about an axis passing through its mid-point and at right angles to its length at 1 rev/s. (0.144 kg m$^2$; 2.84 J)
15. Calculate the M. I. of a solid iron disc of radius 10 cm and thickness 5 cm about an axis passing through its centre and at right angles to its plane. Density of iron = 780 kg/m$^3$.
    $(6.123 \times 10^{-3}\ kg\ m^2)$
16. Calculate the M.I. of a thin uniform rod of mass one kg and length 60 cm about an axis perpendicular to its length and passing through (1) its centre (2) its one end.
    [(1) 0.025 kg m$^2$ ; (2) 0.385 kg m$^2$]

17. A body rotating at 15 rad/s undergoes angular displacement of 40 radians in 5 s. What is the number of revolutions made by it from start to the time it stops? (6.4 rev)

18. A solid cylinder has a height of 12 cm, radius of 4 cm and a mass of 100 g. Find its M.I. about its axis of symmetry and also about an axis passing through its centre and perpendicular to the axis. ($8 \times 10^{-5}$ kgm$^2$ ; $16 \times 10^{-5}$ kg m$^2$)

19. The change in angular velocity of a rotating body from 5 rad/s to 10 rad/s results in a change of angular momentum of 60 kg m$^2$/s. What is the change in KE ? (450 J)

20. A disc of diameter 50 cm and mass 2 kg rotates about an axis passing through its centre and at right angles to its plane with a frequency of 8 rev per sec. Find the angular momentum of the disc and the rotational K.E. (3.14 kg m$^2$/s; 78.88 J)

21. Find the radius of gyration of a uniform circular plate of radius R about an axis perpendicular to its plane and bisecting a radius. ($\sqrt{3}$ R/2)

22. A wheel of M.I. 1 kg m$^2$ velocity at 40 rad/s comes to rest in 10 minutes due to friction when the couple acting on it is withdrawn. Find the work done in overcoming friction and the average frictional torque. What is the angular momentum of wheel two minutes before it stops rotating? ($1.2 \times 10^4$ J; $\frac{1}{15}$ kg m$^2$ ; 8 kg m$^2$/s)

23. The angular velocity of a body changes from 30 rad/s to 60 rad/s. If the change in angular momentum is 100 kg m$^2$/s, find the change in K.E. of rotation of the body. (4500 J)

24. A horizontal disc is freely rotating about a vertical axis passing through its centre at the rate of 120 r.p.m. A blob of wax of mass 20 g falls on the disc and sticks to it at a distance of 5 cm from the axis. If the M.I. of the disc about the given axis is $2 \times 10^{-4}$ kg m$^2$, find the new frequency of rotation of the disc. (96 r.p.m.)

25. A horizontal disc is freely rotating about a vertical transverse axis passing through its centre at the rate of 100 revolutions per minute. A 20 gram blob of wax falls on the disc and sticks to the disc at a distance of 5 cm from its axis. Moment of intertia of the disc about its axis passing through its centre of mass is $2 \times 10^{-4}$ kg m$^2$. Calculate the new frequency of rotation of the disc. (80 r.p.m.)

26. A uniform disc rotating in a horizontal plane about a vertical axis passing through its centre at 20 r.p.m. has a body of mass of 2 kg kept at its centre. If the mass now slides to the edge, what would the angular speed of the disc be? Mass of the disc = 5 kg. Comment on the change in the KE of the disc if any. (10 $\pi$/27 rad/s)

27. A uniform disc of diameter 30 cm is rotating at 300 r.p.m. in a horizontal plane about an axis passing through its centre. The angular velocity of the disc increases to 900 r.p.m. in 10 s. Find (a) angular acceleration assuming it to be uniform (b) number of revolutions made by the disc in this interval (c) final linear velocity of a point on the rim of the disc. (2 $\pi$ rad/s$^2$; 100; 9 $\pi$ m/s)

28. A disc of M.I. 3 kg m$^2$ rotating about an axis passing through its centre and perpendicular to its plane at a speed of 10 rad/s, comes to rest in 5 minutes. Calculate
(1) the frictional couple acting on the disc (2) work done against friction in coming to rest and (3) angular momentum of the disc two minutes before coming to rest. (0.1 Nm; 150 J; 12 kg m$^2$/s)

29. A person is standing at the centre of a turntable rotating at 60 rev/min with his arms outstretched. If he brings his hands close to his body reducing thereby his M.I. to $\frac{3}{5}$ of its original value, what would be his new speed? Neglect friction. (36 r.p.m.)

30. A car running at 72 km/h is brought to rest by the application of brakes which produce constant retardation. The radius of the wheel is 50 cm and the wheel makes 40 rotations before coming to rest after brakes are applied. Find the angular retardation caused. What is the distance covered by the car before it comes to rest? (10/$\pi$ rad; 40 $\pi$ m)

31. The frequency of revolution of an object changes from 2 to 4 Hz in 2 s. What is its angular acceleration? (2 $\pi$ rad/s$^2$)

32. The frequency of a spinning top is 10 Hz. If it is brought to rest in 6.28 s, what is the angular acceleration of a surface particle? (– 10 rad/s$^2$)

33. Due to the action of the brakes producing constant retardation, a car running at 144 km/h is brought to rest in 20 s. What angular retardation is caused in the wheels if the radius of the wheels is 50 cm? (4 rad/s$^2$)

34. A torque of 10 Nm is applied to a flywheel of mass 10 kg and radius of gyration 0.5 m. What is the resultant angular acceleration? (4 rad/s$^2$)

35. To increase the speed of a wheel from 90 r.p.m. to 270 r.p.m. an expenditure of 900 J is required. What is the M.I. of the wheel? (2.54 kg m$^2$)

36. A ceiling fan (M.I. = 40 kg m$^2$) rotates about its axis at a speed of 120 r.p.m. using a power of 125.6 W. How many revolutions will the fan complete before coming to rest after electric power is cut off? (50.24)

37. A ceiling fan (M.I. = 40 kg m$^2$) rotates about its axis at constant speed of 240 r.p.m. using electric motor of 120 W. Find the number of complete revolutions it makes before coming to rest when the power is cut off. (5)

38. The angular momentum of a body changes by 80 kg m$^2$/s when its angular velocity changes from 20 rad/s to 40 rad/s. Find the change in its K.E. of rotation. (2400 J)

39. The angular momentum of a rotating body changes by 120 kg m$^2$/s when its angular velocity changes from 25 rad/s to 45 rad/s. What is the change in the rotational KE of the body? (4200 J)

40. The angular velocity of a disc rotating in its plane changes from 2 rad/s to 10 rad/s in one minute when a constant torque of 2 Nm is applied. What is the M. I. of the disc ? (15 kg m$^2$)

41. The speed of rotation of the body increases from 60 rpm to 90 rpm in 1 minute. Calculate the torque acting on the body, if its M.I. is 500 kg-m$^2$. (26.17 Nm)

42. At the instant when a disc is rotating about an axis passing through its centre at right angles to its plane at 120 r.p.m., the torque acting on the disc is withdrawn. The disc comes to rest in 5 s. Find the torque that was acting on the disc and the work done by the disc before coming to rest. M.I. of the disc = 2.5 kg m$^2$. (2 $\pi$ Nm; 20 $\pi^2$ J)

43. An automobile engine develops 100 hp when rotating at a speed of 1800 r.p.m. What torque does it deliver? (396 Nm)

44. To maintain a rotor at a uniform angular speed of 200 rad/s, an engine needs to transmit a torque of 180 Nm. What engine power is required? (36 kW)

45. A body starts rotating from rest. Due to a couple of 20 Nm it completes 60 revolutions in one minute. Find the moment of inertia of the body. (10/$\pi$ kg – m$^2$)

46. A flywheel rotating with a constant angular velocity of 50 radians per second is brought to rest after completing 100 revolutions by a steady retarding couple of 9.8 Nm. Find the retardation caused and the M. I. of the flywheel. (1.99 rad/s$^2$; 4.925 kg m$^2$)

47. A flywheel of mass 20 kg rotating about an axis passing through its centre and at right angles to its plane loses 4500 J of energy when slowing down from 120 r.p.m. to 60 r.p.m. Find its M. I. and the radius of gyration. What is the change in the angular momentum of the flywheel? (76.07 kg m$^2$; 1.95 m; 477.7 kg m$^2$/s)

48. Two wheels of moments of inertia 4 kg m$^2$ and 2 kg m$^2$ rotate at the rate of 120 rev/min and 240 rev/min respectively and in the same direction. If the two are coupled so as to rotate with a common angular velocity, find the speed of revolution. (160 r.p.m.)

49. Two wheels P and Q rotate side by side on the same axis. Wheel P of M.I. 0.05 kgm$^2$ is set spinning at 600 r.p.m. Wheel Q of M.I. 0.2 kg m$^2$ is initially stationary. A clutch now acts to join P and Q so that they spin together. Find their common speed of rotation. Compare the rotational KE of the system before and after. (120 r.p.m. ; 5)

50. Wheels A and B each of M.I. 5 kg m$^2$ rotate independently side by side about a common axis. Wheel A rotates with a velocity 150 r.p.m. and wheel B rotates at 300 r.p.m. in opposite direction. A clutch acts now to join A and B so that they spin together. What is their common speed of rotation? (75 r.p.m.)

51. A uniform disc rotating about an axis passing through its centre at right angles to its plane loses 50 J of energy when slowing down from 60 r.p.m. to 45 r.p.m. Find its M.I. about its axis of rotation and the change of angular momentum during the period. (5.79 kg m$^2$ ; 9.1 kg m$^2$/s)

52. Calculate the M.I. of the sphere, rotating about an axis passing through a point at a distance 0.1 m from the centre of the sphere. The sphere has a mass of 5 kg and radius 0.2 m.

**(0.13 kg m²)**

53. A thin rod has a length $l$ and mass $m$ per unit length. If the rod rotates about an axis passing through its mid-point and perpendicular to its length with an angular velocity ω, what would be its KE?

**$\left(\frac{1}{24} m\omega^2 l^3\right)$**

54. Calculate the M.I. of a uniform cylinder of mass 100 g and length 8 cm about its axis of symmetry and also about an axis passing through its centre and perpendicular to its length. Take radius of cylinder as 10 cm. **($5 \times 10^{-4}$ kg m² ; $30.33 \times 10^{-5}$ kg m²)**

55. A uniform solid cylinder of radius 2 cm has a mass of 50 g and length of 12 cm. Find its M.I. about an axis passing through its centre and perpendicular to its length. **($1.95 \times 10^{-4}$ kg m²)**

56. The M.I. of a solid cylinder about its geometric axis is the same as its M.I. about an axis passing through its centre of mass and perpendicular to its axis. Find the ratio $l$/R. **($\sqrt{3}$)**

57. A ring of radius 25 cm and mass 10 kg rolls on a horizontal surface at the rate of 100 cm/s. What is the K.E. of the ring in joule? **(10 J)**

58. A solid sphere of mass 1 kg rolls on the horizontal surface of a table with linear speed 2 m/s. find its total kinetic energy. **(2.8 J)**

59. A cylinder is kept on an inclined plane. The cylinder (1) slides without rotating. (2) rolls without slipping. In which case will it take less time to reach the bottom of the plane?

**(1st case)**

60. A sphere rolls without slipping down a rough inclined plane. Find the ratio of its rotational and translational K.Es. **(2 : 5)**

61. A ring and a disc are started rolling and a block with flat bottom is started sliding down an inclined plane from a common height. Which is the order in which they reach the bottom? Write the expression for the velocity at the instant each has fallen through a height $h$.

**(Block, Disc, Ring)**

62. A ring and a disc of equal radius roll down an inclined plane from the same starting point. Compare their velocities when they reach the bottom. **($\sqrt{3} : 2$)**

63. A hoop of radius 30 cm and mass 500 g is thrown up an inclined plane of 30° inclination with an initial velocity of 70 cm/s. Find the distance to which it can go up. **(10 cm)**

64. Compare the time taken by a sphere and a disc, both of the same diameter, to roll from rest down an inclined plane of angle α. **($\sqrt{14/15}$)**