ROTATIONAL

(1) Data :

$M = 10\ kg$

$R = 0.2\ M$

$\tau = 10\ Nm$

$\omega\ /_{t=5s} = ?$

$\omega_0 = 0\ rad/s$

Solution

$\tau = I\alpha$

$\tau = \frac{MR^2}{2} \cdot \alpha$

$10 = \frac{10 \times (0.2)^2}{2} \times \alpha$

$\therefore \alpha = 50\ rad/s^2$

$\omega = \omega_0 + \alpha \cdot t$

$= 0 + 50 \cdot 6$

$= 300\ rad/s$

(2) Data :

$R = 12.5 \times 10^{-2}\ M$

$M = 25\ kg$

$I = ?$

$\omega_0 = 2\ rad/s$

$\omega = 12\ rad/s$

$t = 5s$

$\tau = ?$

Solution

$I = \frac{2}{5} MR^2$

$= \frac{2}{5} (25)(12.5 \times 10^{-2})^2$

$= 0.1562\ kgm^2$

$\tau = I \cdot \alpha \quad \cdots (i)$

$\omega = \omega_0 + \alpha \cdot t$

$12 = 2 + \alpha \cdot 5$

$\therefore 12 - 2 = 5\alpha$

$10 = 5\alpha$

$\therefore \alpha = 2\ rad/s^2$

Subst. in (i)

$\tau = 0.1562 \times 2$

$= 0.3124\ Nm$

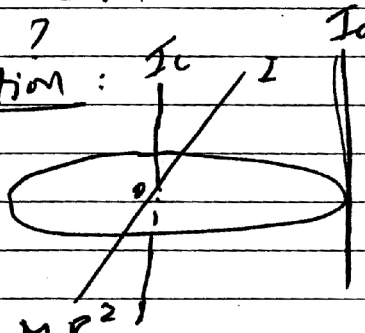
(3) Data :

$M = 0.5\ kg$

$R = 0.5\ M$

$I = ?$

Solution :



$I_c = MR^2$

$= 0.5 \times (0.5)^2$

$I = \frac{I_c}{2} = \frac{0.5 \times 0.25}{2}$

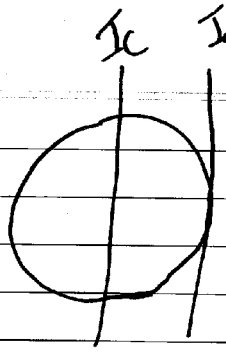
$= 6.25 \times 10^{-2}\ kgm^2$

$I_o = I_c + M(R^2)$

$= 0.5 \times 0.5^2 + 0.5(0.5^2)$

$= 0.25\ kgm^2$

(4)



$I_O = I_C + MR^2$

$= \frac{2}{5}MR^2 + MR^2$

$= \frac{7}{5}MR^2$

$\therefore MK^2 = \frac{7}{5}MR^2$

$\therefore K = \sqrt{\frac{7}{5}}\,R$

(5) Data :

$n_1 = \frac{90}{60}$ Hz

$\therefore \omega_1 = 2\pi n_1 = 2\pi\left(\frac{90}{60}\right)$ rad/s

$I_2 = 0.75\, I_1$

Solution :

$I_1 \omega_1 = I_2 \omega_2$

$I_1\left(2\pi \times \frac{90}{60}\right) = 0.75\, I_1 (2\pi n_2)$

$\therefore n_2 = \frac{90}{60 \times 0.75}$

$= 2$ Hz

$= 2 \times 60$ r.p.m

$= 120$ rpm

(6) Data :

$\tau = 400$ Nm

$m = 40$ kg

$\alpha = 20$ rad/s^2

$I = ?$

$K = ?$

Solution :

$\tau = I \cdot \alpha$

$400 = I \cdot 20$

$\therefore I = 20$ kg m^2

But $I = MK^2$

$\therefore 20 = 40 \times K^2$

$\frac{1}{2} = K^2$

$\therefore K = \frac{1}{\sqrt{2}}\,m$

$= 0.707\, m$

(7) Data :

$R_2 = 2R_1$

$M_2 = M_1$

$\frac{I_1}{I_2} = ?$

Solution :

$\frac{I_1}{I_2} = \frac{\frac{2}{5}MR_1^2}{\frac{2}{5}MR_2^2}$

$= \frac{R_1^2}{R_2^2}$

$= \frac{1}{4}$

(8) Data:

$E = W = 100\ J$

$\omega_0 = 2\pi\left(\frac{60}{60}\right)$

$= 2\pi\ rad/s$

$\omega_1 = 2\pi\left(\frac{30}{60}\right) = \pi\ rad/s$

Solution:

$E = \frac{1}{2} I(\omega_1^2 - \omega_2^2)$

$100 = \frac{1}{2} I((2\pi)^2 - \pi^2)$

$100 = \frac{1}{2} I(4\pi^2 - \pi^2)$

$200 = I(3\pi^2)$

$\therefore I = \frac{200}{3\pi^2}$

$= 6.753\ kgm^2$

$\Delta L = I(\Delta\omega)$

$= 6.753(2\pi - \pi)$

$= 21.21\ kgm^2/s$

(9) Data:

$I_1 = 4\ kgm^2$

$I_2 = 4\ kgm^2$

$\omega_1 = 2\pi\left(\frac{120}{60}\right) = 4\pi\ rad/s$

$\omega_2 = 2\pi\left(\frac{240}{60}\right) = 8\pi\ rad/s$

Solution:

$|I_1\omega_1 - I_2\omega_2| = I\omega$

$|4 \times 4\pi - 4 \times 8\pi| = 8 \times \omega$

$4 \times 4\pi = 8 \times 2\pi n$

$\therefore n = 1\ Hz$

$\therefore n = 60\ rpm$

(10) Data:

$\omega_1 = 2\pi\ \frac{180}{60} = 6\pi\ rad/s$

$r_2 = 25 \times 10^{-2}\ m$

$\omega_2 = 2\pi\left(\frac{120}{60}\right) = 4\pi\ rad/s$

$m_2 = 1.9 \times 10^{-3}\ kg$

Solution:

$I_1\omega_1 = I_1\omega_2 + I_2\omega_2$

$I_1(6\pi) = I_1(4\pi) + (1.9 \times 10^{-3})(25 \times 10^{-2})^2 \times 4\pi$

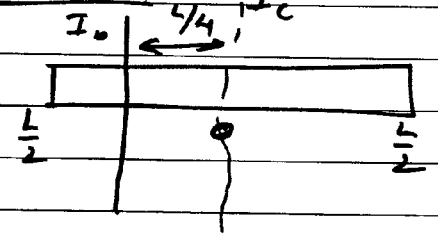
$\therefore I_1 = 2.375 \times 10^{-4}\ kgm^2$

(11) Data:

$L = 1m$

$M = 1kg$

Solution:



$I_o = I_c + M\left(\frac{L}{4}\right)^2$

$= \frac{ML^2}{12} + \frac{ML^2}{16}$

$= \frac{1 \times 1^2}{12} + \frac{1 \times 1^2}{16}$

$= 0.1458\ kg\,m^2$

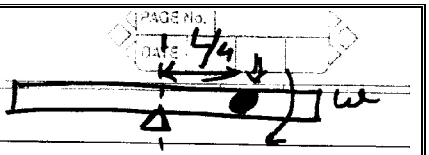
$I_o = Mk^2$

$0.1458 = 1 \times k^2$

$\therefore k = \sqrt{0.1458}$

$= 0.3818m$

(12)



$I_1\omega_1 + I_2\omega_1 = I_1\omega_2 + I_2\omega_2$

$0 + MVR = \frac{ML^2}{12}.\omega + M\left(\frac{L}{4}\right)^2\omega$

$MV\left(\frac{L}{4}\right) = \omega\left[\frac{ML^2}{12} + \frac{ML^2}{16}\right]$

$\frac{V}{4} = \omega\left[\frac{4+3}{48}\right] \cdot L$

$\frac{V}{4} = \omega\left(\frac{7}{48}\right) \cdot L$

$\therefore \omega = \frac{48}{4} \cdot \frac{V}{7L}$

$\therefore \omega = \frac{12V}{7L}$