STATIONARY WAVES

(1) **Data** :

$l = 0.5\,m$

$M = 5\,kg$

$n = 100\,Hz$

Solution:

$T = mg = 5 \times 9.8$
$= 49\,N$

$n = \frac{1}{2l}\sqrt{\frac{T}{M}}$

$100 = \frac{1}{2 \times 0.5}\sqrt{\frac{49}{M}}$

$100 \times 2 \times 0.5 = \sqrt{\frac{49}{m}}$

$100^2 = \frac{49}{m}$

$m = \frac{49}{100^2} = 0.0049\,kg/m$

(2) **Data**:

$l_1 = 100\,cm = 1\,m$

$l_2 = 90\,cm = 0.9\,m$

$|n_2 - n_1| = 8$

Solution :

$\because l_2 < l_1$

$\& \; n \propto \frac{1}{l}$

$\therefore n_2 > n_1$

$\therefore n_2 - n_1 = 8$ —(1)

$\frac{n_2}{n_1} = \frac{\frac{1}{2l_2}\sqrt{\frac{T}{m}}}{\frac{1}{2l_1}\sqrt{\frac{T}{m}}}$

$\frac{n_2}{n_1} = \frac{l_1}{l_2}$

$\frac{n_2}{n_1} = \frac{1}{0.9}$

$\therefore n_1 = 0.9\,n_2$ —(2)

Subst in (1)

$n_2 - 0.9 n_2 = 8$

$0.1 n_2 = 8$

$n_2 = 80\,Hz$

Subst in (2)

$\therefore n_1 = 0.9\,n_2$
$= 0.9 \times 80$
$= 72\,Hz$

$\because$ Fork was in resonance with l_1 $\therefore N = n_1 = 72\,Hz$

(3) **Data** :

$f_1 = f_2$

$r_1 = r_2$

$\frac{n}{n'} = \frac{1}{2}$

$\frac{T}{T'} = \frac{1}{8}$

$\frac{l}{l'} = ?$

Solution :

$$m = \frac{M}{l} = \frac{V\rho}{l} = \frac{Al\rho}{l}$$

$\because r_1 = r_2$ & $\rho_1 = \rho_2$

$\therefore A_1 = A_2$ & $\rho_1 = \rho_2$

$\therefore m_1 = m_2$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1}{T_2}}\sqrt{\frac{m_2}{m_1}}$$

$$\frac{1}{2} = \frac{l_2}{l_1}\sqrt{\frac{1}{8}}\sqrt{\frac{1}{1}}$$

$$\therefore \frac{l_1}{l_2} = \frac{2}{2\sqrt{2}}$$

$$\frac{l_1}{l_2} = \frac{1}{\sqrt{2}}$$

$\therefore l_1 : l_2 = 0.7072 : 1$

(4) Data :

$l = 1m$

$M = \frac{2}{1000}\ kg$

$n = 300\ Hz$

$T = ?$

Solution :

$$m = \frac{M}{l} = \frac{0.002}{1}$$

$= 0.002\ kg/m$

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$300 = \frac{1}{2\times 1}\sqrt{\frac{T}{0.002}}$$

$$600^2 = \frac{T}{0.002}$$

$600^2 \times 0.002 = T$

$\therefore T = 720\ N$

(5) ~~Data~~

$n = N$

$l' = l + \frac{4}{100}l = 1.04l$

$|n - n'| = 6$

Solution:

$\because n \propto \frac{1}{l}$

& $l' > l$

$\therefore n' < n$

$\therefore n - n' = 6$ —(1)

$$\frac{n}{n'} = \frac{\frac{1}{2l}\sqrt{\frac{T}{m}}}{\frac{1}{2l'}\sqrt{\frac{T}{m}}}$$

$$\frac{n}{n'} = \frac{l'}{l} = \frac{1.04l}{l} = 1.04$$

$n = 1.04n'$ — (2)

Subst in (1)

$n - n' = 6$

$1.04n' - n' = 6$

$0.04n' = 6$

$n' = \frac{600}{4}$

$n' = 150\,Hz$

Subst. in (2)

$\therefore n = 1.04n'$

$= 1.04 \times 150$

$= 156\,Hz$

$\therefore N = n = 156\,Hz$

(6) Data:

$T = 1\,kg \times 9.8$

$= 9.8\,N$

$v = 68\,m/s$

$\rho = 7900\,kg/m^3$

$A = ?$

Solution:

$v = 68$

$\therefore \sqrt{\frac{T}{m}} = 68$

$\therefore \sqrt{\frac{9.8}{m}} = 68$

$m = \frac{9.8}{68^2}$

$\frac{M}{L} = \frac{9.8}{68^2}$

$\frac{V\rho}{L} = \frac{9.8}{68^2}$

$\frac{A\cancel{L}\rho}{\cancel{L}} = \frac{9.8}{68^2}$

$A\rho = \frac{9.8}{68^2}$

$A = \frac{9.8}{68^2 \times 7900}$

$= 0.2683 \times 10^{-6}\,m^2$

$= 0.2683\,mm^2$

(7) Data:

$l = 0.7\,m$

$v = \sqrt{\frac{T}{m}} = ?$

$n_3 = 300\,Hz$

Soln:

$n_3 = 3\,\frac{1}{2l}\sqrt{\frac{T}{m}}$

$\therefore v = \sqrt{\frac{T}{m}} = \frac{2n_3 l}{3}$

$= \frac{2 \times 300 \times 0.7}{3}$

$= 140\,m/s$

(8) Data :

$\frac{T}{l^2} = \frac{360 \times 10^{-5}}{10^{-4}}$

$= 36 \; N/m^2$

$n = 300 \, Hz$

$m = ?$

Solution

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

$n^2 = \frac{1}{4l^2}\frac{T}{m}$

$m = \frac{1}{4}\frac{T}{l^2} \times \frac{1}{n^2}$

$= \frac{1}{4} \times 36 \times \frac{1}{90000}$

$= 10^{-4} \; kg/m$

(9) Data :

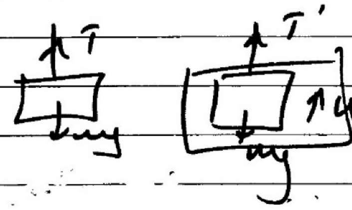
$n = N = 250 \, Hz$

$|n' - n| = 10$

$\rho = ?$

$\rho_{H_2O} = 1000 \; kg/m^3$

Solution :



$\because T' < T$

$\therefore n' < n$

$\therefore n - n' = 10$

$250 - n' = 10$

$\therefore n' = 240 \, Hz$

$\frac{n'}{n} = \frac{\frac{1}{2l}\sqrt{\frac{T'}{m}}}{\frac{1}{2l}\sqrt{\frac{T}{m}}}$

$\therefore \frac{n'}{n} = \sqrt{\frac{T'}{T}}$

$\frac{240}{250} = \sqrt{\frac{mg - U}{mg}}$

$\frac{240}{250} = \sqrt{\frac{V_s\rho_s g - V_s\rho_L g}{V_s\rho_s g}}$

$\frac{240}{250} = \sqrt{\frac{V_s\rho_s - V_s\rho_L}{V_s\rho_s}}$

$\frac{240}{250} = \sqrt{\frac{\rho_s - \rho_L}{\rho_s}}$

$$\frac{24^2}{25^2} = \frac{\rho_s - 1}{\rho_s}$$

$$576\rho_s = 625\rho_s - 625$$

$$625 = 49\rho_s$$

$$\rho_s = \frac{625}{49}$$

$$\rho_s = 12.76\, g/cm^3$$

(10) (i) $A = 2a$
$= 2 \times 2$
$= 4\, cm$

(ii) dist $= \lambda/4$

Comparing with

$$y = A \sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$$

we get $\lambda = 60\, cm$

$\therefore$ dist $= \frac{\lambda}{4} = \frac{60}{4} = 15\, cm$

(iii) loop length $= \frac{\lambda}{2}$
$= \frac{60}{2} = 30\, cm$

(iv) Wave vel
$= n\lambda$

$= (100\ \ ) \times \left(\frac{60}{100}\right)$

$= 60\, m/s$

(11) $y = 0.02 \cos(\pi x) \sin(100\pi t)$

Compare with

$$y = 2a \cos(2\pi x/\lambda) \sin(2\pi n t)$$

$2a = 0.02$

$\therefore \boxed{a = 0.01\, m}$

$\frac{2\pi x}{\lambda} = \pi x$

$\therefore \boxed{\lambda = 2\, m}$

$2\pi n t = 100\pi t$

$\therefore \boxed{n = 50\, Hz}$

& $T = \frac{1}{n} = \frac{1}{50}$

$\therefore \boxed{T = 0.02\, s}$

$v = n\lambda = 50 \times 2$

$\boxed{v = 100\, m/s}$

(12) Data :

$p_1 = 4$; $p_2 = 2$

$T_1 = 1960$ dyne

$m_{pan} = 1g$

Solution :

$Tp^2 = $ const

$T_1 p_1^2 = T_2 p_2^2$

$1960 \times 4^2 = T_2 \times 2^2$

$\therefore T_2 = \frac{1960 \times 16}{4}$

$T_2 = 7840$ dyne

$\therefore T_2 = \frac{7840}{980} g$

$= 8g \cdot wt$

pan = 1g

$\therefore$ extra weights to be added $= 8 - 1$

$= 7g$ wt.

(13) Data :

parallel position : $p = 6$

$l = 7.2m$

$T = \frac{10}{1000} \times 9.8$

$= 0.098N$

$M = 14.4 \times 10^{-2} g$

$n = ?$

Solution :

$m = \frac{M}{L} = \frac{14.4 \times 10^{-2} \times 10^{-3}}{7.2}$

$= 2 \times 10^{-5}$ kg/m

$\because$ parallel position

$\therefore N = 2n$

$\therefore N = 2 \times \frac{p}{2l}\sqrt{\frac{T}{m}}$

$= \frac{6}{7.2}\sqrt{\frac{0.098}{2 \times 10^{-5}}}$

$= 58.33 Hz.$

(14) Data.

open at one end

$n_{11} = ?$

$L = \frac{42.10}{100} = 0.4210m$

$V = 350$ m/s

$d = 3.5 \times 10^{-2} m$

Solution

$e = 0.3d$

$= 0.3 \times 3.5 \times 10^{-2}$

$= 1.05 \times 10^{-2} m$

$n_{11} = 11\left(\frac{V}{4(L+e)}\right)$

$= \frac{11 \times 350}{4(0.4210 + 0.0105)}$

$= 2231 Hz$

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(15) Data:

open both ends

$n - n' = 5$

$L = 0.25m$

$L' = ?$

$v = 350 m/s$

$e = 0.015m$

Solution.

$n - n' = 5$

$\frac{v}{2L} - \frac{v}{2L'} = 5$

$\frac{v}{2(L+2e)} - \frac{v}{2(L'+2e)} = 5$

$\frac{350}{2(0.25+0.03)} - \frac{350}{2(L'+0.03)} = 5$

$\therefore L' = 0.2523m$

(16) Data:

$n_{1c} = n_{4o}$

Solution

$n_1^c = n_4^o$

$\frac{v}{4L} = \frac{4 \cdot v}{2L'}$

$\frac{L}{L'} = \frac{1}{8}$

$L : L' = 1 : 8$

(17) $n_1 = \frac{v}{2(L_1+2e)}$

$n_2 = \frac{v}{2(L_2+2e)}$

$\therefore \frac{n_1}{n_2} = \frac{L_2+2e}{L_1+2e}$

$\therefore n_1L_1 + 2en_1 = L_2n_2 + 2en_2$

$\therefore 2e(n_2 - n_1) = n_1L_1 - n_2L_2$

$\therefore e = \frac{n_1L_1 - n_2L_2}{2(n_2 - n_1)}$

(18) Data:

$L = 10cm$

$L' = 32.2cm$

Solution:

$L' - L = \frac{\lambda}{2}$

$\therefore \lambda = 2(32.2 - 10)$

$= 2 \times 22.2$

$= 44.4 cm$

$e = \frac{32.2 - 3 \times 10}{2}$

$= \frac{2.2}{2}$

$= 1.1 cm.$