Differentiate between elastic and plastic body

Elastic body	**Plastic body**
It offers resistance to deformation	It offers no resistance to deformation
It regains its original size & shape when deforming forces are removed	It does not regain its original size & shape on the removal of deforming forces
Hooke's law can be applied	Hooke's law cannot be applied
e.g. Iron, steel, copper	e.g. Mud, clay

Definition:

Stress: Stress is defined as the internal restoring force per unit area

$$\text{Stress} = \frac{\text{Internal restoring force}}{\text{Area}}$$

Since internal force cannot be measured and in equilibrium condition the magnitude of internal force is equal to that of the deforming force, therefore, we can write

$$\text{Stress} = \frac{\text{Applied Force}}{\text{Area}}$$

Units: CGS : dyne cm^{-2} SI : N m^{-2} Dimensions: $[M^1L^{-1}T^{-2}]$

NOTE: Stress is not a vector quantity. Unlike a force, it does not hav particular direction. **Stress is a tensor quantity**.

Strain: Strain is defined as change in dimension of a body produced per unit original value of that dimension.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original Value of that dimension}}$$

It is Unitless and dimensionless

Elastic limit: The maximum stress to which a body can be subjected without permanent deformation is called Elastic Limit.

Tensile Stress or Longitudinal Stress: It is defined as the internal elastic force acting normally per unit area of cross section of the wire.

$$\text{Tensile Stress} = \frac{F}{A}$$

Tensile Strain or Longitudinal Strain: It is defined as the change in length produced per unit original length of the wire

$$\text{Tensile Strain} = \frac{\text{Change in Length}}{\text{Original Length}} = \frac{\Delta L}{L}$$

Volume Stress: It is defined as compressing force per unit area of the body

$$\text{Volume Stress} = \frac{\text{Compressing force}}{\text{Area of the body}} = \frac{F}{A}$$

Volume Strain: It is defined as the change in volume produced per unit original volume of the body

$$\text{Volume Strain} = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{dV}{V}$$

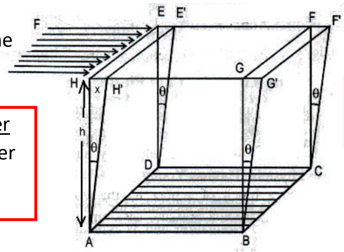
Shearing Stress: It is defined as deforming tangential force per unit area of the surface on which it is applied

$$\text{Shear Stress} = \frac{\text{tangential force applied}}{\text{Area of the surface}} = \frac{F}{A}$$

Shearing Strain: It is defined as the ratio of the lateral displacement of any layer to the perpendicular distance from the fixed layer

$$\text{Shear Strain} = \frac{\text{lateral displacement of any layer}}{\text{Its distance from the fixed layer}} = \frac{x}{h} = \tan\theta \approx \theta$$

where θ is small and measured in radians.



Hooke's Law: Within elastic limit, the stress is directly proportional to the strain

Stress α Strain

Stress = Constant x Strain

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant (called modulus of elasticity of the body)}$$

NOTE: Modulus of elasticity is a property of the material of the body

Within elastic limit, the ratio of the stress to the strain produced is called the Modulus of Elasticity of the body.

Units: dyne cm^{-2} (CGS) and N m^{-2} (SI) Dimensions: $[M^1L^{-1}T^{-2}]$

Young's Modulus: The ratio of tensile stress to the tensile strain produced within the elastic limit is called Young's modulus of the body.

NOTE: It is a property of solids ONLY.

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

where F : Force applied , L : Original Length, A : Area of cross section of wire, ΔL : extension

Units: dyne cm^{-2} (CGS) and N m^{-2} (SI) Dimensions: $[M^1L^{-1}T^{-2}]$

Bulk Modulus: The ratio of the volume stress to the volume strain produced within the elastic limits is called Bulk Modulus of the body

NOTE: It is a property of solids, liquids and gases

$$B \text{ or } K = \frac{\text{Volume Stress}}{\text{Volume Strain}} = -\frac{dP}{dV/V} = -\frac{dP\,V}{dV}$$

[The –ve sign is used to make K positive since dP & dV will be of opp. sign]

Units: dyne cm^{-2} (CGS) and N m^{-2} (SI) Dimensions: $[M^1L^{-1}T^{-2}]$.

Shear Modulus (Modulus of Rigidity): The ratio of shearing stress to the shearing strain within elastic limit is called shear modulus of elasticity or Modulus of rigidity

NOTE: It is a property of solids ONLY.

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{F/A}{x/h} = \frac{Fh}{Ax} = \frac{F}{A\theta} \quad (\theta \text{ is in radians})$$

Units: dyne cm^{-2} (CGS) and N m^{-2} (SI) Dimensions: $[M^1L^{-1}T^{-2}]$.

Poisson's Ratio: Consider a wire of length L suspended from a support. Weights are attached to one end using a hanger. The wire extends by ΔL. Simultaneously the wire suffers a contraction in diameter. Let d be the original diameter and Δd be the contraction suffered.

$$\text{Lateral strain} = \frac{\Delta d}{d} \qquad \text{Longitudinal Strain} = \frac{\Delta L}{L}$$

It is experimentally observed that within the elastic limit the ratio of lateral strain to longitudinal strain remains constant for a given material and is called Poisson ratio.

$$\text{Poisson Ratio} = \sigma = -\frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\Delta d/d}{\Delta L/L} = -\frac{\Delta d.L}{\Delta L.d}$$

[-ve sign is used to make σ positive as Δd and ΔL are opposite sign]

Units: No units Dimensions: No Dimensions

NOTE: $Y = 3K(1-2\sigma)$ $\quad Y = 2\eta(1+\sigma)$ $\quad \frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$

Thermal Stress

Consider a metal rod of length L_o at θ_1°C and cross sectional area A, and is heated through a temperature of Δθ then its length increases by

$\Delta L = L_t - L_o = L_o.\alpha.\Delta\theta$

where Lt : Final length, α: coefficient of linear expansion of the rod

Thus, tensile strain = $\Delta L/L = \alpha\Delta\theta$

Thus the stress produced = Y . strain = $Y.\alpha.\Delta\theta$

The force developed = $Y.A.\alpha.\Delta\theta$

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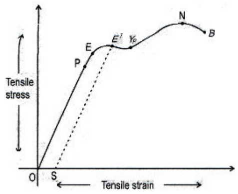
Behavior of a wire:

Consider a long uniform wire fixed to a rigid support. On the other end increasing loads are added for stretching the wire. For each load stress and strain is calculated and a graph of stress v/s strain is plotted.



Initial part of the graph OE is a straight line, which shows that stress is proportional to strain (Hooke's law). Stress at point E is the elastic limit of the material of the wire. If the applied load is completely removed, the wire completely regains its original length.

If the stress is further increased then the graph is no longer a straight line(Hooke's law not obeyed). With a small increase in stress the stain increases faster. If the load is removed, the wire is unable to return to its original length. However the wire yet has elastic properties. A permanent strain OS called set is introduced in the wire. But the graph is again is a straight line from S.

If the load is increased further, a point Y_p is reached, at which the tangent to the curve becomes parallel to the strain axis. This means that wire begins to increase without increase in the load (stress). The wire is said to 'flow' (plastic flow). Point Y_p is called yield point and the corresponding stress is called Yield stress.

Now the wire begins to flow and the cross section starts decreasing resulting in a neck formation. Point N represents the maximum stress which the wire cam withstand (called breaking stress). Even if the stress is now decreased, the strain increases and finally wire breaks (point B).

Work done in stretching a wire / Strain Energy / Strain energy per unit Volume:

Consider a wire of length L and radius r, which is suspended from a rigid support. Let the free end of wire be loaded by a weight F = Mg. Let L + l be the final length of the wire. Then,

Longitudinal stress = F/A Longitudinal Strain = l/L

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{l/L}$$

Thus, $F = \frac{YAl}{L}$

The force increased from 0 to F during the elongation from 0 to l

Let f be force and x be the corresponding elongation at certain instant.

Then, $f = \frac{YAx}{L}$

Let dW be the work done for a small extension of dx

$dW = f \cdot dx = \frac{YAx}{L} dx$

Total work done in stretching it from 0 to l can be obtained by integration

$$W = \int_0^l dW = \int_0^l \left(\frac{YAx}{L}\right) dx = \frac{YA}{L}\int_0^l x dx = \frac{YA}{L}\left[\frac{x^2}{2}\right]_0^l = \frac{YA}{L}\left[\frac{l^2}{2} - \frac{0}{2}\right]$$

$$W = \frac{YAl^2}{2L} = \frac{1}{2}\frac{YAl}{L} l = \frac{1}{2}Fl = \frac{1}{2}(load)(extension)$$

$$Thus, strain\ energy = \frac{1}{2}Fl = \frac{1}{2}(load)(extension)$$

Strain energy is defined as an elastic potential energy gained by a wire during elongation by stretching force.

Work done per unit volume = $\frac{\text{work done in stretching a wire}}{\text{volume of wire}}$

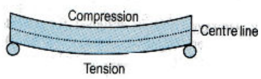
$= \frac{1}{2}\frac{Fl}{V} = \frac{1}{2}\frac{Fl}{AL} = \frac{1}{2}\frac{F}{A}\frac{l}{L} = \frac{1}{2}$(stress)(strain)

Strain energy per unit volume = $\frac{1}{2}$(stress)(strain) = $\frac{1}{2}$ Y (strain)^2 = $\frac{1}{2}\frac{(stress)^2}{Y}$

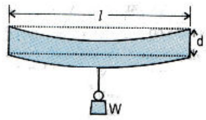
Sagging/Buckling and I – beam:

In construction of buildings, use of beams and columns is very common

Consider a steel bar of length L, breadth b and depth d loaded at centre by a load W. Then the sag (δ) of the CG of beam is given by



$$\delta = \frac{WL^3}{4Ybd^3}$$



Sag δ is, inversely proportional to d^3
inversely proportional to b
directly proportional at L^3
inversely proportional to Y

Thus bending can be reduced by using a material of higher Y, smaller L, larger b and d.

The I beam minimizes both weight and stress. An I beam is much lighter and prevents buckling. A hollow circular pole or tube is also resistant to bending.