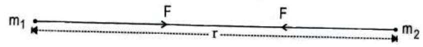
GRAVITATION

Newton's Law of Gravitation

Statement: Every particle of matter attracts every other particle of matter with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them



$F \propto m_1m_2$ and $F \propto \frac{1}{r^2}$, Therefore, $F \propto \frac{m_1m_2}{r^2}$ Thus, $F = \frac{Gm_1m_2}{r^2}$

G= Universal Gravitational Constant = 6.673×10^{-11} Nm^2/kg^2

SI Unit: Nm^2/Kg^2 CGS Unit: dyne cm^2/g^2 Dimensions: $[M^{-1}L^3T^{-2}]$

The Gravitational Force (F) acts along the line joining the two particle, its attractive in nature, Equal and Opposite, **Independent of the medium in between.**

Variation of g due to altitude:

M : mass of the earth; R : Radius of the earth; m : mass on the surface

Then , Weight of object = Gravitational Force

$mg = \frac{GMm}{R^2}$; Therefore, $g = \frac{GM}{R^2}$

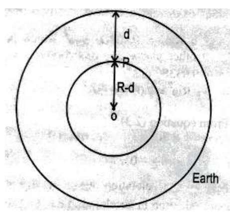
At a height h above the surface of earth, $mg_h = \frac{GMm}{(R+h)^2}$; Thus $g_h = \frac{GM}{(R+h)^2}$

NOTE: both g and g_h are independent of mass of object

Dividing the two we get, $\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$, Thus $g_h = \frac{gR^2}{(R+h)^2}$

If h<<R, then $g_h = g\left[1 - \frac{2h}{R}\right]$

Variation of g due to depth:



M : mass of the earth; R : Radius of the earth

On the surface of the Earth, $g = \frac{GM}{R^2}$

Mass = Volume x Density = $\frac{4}{3}\pi R^3 \times \rho$

Thus, $g = \frac{G \times 4/3\, \pi R^3 \times \rho}{R^2} = \frac{4\pi R\rho G}{3}$ .....(i)

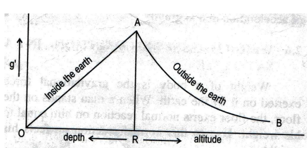
Consider a point P at a depth d below the surface of the earth and let g_d be its acceleration due to gravity. (R – d) is its distance from the center of the earth. Thus, $g_d = \frac{GM'}{(R-d)^2}$, where M' = mass of the inner solid sphere of radius (R – d)

M'= volume x density = $\frac{4}{3}\pi(R-d)^3 \rho$

Thus, $g_d = \frac{G \times 4/3\, \pi(R-d)^3\rho}{(R-d)^2} = \frac{4}{3}\pi(R-d)\rho G$........(ii)

Dividing (i) and (ii) we get, $\frac{g_d}{g} = \frac{R-d}{R}$

Thus, acceleration due to gravity decreases with depth.



NOTE: At the centre of the earth, (d = R) , $g_d = 0$

Variation of g due to latitude :

Earth rotates on its polar axis from west to east with constant ω. So everyone is moving in a circle. Consider a point object P on the surface of earth.

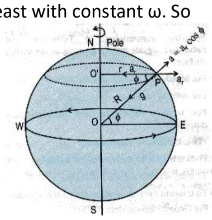
Φ = latitude angle = ∠EOP=∠OPO'

R = radius of earth

PO' = r = radius of circular path = RcosΦ

a_r = centripetal acceleration=$r\omega^2$ = $R\cos\Phi\,\omega^2$

Thus, centrifugal acceleration=$R\omega^2\cos\Phi$



Radial component of centrifugal acceleration= a = $a_r\cos\Phi = R\omega^2\cos^2\Phi$

The effective acceleration g' due to gravity at P directed to the centre of earth = g' = g – a = $g - R\omega^2\cos^2\Phi$

NOTE: as latitude increases (equator to pole), Φ increases, cosΦ decreases, g' increases. g' is minimum at equator & maximum at the poles.

Case 1: At equator, Φ=0, cosΦ = 1, g' = g – Rω²

$\omega = \frac{2\pi}{24 \times 60 \times 60} = 7.275 \times 10^{-5}$ rad/s. Thus, $g - g' = R\omega^2 = 0.03386 m/s^2$

Case 2: At poles, Φ=90°, cosΦ = 0, g' = g

Projection of a Satellite :

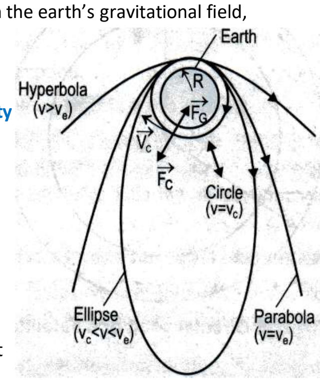
For projection of satellite **minimum two stage** rocket is used. The satellite is kept at the tip of the rocket. The fuel of the first stage raises the rocket to a desired height above the surface of the earth. Now by remote control, the empty first stage is detached and the rocket is rotated by 90° so that it becomes horizontal. Then the fuel of the second stage is burnt. Then, the empty second stage gets detached. The resulting motion of the satellite depends on this velocity of projection given to it.

Definition :

The minimum horizontal velocity of projection that must be given to a satellite at a certain height, so that it can revolve in a circular orbit round the earth is called **critical velocity or orbital velocity**.

The minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the earth's gravitational field, is called the **escape velocity**.

Possible cases of projection of satellite :



>>If **projection velocity (V) < Critical velocity** satellite moves in elliptical orbit. Apogee being the point of projection and perigee lying at 180°. If it enters the atmosphere while coming towards perigee it will loose energy and spiral down. If it does not enter the atmosphere, it will continue in an elliptical orbit.

>>If **projection velocity is equal to critical velocity**, the satellite moves in circular orbit

>>If **projection velocity is greater than the critical velocity**, the satellite move in an elliptical orbit. Apogee will be greater than projection height.

>>If **projection velocity is equal to escape velocity**, the satellite moves in parabolic path

>> If **projection velocity is more than escape velocity**, then orbit will be hyperbolic and will escape the gravitational pull of the earth.

Critical Velocity of a satellite :

The minimum horizontal velocity of projection that must be given to a satellite at a certain height, so that it can revolve in a circular orbit round the earth is called **critical velocity or orbital velocity**.

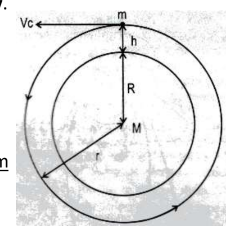
Let m : mass of the orbiting satellite

h : height of the revolving satellite above the surface of the earth

M : Mass of earth, R: radius of earth,

Vc : critical velocity

Centripetal Force = Gravitational Force, $\frac{mV_c^2}{r} = \frac{GMm}{r^2}$



$V_c = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}}$ **Critical velocity is independent of mass of satellite**

But, $g_h = \frac{GM}{(R+h)^2}$, Thus, $GM = g_h(R+h)^2$ substitute in above equation gives

$V_c = \sqrt{\frac{g_h(R+h)^2}{R+h}} = \sqrt{g_h(R+h)}$, g_h : acceleration due to gravity at ht h.

If satellite revolves close to the surface of the earth the h→0,

$V_c = \sqrt{\frac{GM}{R}}$, But, g = GM/R². Thus, GM = gR² . Thus $V_c = \sqrt{(gR^2/R)} = \sqrt{gR}$

g : acceleration due to gravity at the earth's surface

Time Period of a Satellite :

The time taken by the satellite to complete one revolution round the earth is called its period or periodic time or time period.

Let a satellite of mass m be revolving around the earth, of mass M and radius R, at a height h above the surface of the earth. Then,

$V_c = \frac{\text{circumference}}{\text{time period}} = \frac{2\pi r}{T}$

$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$. Squaring we get, $\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$. Thus, $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

Therefore, $T^2 \propto r^3$.

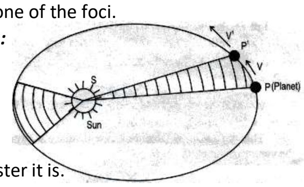
Thus, square of the period of revolution of the satellite is directly proportional to the cube of the radius of its orbit.

$T = 2\pi\sqrt{\frac{r^3}{GM}}$. **Time period is independent of the mass of satellite**

Kepler's Laws :

Kepler's first law (Law of orbits): Every planet revolves in an elliptical orbit around the sun, with the sun situated at one of the foci.

Kepler's second law (Law of equal areas):



The radius vector drawn from the sun to any planet sweeps out equal areas in equal intervals of time. The areal velocity of the radius vector is constant.

NOTE: Closer the planet to the sun the faster it is.

Kepler's third law (Law of period): The square of the period of revolution of the planet around the sun is directly proportional to the cube of the semi – major axis of the elliptical orbit.

Binding Energy :

The minimum amount of energy required for a satellite to escape from the earth's gravitational influence is called its binding energy.

Binding Energy of a revolving satellite

m : mass of satellite, r = R+h : radius of orbit of satellite,

h: ht above earth's surface, M : Mass of earth, R : radius of earth

Vc : critical velocity of satellite

K.E. = $\frac{1}{2} mV_c^2 = \frac{1}{2} m.\frac{GM}{r} = \frac{GMm}{2r}$ …………..(i)

P.E. = mass x Gravitational Potential = m x $\left(-\frac{GM}{r}\right) = -\frac{GMm}{r}$ …..(ii)

T.E. = K.E. + P.E. = $\frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r}$ (– ve sign indicates satellite is bound to earth)

Binding energy of the satellite = |Total Energy| = $\frac{GMm}{2r}$

Binding Energy of a satellite at rest on the surface of the earth :

KE = 0, PE = mass x potential = m $\left(-\frac{GM}{R}\right) = -\frac{GMm}{R}$

T.E. = KE + PE = $-\frac{GMm}{R}$ (–ve sign indicates the satellite is bound to the surface of the earth)

B.E. = |T.E.| = $\frac{GMm}{R}$.

Escape Velocity :

The minimum velocity with which a body should be projected from the surface of the earth, so that it escapes from the earth's gravitational field, is called the **escape velocity** of the body.

Let m be the mass of the satellite steady on the earth's surface. Its binding energy is given by

B.E. = $\frac{GMm}{R}$

To escape the satellite from the earth, binding energy is to be provided in the form of kinetic energy.

Kinetic energy of projection = binding energy

$\frac{1}{2} mV_e^2 = \frac{GMm}{R}$. Thus, $V_e^2 = \frac{2GM}{R}$

Therefore, $V_e = \sqrt{\frac{2GM}{R}}$ **NOTE: It is independent of the mas of satellite**

Alternately, $GM = gR^2$ Substitute in the above equation

$V_e = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$

Weightlessness in a Satellite :

An astronaut of mass m in a satellite is moving with a constant speed along the orbit. Both, the satellite and the astronaut are attracted to the centre of the earth with an acceleration same as the acceleration due to gravity at that place. The astronaut is unable to exert weight on floor of the satellite, in turn the satellite does not provide normal reaction on the astronaut. Therefore he feels weightlessness.

If 'a' is the centripetal acceleration, then mg – N = ma, where g is the acceleration due to gravity at that place. Since a=g, therefore N = 0.

Hence he feels weightlessness. Both the astronaut and satellite are in a state of free fall towards the earth.

Communication or Geostationary Satellites :

An artificial satellite revolving in a circular orbit round the earth in the equatorial plane, in the same sense of rotation of the earth and having same period of revolution as the period of rotation of the earth (1day = 24 hours = 86400 s) is called geo-stationary or geo-synchronous satellite. The relative velocity of the satellite with respect to the earth is zero, it appears stationary from the earth's surface. It is mainly used for communication purpose so it is also called communication satellite.

Uses of satellites

>> Transmission of television and radio waves over large areas of the earth
>> Broadcasting telecommunication
>> Military purpose
>> Weather forecasting and meteorological purpose
>> Astronomical observations
>> Study of solar and cosmic radiations
>> Relay distress signals from ships
>> Transmit cyclone warnings
>> GPS (Geopositions system)